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Mean Time to Recruitment for a Multi Grade Manpower System with Single Threshold, Single Source of Depletion When Wastages form a Geometric Process and Inter Decision Time Forms Ordinary Renewal Process, Order Statistics

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Abstract: ***In this paper, for a marketing organization consisting of multi grades subject to the depletion of manpower (wastages) due to policy decisions with high or low attrition rate, is considered. An important system characteristic namely the mean time to recruitment is obtained for a suitable policy of recruitment when (i) wastages are independent and identically distributed Geometric process and (ii) threshold for each grade has single components with exponential distribution, and (iii) the inter-policy decisions form an ordinary renewal process or order statistics is considered.***

Keywords: Geometric process, Two types of policy decisions with high or low attrition rate, Hyper exponential, Mean time to recruitment

1. Introduction

Exits of personal which is in other words known as wastage, is an importantaspects in the study of manpower planning.Many models have been discussed using different types of wastages and also different types of distribution for the loss of man powers, the thresholds and inter decision times.Such models are seen in [1] and [2].In [3],[4],[5] and [6] theauthors have obtained the mean time to recruitment in a two grade manpower system based on order statistics by assuming different distribution for thresholds.In [8] for a two grade manpower system with two types of decisions when the wastages form a geometric process is obtained.The problem of time to recruitment is studied by several authors for the organizations consisting of single grade/two grade/ three grades .More specifically for a two grade system, in all the earlier work, the threshold for the organization is minimum or maximum or sum of the thresholds for the loss of manpower in each grades, no attempt has been made so far to design a comprehensive recruitment policy for a system with two or three grades. In [10],[11]&[12] a new design for a comprehensive univariate CUM recruitment policy of manpower system is used with n grades in order to bring results proved independently for maximum, minimum model as a special case.In all previous work , the problem of time to recruitment is studied for only an organization consisting of atmost three grades.In[11],[12] author has worked on this comprehensive univariate policy when wastages form ordinary renewal process and interdecision time form geometric and order statistics. In this paper an organization with n-grades is considered and the mean time to recruitment are obtained using an appropriate univariate CUM policy of recruitment (i.e)**"The organization survives iffatleast$r$, $(1 \le r \le n)$out of n-grades survives in the sense that threshold crossing has not take place in these grades",**when wastages form geometric process.

2. Model Description and Assumptions

1) An organization having two grades in which decisions are taken at random epochs in $(0,\infty)$ and at every decision making epoch a random number of persons quit the organization. There is an associated loss of man hours to the organization if a person quits.
2) It is assumed that the loss of man hours is linear and cumulative.
3) The loss of manpower at any decision epoch forms a sequence of independent and identically distributed random variables which form geometric process.
4) The inter-decision times are independent and identically distributed random variables.
5) The loss of manpower process and the process of inter-decision times are statistically independent.
6) The thresholds for the n-grades are independent and identically distributed exponential random variable.
7) **Univariate CUM policy of recruitment: "The organization survives iffatleast$r$, $(1 \le r \le n)$out of n-grades survives in the sense that threshold crossing has not take place in these grades"**

3. Notations

X_i:the continuous random variable denoting the amount of depletion caused to the organization due to the exit of persons corresponding to the j^th^decision,j=1,2,3… and X_i's form a geometric process.

$G(x)$: distribution function ofX_1 such that $G(x) = 1 - ce^{-cx}$ $g(x)$: probability density function.

$G_k(.)$: The distribution function of $\sum_{i=1}^{k} X_i$

$g_k(.)$its probability density function.

U_i: $i = 1,2,3$ ...the inter decision time between $(i-1)^{th}$ and i^{th}decision.

$F(.)[f(.)]$:distribution (density) function.
$F_k(.), f_k(.)$: The distribution(density) function of $\sum_{i=1}^{k} U_i$.
$V_k(t)$: The probability that there are exactly k decision making epoch in $(0, t]$.
Y_j:The continuous random variable denoting the thresholds for the j[th]grade.
Y: The continuous random variable denoting the thresholds for the organization.
$H(.)$:the distribution function of Y.
T_j: Time taken for threshold crossing in the j^{th} grade,j=1,2,3…n.
T :Time to recruitment of the organization
$E(T)$:mean time to recruitment.

