Regular Generalized Closed Sets in Grill Topological Spaces

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Abstract: ***The purpose of this paper is to introduce and study a new class of regular generalized closed sets and functions in a topological space X, defined in terms of a grill G on X. The characterization of such sets along with certain other properties of them are obtained.***

Keywords: $rg-$ closed, topology τ_G, operator Φ, $G-rg$ closed

Introduction

It is found from literature that during recent years many topologists are interested in the study of generalized types of closed sets. For instance, a certain form of generalized closed sets was initiated by Levine [6], whereas the notion of rg – closed sets was studied by Palaniappan and Rao [7]. Following the trend, we have introduced and investigated a kind of generalized closed sets, the definition being formulated in terms of grills. The concept of grill was first introduced by Choquet [1] in the year 1947. From subsequent investigations it is revealed that grills can be used as an extremely useful device for investigation of a number of topological problems.

Preliminaries

Definition 2.1: A nonempty collection G of non-empty subsets of a topological space X is called a *Grill* [1] if $(i) A \in G$ and $A \subseteq B \subseteq X \Rightarrow B \in G$ and $(ii) A, B \subseteq X$ and $A \bigcup B \in G \Rightarrow A \in G$ or $B \in G$.

Let G be a grill on a topological space (X,τ). In [8] an operator $\Phi : P(X) \to P(X)$ was defined by $\Phi(A) = \{x \in X / U \bigcap A \in G, \forall U \in \tau(x)\}$, $\tau(x)$ denotes the neighborhood of x. Also the map $\Psi : P(X) \to P(X)$, given by $\Psi(A) = A \bigcup \Phi(A)$ for all $A \in P(X)$. Corresponding to a grill G, on a topological space (X,τ) there exists a unique topology τ_G on X given by $\tau_G = \{U \subseteq X / \Psi(X \setminus U) = X \setminus U\}$ where for any $A \subseteq X$, $\Psi(A) = A \bigcup \Phi(A) = \tau_G - cl(A)$. Thus a subset A of X is τ_G - closed (resp. τ_G -dense in itself) if $\Psi(A) = A$ or equivalently if $\Phi(A) \subseteq A$ (resp. $A \subseteq \Phi(A)$).

In the next section we introduce and study a new class of generalized closed sets, termed $G-rg$ closed, in terms of a given grill G, the definition having a close bearing to the above operator Φ. This class of $G-rg$ closed sets will be seen to properly contain the class of rg closed sets as introduced in [7]. An explicit form of such a $G-rg$ closed set is also obtained. In section 4, we introduce and investigate the notion regular generalized continuous functions in grill topological spaces. Also, we investigate its relationship with other functions.

Throughout the paper, by a space X we always mean a topological space (X,τ) with no separation properties assumed. If $A \subseteq X$, we shall adopt the usual notations $\text{int}(A)$ and $cl(A)$ respectively for the interior and closure of A in (X,τ). Again $\tau_G - cl(A)$ and $\tau_G - \text{int}(A)$ will respectively denote the closure and interior of A in (X,τ_G). Similarly, whenever we say that a subset A of a space X is open (or closed), it will mean that A is open (or closed) in (X,τ). For open and closed sets with respective to any other topology on X, eg. τ_G, we shall write τ_G - open and τ_G - closed. The collection of all open neighborhoods of a point x in (X,τ) will be denoted by $\tau(x)$. A subset A of a space (X,τ) is said to be regular open [9] (resp. regular closed) if $A = \text{int}\, cl(A)$ (resp. $A = cl\, \text{int}(A)$).

We now append a few definitions and results that will be frequently used in the sequel.

Definition 2.2: A subset A of a space (X,τ) is said to be $G-g$ *closed* [2] (ie. generalized closed set in grill topological space) if $\Phi(A) \subseteq U$ whenever $A \subseteq U$ and U is open in X.

The complement of this set is called $G-g$ *open.*

Theorem 2.1: [8] Let (X,τ) be a topological space and G be a grill on X. Then for any $A,B \subseteq X$ the following hold:

(a) $A \subseteq B \Rightarrow \Phi(A) \subseteq \Phi(B)$

(b) $\Phi(A \cup B) = \Phi(A) \cup \Phi(B)$

(c) $\Phi(\Phi(A) \subseteq \Phi(A) = cl(\Phi(A)) \subseteq cl(A)$

Definition 2.3: A subset A of a topological space X is said to be $\theta-$*closed* [10] if $A = \theta cl(A)$ where $\theta cl(A)$ is defined as

$\theta cl(A) = \{x \in X / cl(U) \cap A \neq \phi \forall U \in \tau \,\&\, x \in U\}$.

Definition 2.4: A subset A of a topological space X is said to be $\theta-$*open* [10] if $X \setminus A$ is $\theta-$*closed*.

Definition 2.5: A subset A of a topological space X is said to be $\delta-$*closed* [10] if $A = \delta cl(A)$ where $\delta cl(A)$ is defined as

$\delta cl(A) = \{x \in X / \operatorname{int} cl(U) \cap A \neq \phi \forall U \in \tau \,\&\, x \in U\}$.

Definition 2.6: A subset A of a topological space X is said to be $\delta-$*open* [10] if $X \setminus A$ is $\delta-$*closed*.

Definition 2.7: A subset A of a topological space X is said to be θ_g-*closed* [4] if $\theta cl(A) \subseteq U$ whenever $A \subseteq U$ and U is open in X.

Definition 2.8: A subset A of a topological space X is said to be δ_g-*closed* [3] if $\delta cl(A) \subseteq U$ whenever $A \subseteq U$ and U is open in X.

Definition 2.9: A subset A of a topological space X is said to be θ_g- *open*[4] (δ_g- *open* [3]) if $X \setminus A$ is θ_g- *closed*(δ_g-*closed*) in X.

Definition 2.10: A function $f:(X,\tau) \to (Y,\sigma)$ is said to be

(1) $\theta-$continuous [5] if $f^{-1}(V)$ is $\theta-$closed of (X,τ) for every closed set V of (Y,σ).

(2) $\delta-$ continuous if $f^{-1}(V)$ is $\delta-$ closed of (X,τ) for every closed set V of (Y,σ).

(3) θ_g- continuous if $f^{-1}(V)$ is θ_g-closed of (X,τ) for every closed set V of (Y,σ).

(4) δ_g- continuous if $f^{-1}(V)$ is δ_g- closed of (X,τ) for every closed set V of (Y,σ).

(5) $rg-$ *continuous* if $f^{-1}(V)$ is $rg-$ closed of (X,τ) for every closed set V of (Y,σ).

Definition 2.11: A function $f:(X,\tau) \to (Y,\sigma)$ is said to be (1) $\theta-$*closed* [10] if $f(F)$ is $\theta-$closed set of (Y,σ) for every closed set F of (X,τ).

(2) $\delta-$*closed* [10] if $f(F)$ is $\delta-$closed set of (Y,σ) for every closed set F of (X,τ).

(3) $\theta g-$*closed* if $f(F)$ is $\theta g-$closed set of (Y,σ) for every closed set F of (X,τ).

(4) $\delta g-$*closed* if $f(F)$ is $\delta g-$closed set of (Y,σ) for every closed set F of (X,τ).

(5) rg *closed* if $f(F)$ is rg closed set of (Y,σ) for every closed set F of (X,τ).

Definition 2.12: A function $f:(X,\tau,G) \to (Y,\sigma)$ is said to be $Gg-$ continuous if $f^{-1}(V)$ is $Gg-$ closed of (X,τ) for every closed set V of (Y,σ).

Definition 2.13: A function $f:(X,\tau) \to (Y,\sigma,G)$ is said to be $Gg-$ closed if $f(V)$ is $Gg-$ closed of (X,τ) for every closed set V of (Y,σ).

3. $rg-$closed sets with respect to a grill

Definition 3.1: Let (X,τ) be a topological space. A subset A of (X,τ) is called *regular generalized closed* (rg closed) set [3] if $cl(A) \subseteq U$ whenever $A \subseteq U$ and U is regular open in X.

Definition 3.2: Let (X,τ) be a topological space and G be a grill on X. Then the subset A of (X,τ) is said to be $rg-$ *closed sets with respect to a grill* ($G-rg$ closed) if $\Phi(A) \subseteq U$ whenever $A \subseteq U$ and U is regular open in X. A subset A of X is $G-rg$ open if $X \setminus A$ is $G-rg$ closed.

Lemma 3.1: Let (X,τ) be a topological space and G be a grill on X. If A is $G-rg$ closed, then $\Phi(A)$ is $G-rg$ closed.

Proposition 3.1: For a topological space (X,τ) and G be a grill on X, then the following holds:

(a) Every closed set in X is $G-rg$ closed.

(b) For any subset A in X, $\Phi(A)$ is $G-rg$ closed.

(c) Every τ_G-closed set is $G-rg$ closed.

(d) Any non member of G is $G-rg$ closed.

(e) Every $rg-$closed set is $G-rg$ closed.

(f) Every $g-$closed set is $G-rg-$ closed.

(g) Every $\theta-$closed set is $G-rg-$ closed.

(h) Every $\delta-$closed set is $G-rg-$ closed.

Proof: (a) Let A be a closed set and U be any regular open set in X such that $A \subseteq U$.

Then $\Phi(A) \subseteq cl(A) = A \subseteq U$ .Therefore, A is $G-rg$ closed.