4. Main Result

The survival function of the time to recruitment is given by

$$P(T > t) = \sum_{k=0}^{\infty} P(Exactly k decision epoch (0,t] and the threshold level Y is not crossed by the total loss of manhours in these k decisions in at least r grades)$$

$$i.e P(T > t) = \sum_{k=0}^{\infty} V_k(t) P(\sum_{i=1}^{k} X_i < Y) \quad (1)$$

By the law of total probability

$$P(\sum_{i=}^{k} X_i < Y) = \int_0^{\infty} P[Y > \sum_{i=1}^{k} x_i \Big/ \sum_{i=1}^{k} x_i = x]\, g_k(x) dx \quad (2)$$

$$= \int_0^{\infty} g_k(x)\,[1 - H(x)] dx.$$

$$= \int_0^{\infty} g_k(x) \sum_{i=r}^{n} nC_i\, [1 - H(x)]^i [H(x)]^{n-i} dx.$$

$$= \int_0^{\infty} g_k(x) \sum_{i=r}^{n} nC_i\, [e^{-\theta x}]^i [1 - e^{-\theta x}]^{n-i} dx.$$

$$= \sum_{i=r}^{n} nC_i \int_0^{\infty} g_k(x) e^{-i\theta x}\, [1 - e^{-\theta x}]^{n-i} dx$$

Using binomial expansion

$$= \sum_{i=r}^{n} nC_i \int_0^{\infty} g_k(x) e^{-i\theta x}\, [1 - (n-i)C_1 e^{-\theta x} + (n-i)C_2 e^{-(i+2)\theta x} + \; ... + (-1)^{n-i} e^{-n\theta x}] dx.$$

$$= \sum_{i=r}^{n} nC_i \int_0^{\infty} g_k(x)\, [e^{-i\theta x} - (n-i)C_1 e^{-(i+1)\theta x} + (n-i)C_2 e^{-(i+2)\theta x} + \; ... + (-1)^{n-i} e^{-n\theta x}] dx.$$

$$= \sum_{i=r}^{n} nC_i [\overline{g_k}(i\theta) - (n-i)C_1 \overline{g_k}((i+1)\theta) + (n-i)C_2 \overline{g_k}((i+2)\theta) + \; ... (-1)^{n-i} \overline{g_k}(n\theta)] \quad (3)$$

From renewal theory $V_k(t) = F_k(t) - F_{k+1}(t)$ with

$$F_0(t) = 1 \quad (4)$$

Substituting (3) and (4) in (1) we get,

$$P(T > t) = \sum_{k=0}^{\infty} [F_k(t) - F_{k+1}(t)] \sum_{i=r}^{n} nC_i [\overline{g_k}(i\theta) - (n-i)C_1 \overline{g_k}((i+1)\theta) + (n-i)C_2 \overline{g_k}((i+2)\theta) + \; ... (-1)^{n-i} \overline{g_k}(n\theta)] \quad (5)$$

Now , we evaluate $\overline{g_k}(i\theta)$.

As $X_i, i = 1,2,3 \ldots$ form a geometric process with rate 'a',

$$\overline{g_k}(i\theta) = \prod_{j=1}^{k} \bar{g}\left(\frac{i\theta}{a^{i-1}}\right) \quad (6)$$

Since $g(x) = ce^{-cx}$

$$\overline{g_k}(i\theta) = \prod_{j=1}^{k} \int_0^{\infty} e^{-\left(\frac{i\theta}{a^{i-1}}\right)x} ce^{-cx} dx$$

$$= \prod_{j=1}^{k} \int_0^{\infty} ce^{-\left(c + \frac{i\theta}{a^{i-1}}\right)x} dx$$

$$= \prod_{j=1}^{K} \left[\frac{ca^{j-1}}{ca^{j-1} + i\theta}\right] = V(i\theta, k) \text{ where } V(i\theta, k)$$

$$= \prod_{j=i}^{k} \left[\frac{ca^{j-1}}{ca^{j-1} + i\theta}\right] \quad (7)$$

$$P(T > t) = \sum_{k=0}^{\infty} [F_k(t) - F_{k+1}(t)] \sum_{i=r}^{n} nC_i [V(i\theta, k) - (n-i)C_1 V((i+1)\theta, k) + (n-i)C_2 V((i+2)\theta, k) + \; ... (-1)^{n-i} V(n\theta, k)] \quad (8)$$

$$L(t) = 1 - P(T > t) = 1 - \sum_{k=0}^{\infty} [F_k(t) - F_{k+1}(t)] \times \sum_{i=r}^{n} nC_i [V(i\theta, k) - (n-i)C_1 V((i+1)\theta, k) + (n-i)C_2 V((i+2)\theta, k) + \; ... (-1)^{n-i} V((n\theta, k)] \quad (9)$$