(b) Let $A \subseteq X$ and U be any regular open set in X such that $A \subseteq U$. Then by Lemma 3.1, $\Phi(\Phi(A)) \subseteq \Phi(A) \subseteq U$. Therefore $\Phi(A)$ is $G-rg$ closed.

(c) Let A be a τ_G -closed set and U be any regular open set in X such that $A \subseteq U$. Then $\Psi(A) = A \Rightarrow A \bigcup \Phi(A) = A \Rightarrow \Phi(A) \subseteq A \subseteq U$. Therefore A is $G-rg$ closed.

(d) Let $A \notin G$ and U be regular open set such that $A \subseteq U$ .Then $\Phi(A) \subseteq \phi \subseteq U$ .Therefore A is $G-rg$ closed.

(e) Let A be a $rg-$ closed set and U be a regular open set such that $A \subseteq U$ .Then $\Phi(A) \subseteq cl(A) \subseteq U$ Therefore A is $G-rg$ closed.

(f) Let A be a $g-$closed set and U be a open set such that $A \subseteq U$. Since every open set is regular open in X, then $\Phi(A) \subseteq cl(A) \subseteq U$. Therefore A is $G-rg$ closed.

(g) Let A be a $\theta-$closed set. Then $A = \theta cl(A)$. Let U be a regular open set in X such that $A \subseteq U$. Then by Theorem 2.1, $\Phi(A) \subseteq cl(A) \subseteq \theta cl(A) \subseteq U$. Therefore A is $G-rg$ closed.

(h) Let A be a $\delta-$closed set. Then $A = \delta cl(A)$. Let U be a regular open set in X such that $A \subseteq U$. Then by Theorem 2.1, $\Phi(A) \subseteq cl(A) \subseteq \delta cl(A) \subseteq U$. Therefore A is $G-rg$ closed.

The converse of the above proposition is not true in general as seen from the following examples.

Example 3.1: Let $X = \{a,b,c\}, \tau = \{\phi, \{a\}, \{b\}, \{a,b\}, X\}$ and $G = \{\{a\}, \{a,c\}, X\}$. Then (X,τ) is a topological space and G is grill on X. Then it is easy to verify that
(a) $\{a,b\}$ is not closed but is $G-rg$ closed.
(b) $\{a,c\}$ is not τ_G -closed but is $G-rg$ closed.
(c) $\{a,b\}$ is not a grill but is $G-rg$ closed.
(d) $\{b\}$ is not $rg-$ closed but is $G-rg$ closed.
(e) $\{a,b\}$ is not $g-$closed set but is $G-rg-$ closed.
(f) $\{a,b\}$ is not $\theta-$closed set but is $G-rg-$ closed.
(g) $\{a,b\}$ is not $\delta-$closed set but is $G-rg-$ closed.

Example 3.2: In example 3.1, the set $\{b\}$ is $G-rg-$ closed but not $\theta_g -$closed and $\delta_g -$ closed.

Remark 3.1: From the above discussions and known results we have the following implications:

$$\begin{array}{ccccccccc} \theta-closed & & & & & & \theta_g-closed & & \\ \downarrow & & & & & & \downarrow & & \\ closed & \longrightarrow & g\ closed & \longrightarrow & rg\ closed & \longrightarrow & G-rg\ closed & \longleftarrow & Gg-closed \\ \uparrow & & & & & & \uparrow & & \\ \delta-closed & & & & & & \delta_g-closed. & & \end{array}$$

Definition 3.3: Let X be a space and $(\phi \neq) A \subseteq X$. Then $[A] = \{B \subseteq X / A \bigcap B \neq \phi\}$ is a grill on X, called the *Principal grill* generated by A.

Lemma 3.2: Let (X,τ) be a space and G be a grill on X. If $A \subseteq X$ is τ_G - dense in itself, then $\Phi(A) = cl\Phi(A) = \tau_G - cl(A) = cl(A)$.

Proposition 3.2: In the case of principle grill $[X]$ generated by X, it is known that $\tau = \tau_{[X]}$ and any rg closed is $[X]-rg$ closed.

Proof: It follows from Proposition 3.1 (e).

Theorem 3.2: Let (X,τ) be a topological space and G be a grill on X . Then for $A \subseteq X$, A is $G-rg$ closed iff $\tau_G - cl(A) \subseteq U, A \subseteq U$ and U is regular open.

Proof: Suppose A is $G-rg$ closed. Then $\Phi(A) \subseteq U \Rightarrow A \bigcup \Phi(A) \subseteq U$. Therefore $\tau_G - cl(A) \subseteq U, A \subseteq U$ and U is regular open. Conversely, $\tau_G - cl(A) \subseteq U, A \subseteq U$ and U is regular open. Therefore $A \bigcup \Phi(A) \subseteq U \Rightarrow \Phi(A) \subseteq U$. Hence A is $G-rg$ closed.