$Diff.$ with to respect to t

$$l(t) = -\sum_{k=0}^{\infty} [f_k(t) - f_{k+1}(t)] \sum_{i=r}^{n} nC_i [V(i\theta, k) - (n-i)C_1 V((i+1)\theta, k) + (n-i)C_2 V((i+2)\theta, k) + \; ... (-1)^{n-i} V(n\theta, k)] \quad (10)$$

Taking Laplace Transform on both sides

$$\bar{l}(s) = -\sum_{k=0}^{\infty}[\bar{f}_k(s)-\bar{f}_{k+1}(s)]\sum_{i=r}^{n} nc_i[V(i\theta,k) - (n-i)C_1V((i+1)\theta,k)+(n-i)C_2V((i+2)\theta,k) + \ldots(-1)^{n-i}V(n\theta,k)] \quad (11)$$

Case(i)

$\{U_i\}_{i=1}^{\infty}$ form an ordinary renewal process.

The inter decision times are assumed to be independent and identically distributed hyper exponential random variable with probability density function $f(t) = pe^{-\lambda_h t} + qe^{-\lambda_l t}, p+q = 1$. Where λ_h, λ_l are high and low attrition rate ,p,q are the proportion of decisions having high and low attrition.

Since $U_i^{'}s$ are i.i.d random variable,

$$\bar{l}(s) = -\sum_{k=0}^{\infty}\left[[\bar{f}(s)]^k-[\bar{f}(s)]^{k+1}\right]\sum_{i=r}^{n} nC_i[V(i\theta,k) - (n-i)C_1V((i+1)\theta,k)+(n-i)C_2V((i+2)\theta,k) + \ldots(-1)^{n-i}V(n\theta,k)] \quad (12)$$

$$\frac{d}{ds}\left(\bar{l}(s)\right) = -\sum_{k=0}^{\infty}\left[k[\bar{f}(s)]^{k-1}-(K+1)[\bar{f}(s)]^{k}\right]\frac{d}{ds}\left(\bar{f}(s)\right) \times\sum_{i=r}^{n} nC_i[V(i\theta,k)-(n-i)C_1V\left((i+1)\theta,k\right) + (n-i)C_2V\left((i+2)\theta,k\right) + \ldots(-1)^{n-i}V(n\theta,k)] \quad (13)$$

$$Since \left(\bar{f}(s)\right)_{s=0} = 1$$

$$\frac{d}{ds}\left(\bar{l}(s)\right)_{s=0} = \sum_{k=0}^{\infty}\left[\frac{d}{ds}\left(\bar{f}(s)\right)\right]_{s=0}\sum_{i=r}^{n} nC_i[V(i\theta,k) - (n-i)C_1V((i+1)\theta,k)+(n-i)C_2V((i+2)\theta,k) + \ldots(-1)^{n-i}V(n\theta,k)] \quad (14)$$

The mean time to recruitment is

$$E(T) = -\left[\frac{d}{ds}\left(\bar{l}(s)\right)\right]_{s=0} = -\sum_{k=0}^{\infty}\left[\frac{d}{ds}\left(\bar{f}(s)\right)\right]_{s=0}\sum_{i=r}^{n} nC_i[V(i\theta,k) - (n-i)C_1V((i+1)\theta,k) + (n-i)C_2V((i+2)\theta,k) + \ldots(-1)^{n-i}V(n\theta,k)] \quad (15)$$

$$E(T) = \sum_{k=0}^{\infty}\left(\frac{p\lambda_h+q\lambda_l}{\lambda_h\lambda_l}\right)\sum_{i=r}^{n} nC_i[V(i\theta,k) - (n-i)C_1V((i+1)\theta,k)+(n-i)C_2V((i+2)\theta,k) + \ldots(-1)^{n-i}V(n\theta,k)] \quad (16)$$

Case(ii)

Consider the population $\{U_i\}_{i=1}^{\infty}$ of independent and identically distributed interdecision times with hyper exponential cumulative distribution $F(t) = 1 - pe^{-\lambda_h t} - qe^{-\lambda_l t}$ and the corresponding density function f(t).Assume that $\{U_i\}_{i=1}^{m}$ be a sample of size m selected from this population .Let $U_1, U_2, \ldots, U_m$ be the order statistics corresponding to this sample with respective probability density function $f_{U_1}, f_{U_2}, \ldots\ldots f_{U_m}$. U_1 is the first order statistics and U_m is the mth order statistics such that $U_1 \le U_2 \le \cdots\ldots \le U_m$ and hence not independent.

The probability density function of jth order statistics is given by[Sheldon M.Ross2005]

$$f_{U_j}(t) = j\binom{m}{j}[F(t)]^{j-i} f(t)[1-F(t)]^{m-j}, j = 1,2,3\ldots m$$

Therefore the probability density function of U_1 and U_m are given by

$$f_{U_1}(t) = m\,f(t)[1-F(t)]^{m-1}$$
$$f_{U_m}(t) = m\,f(t)[F(t)]^{m-1}$$

SubCase(i)

$$f(t) = f_{U_1}(t)$$

$$\bar{f}(s) = \bar{f}_{U_1}(s) = \int_0^{\infty} e^{-st}\, mf(t)[1-F(t)]^{m-1}dt$$

$$= \int_0^{\infty} e^{-st}\,[-[d(1-F(t))]^m]$$

$$\bar{f}(s) = \int_0^{\infty} e^{-st}\,[-\,(d(pe^{-\lambda_h t}+qe^{-\lambda_l t})^m]$$

By using binomial expansion

$$\bar{f}(s) = \int_0^{\infty} e^{-st}\,[-d(\sum_{r_1=0}^{m}\binom{m}{r_1}p^{r_1}q^{m-r_1}e^{-(\lambda_h r_1-\lambda_l r_1+\lambda_l m)t})]$$

$$= \sum_{r_1=0}^{m}\binom{m}{r_1}p^{r_1}q^{m-r_1}\int_0^{\infty} e^{-st}\,(\lambda_h r_1-\lambda_l r_1+\lambda_l m) \times e^{-(\lambda_h r_1-\lambda_l r_1+\lambda_l m)t}dt$$

$$= \sum_{r_1=0}^{m}\binom{m}{r_1}p^{r_1}q^{m-r_1}(\lambda_h r_1-\lambda_l r_1+\lambda_l m) \times\int_0^{\infty} e^{-(s+\lambda_h r_1-\lambda_l r_1+\lambda_l m)t}dt$$

$$= \sum_{r_1=0}^{m}\binom{m}{r_1}p^{r_1}q^{m-r_1}(\lambda_h r_1-\lambda_l r_1+\lambda_l m) \times\left[\frac{e^{-(s+\lambda_h r_1-\lambda_l r_1+\lambda_l m)t}}{-(s+\lambda_h r_1-\lambda_l r_1+\lambda_l m)}\right]_0^{\infty}$$

$$= \sum_{r_1=0}^{m}\binom{m}{r_1}p^{r_1}q^{m-r_1}(\lambda_h r_1-\lambda_l r_{1+}+\lambda_l m) \times\left[\frac{1}{(s+\lambda_h r_1-\lambda_l r_1+\lambda_l m)}\right]$$

$$\frac{d}{ds}\left(\bar{f}(s)\right) = \sum_{r_1=0}^{m}\binom{m}{r_1}p^{r_1}q^{m-r_1}(\lambda_h r_1-\lambda_l r_1+\lambda_l m) \times\left[\frac{-1}{(s+\lambda_h r_1-\lambda_l r_1+\lambda_l m)^2}\right]$$

$$\left[\frac{d}{ds}\left(\bar{f}(s)\right)\right]_{s=0} = -\sum_{r_1=0}^{m}\binom{m}{r_1}p^{r_1}q^{m-r_1}\left[\frac{1}{(\lambda_h r_1-\lambda_l r_1+\lambda_l m)}\right] \quad (17)$$

Substituting (17) in (15) we get

$$E(T) = \sum_{k=0}^{\infty} \sum_{r_1=0}^{m} \binom{m}{r_1} p^{r_1} q^{m-r_1} \left[\frac{1}{(\lambda_h r_1 - \lambda_l r_1 + \lambda_l m)}\right] \times \sum_{i=r}^{n} nC_i [V(i\theta,k) - (n-i)C_1 V((i+1)\theta,k) + (n-i)C_2 V((i+2)\theta,k) + \ldots (-1)^{n-i} V(n\theta,k)] \quad (18)$$

Subcase(ii)

$$f(t) = f_{U_m}(t) = mf(t)[F(t)]^{m-1}$$

$$\bar{f}(s) = \bar{f}_{U_m}(s) = \int_0^{\infty} e^{-st}\, mf(t)[F(t)]^{m-1} dt$$

$$= \int_0^{\infty} e^{-st}\, d(F(t))^m = \int_0^{\infty} e^{-st} \left[d(1 - pe^{-\lambda_h t} - qe^{-\lambda_l t})\right]^m$$

$$= \int_0^{\infty} e^{-st}\, d(\sum_{r_1=0}^{m} \binom{m}{r_1} (-1)^{m-r_1} 1^{r_1} (pe^{-\lambda_h t} + qe^{-\lambda_l t})^{m-r_1}$$

$$= \int_0^{\infty} e^{-st}\, d(\sum_{r_1=0}^{m} \sum_{r_2=0}^{m-r_1} (-1)^{m-r_1} \frac{m!}{r_1!\, r_2!\, (m-r_1-r_2)!} \times p^{r_2} q^{m-r_1-r_2} e^{-(\lambda_h r_2 - \lambda_l r_2 - \lambda_l r_1 + \lambda_l m)t})$$

$$= \int_0^{\infty} e^{-st} \sum_{r_1=0}^{m} \sum_{r_2=0}^{m-r_1} \frac{(-1)^{m-r_1} m!\, p^{r_2} q^{m-r_1-r_2}}{r_1!\, r_2!\, (m-r_1-r_2)!} (\lambda_h r_2 - \lambda_l r_2 - \lambda_l r_1 + \lambda_l m)\; e^{-(\lambda_h r_2 - \lambda_l r_2 - \lambda_l r_1 + \lambda_l m)t} dt$$

$$= \sum_{r_1=0}^{m} \sum_{r_2=0}^{m-r_1} (-1)^{m-r_1+1} \frac{m!}{r_1!\, r_2!\, (m-r_1-r_2)!} p^{r_2} q^{m-r_1-r_2} \times (\lambda_h r_2 - \lambda_l r_2 - \lambda_l r_1 + \lambda_l m) \times \int_0^{\infty} e^{-(s+\lambda_h r_2 - \lambda_l r_2 - \lambda_l r_1 + \lambda_l m)t}\, dt$$

$$\bar{f}(s) = \sum_{r_1=0}^{m} \sum_{r_2=0}^{m-r_1} (-1)^{m-r_1+1} \frac{m!}{r_1!\, r_2!\, (m-r_1-r_2)!} p^{r_2} q^{m-r_1-r_2} \times \frac{(\lambda_h r_2 - \lambda_l r_2 - \lambda_l r_1 + \lambda_l m)}{(s + \lambda_h r_2 - \lambda_l r_2 - \lambda_l r_1 + \lambda_l m)}$$

$$\left[\frac{d}{ds}\left(\bar{f}(s)\right)\right]_{s=0} = -\left[\sum_{r_1=0}^{m} \sum_{r_2=0}^{m-r_1} (-1)^{m-r_1+1} \frac{m!}{r_1!\, r_2!\, (m-r_1-r_2)!} p^{r_2} q^{m-r_1-r_2} \times \frac{(\lambda_h r_2 - \lambda_l r_2 - \lambda_l r_1 + \lambda_l m)}{(s + \lambda_h r_2 - \lambda_l r_2 - \lambda_l r_1 + \lambda_l m)^2}\right]_{s=0}$$

$$\left[\frac{d}{ds}\left(\bar{f}(s)\right)\right]_{s=0} = -\sum_{r_1=0}^{m} \sum_{r_2=0}^{m-r_1} (-1)^{m-r_1+1} \frac{m!}{r_1!\, r_2!\, (m-r_1-r_2)!} p^{r_2} q^{m-r_1-r_2} \times \frac{1}{(\lambda_h r_2 - \lambda_l r_2 - \lambda_l r_1 + \lambda_l m)} \quad (19)$$

Substituting (19) in (15) the mean time to recruitment is

$$E(T) = \sum_{k=0}^{\infty} \sum_{i=r}^{n} nC_i \left[V(i\theta,k) - (n-i)C_1 V((i+1)\theta,k) + (n-i)C_2 V((i+2)\theta,k) + \cdots (-1)^{n-i} V(n\theta,k)\right) \sum_{r_1=0}^{m} \sum_{r_2=0}^{m-r_1} (-1)^{m-r_1+1} \frac{m!}{r_1!\, r_2!\, (m-r_1-r_2)!} \times p^{r_2} q^{m-r_1-r_2} \frac{1}{(\lambda_h r_2 - \lambda_l r_2 - \lambda_l r_1 + \lambda_l m)} \quad (20)$$

5. Numerical Illustration

The behavior of the performance measure due to the change in parameter is analyzed numerically for different values of n and r.

Case(i) n=3,r=1

From equation (16) the mean time for recruitment is given by

$$E(T) = \sum_{k=0}^{\infty} \left(\frac{p\lambda_l + q\lambda_h}{\lambda_h \lambda_l}\right) \{3V(\theta,k) + 9V(2\theta,k) + 7V(3\theta,k)\}$$

Case(ii) n=3,r=2

From equation (16) the mean time for recruitment is given by

$$E(T) = \sum_{k=0}^{\infty} \left(\frac{p\lambda_l + q\lambda_h}{\lambda_h \lambda_l}\right) \{3V(2\theta,k) - 2V(3\theta,k)\}$$

Case(iii) n=3,r=3

From equation (16) the mean time for recruitment is given by

$$E(T) = \sum_{k=0}^{\infty} \left(\frac{p\lambda_l + q\lambda_h}{\lambda_h \lambda_l}\right) \{V(3\theta,k\}$$

Case(iv) n=3,r=1

From equation (18) the mean time for recruitment is given by

$$E(T) = \sum_{k=0}^{\infty} \sum_{r_1=0}^{m} \binom{m}{r_1} p^{r_1} q^{m-r_1} \left[\frac{1}{(\lambda_h r_1 - \lambda_l r_1 + \lambda_l m)}\right] \times \{3V(\theta,k) + 9V(2\theta,k) + 7V(3\theta,k)\}$$

Case(v) n=3,r=2

From equation (18) the mean time for recruitment is given by

$$E(T) = \sum_{k=0}^{\infty} \sum_{r_1=0}^{m} \binom{m}{r_1} p^{r_1} q^{m-r_1} \left[\frac{1}{(\lambda_h r_1 - \lambda_l r_1 + \lambda_l m)}\right] \times \{3V(2\theta,k) - 2V(3\theta,k\}$$

Case(vi) n=3,r=3

From equation (18) the mean time for recruitment is given by

$$E(T) = \sum_{k=0}^{\infty} \sum_{r_1=0}^{m} \binom{m}{r_1} p^{r_1} q^{m-r_1} \left[\frac{1}{(\lambda_h r_1 - \lambda_l r_1 + \lambda_l m)}\right] \times \{V(3\theta,k\}$$

Case(vii) n=3,r=1

From equation (20 the mean time for recruitment is given by

$$E(T) = \sum_{k=0}^{\infty}\{3V(\theta,k) + 9V(2\theta,k) + 7V(3\theta,k)\} \times \sum_{r_1=0}^{m}\sum_{r_2=0}^{m-r_1}(-1)^{m-r_1+1}\frac{m!}{r_1!\,r_2!\,(m-r_1-r_2)!} \times p^{r_2}q^{m-r_1-r_2}\frac{1}{(\lambda_h r_2 - \lambda_l r_2 - \lambda_l r_1 + \lambda_l m)}$$

Case(viii) n=3,r=2

From equation (20) the mean time for recruitment is given by

$$E(T) = \sum_{k=0}^{\infty}\{3V(2\theta,k) - 2V(3\theta,k)\} \times \sum_{r_1=0}^{m}\sum_{r_2=0}^{m-r_1}(-1)^{m-r_1+1}\frac{m!}{r_1!\,r_2!\,(m-r_1-r_2)!} \times p^{r_2}q^{m-r_1-r_2}\frac{1}{(\lambda_h r_2 - \lambda_l r_2 - \lambda_l r_1 + \lambda_l m)}$$

Case(ix) n=3,r=3

From equation (20) the mean time for recruitment is given by

$$E(T) = \sum_{k=0}^{\infty}\{V(3\theta,k)\}\sum_{r_1=0}^{m}\sum_{r_2=0}^{m-r_1}(-1)^{m-r_1+1}\frac{m!}{r_1!\,r_2!\,(m-r_1-r_2)!} \times p^{r_2}q^{m-r_1-r_2}\frac{1}{(\lambda_h r_2 - \lambda_l r_2 - \lambda_l r_1 + \lambda_l m)}$$

6. Conclusion

From the present work we can study about two grade and three grade manpower system. This work also can be extended in two sources of depletion.The influence of the hypothetical parameter on the performance measure can be studied numerically with the help of MATLAB.

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