Theorem 3.3: Let G be a grill on a space (X,τ). If A is τ_G -dense in itself and $G-rg$ closed iff A is rg -closed.

Proof: Let A be τ_G -dense in itself, Then by Lemma 3.2, $\Phi(A) = cl(A)$. Since A is $G-rg$ closed, $\Phi(A) \subseteq U$ where U is regular open in X and $A \subseteq U$. Therefore $cl(A) \subseteq U$ where U is regular open in X and $A \subseteq U$. Hence A is rg -closed.

Conversely, A is rg -closed, $cl(A) \subseteq U$ where U is regular open in X and $A \subseteq U$. Therefore $\Phi(A) \subseteq U$ where U is regular open in X and $A \subseteq U$. Therefore $\Phi(A) = cl(A)$. Hence A is τ_G -dense in itself and $G-rg$ closed.

Theorem 3.4: For any grill G on a space (X,τ) .The following are equivalent.

(a) Every subset of X is $G-rg$ closed.

(b) Every regular open subset of (X,τ) is τ_G -closed.

(c) **Proof** : $(a) \Rightarrow (b)$ Let A be regular open in (X,τ) . Then by (a), A is $G-rg$ closed, so that $\Phi(A) \subseteq A$. Therefore A is τ_G -closed.

$(b) \Rightarrow (a)$ Let $A \subseteq X$ and U be regular open in (X,τ) such that $A \subseteq U$. Then by (b), $\Phi(U) \subseteq U$. Also, $A \subseteq U$ $\Rightarrow \Phi(A) \subseteq \Phi(U) \subseteq U$. Therefore A is $G-rg$ closed.

Theorem 3.5: Let (X,τ) be a topological space and G be a grill on X and A, B be subsets of X such that $A \subseteq B \subseteq \tau_G - cl(A)$. If A is $G-rg$ closed, then B is $G-rg$ closed.

Proof: Suppose $B \subseteq U$ and U is regular open in X .
Since A is $G-rg$ closed,

$\Phi(A) \subseteq U \Rightarrow \tau_G - cl(A) \subseteq U \rightarrow (1)$.

Now, $A \subseteq B \subseteq \tau_G - cl(A)$ which implies

$\tau_G - cl(A) \subseteq \tau_G - cl(B) \subseteq \tau_G - cl(A)$.

Therefore $\tau_G - cl(A) = \tau_G - cl(B)$.

Therefore by (1) $\tau_G - cl(B) \subseteq U$.

Hence B is $G-rg$ closed.

Corollary 3.1: τ_G - closure of every $G-rg$ closed set is $G-rg$ closed.

Theorem 3.6: Let G be a grill on a space (X,τ) and A, B be subsets of X such that $A \subseteq B \subseteq \Phi(A)$. If A is $G-rg$ closed, then A and B are rg closed.

Proof: Let $A \subseteq B \subseteq \Phi(A)$. Then $A \subseteq B \subseteq \tau_G - cl(A)$. By Theorem 3.5, B is $G-rg$ closed. Again $A \subseteq B \subseteq \Phi(A) \Rightarrow \Phi(A) \subseteq \Phi(B) \subseteq \Phi(\Phi(A)) \subseteq \Phi(A)$. This implies that $\Phi(A) = \Phi(B)$. By Theorem 3.3, A and B are τ_G - dense in itself. Therefore A and B are rg closed.

Theorem 3.7: Let G be a grill on a space (X,τ). Then a subset A of X is $G-rg$ open iff $F \subseteq \tau_G - \text{int}(A)$ whenever $F \subseteq A$ and F is closed.

Proof: Let A be $G-rg$ open set and $F \subseteq A$ where F is closed. Then $X \setminus A \subseteq X \setminus F$. This implies that $\Phi(X \setminus A) \subseteq \Phi(X \setminus F) = X \setminus F$.

Hence $\tau_G - cl(X \setminus A) \subseteq X \setminus F$ which implies $F \subseteq \tau_G - \text{int}(A)$.

Conversely, $(X \setminus A) \subseteq U$ where U is open in (X,τ). Then $X \setminus U \subseteq \tau_G - \text{int}(A)$. Therefore A is $G-rg$ open.

4. $G-rg$ continuous and $G-rg$ closed function

Definition

4.1: A function $f:(X,\tau,G) \rightarrow (Y,\sigma)$ is said to be $G-rg$ continuous (resp. rg continuous) if $f^{-1}(V)$ is $G-rg$ open (resp. rg open) for each $V \in \sigma$.

Remark 4.1:
(a) Every continuous function is rg continuous.
(b) Every $rg-$ continuous function is $G-rg$ continuous.

Example 4.1: Let $X = \{a,b,c,d\}$

$\tau = \{\phi, X, \{a\}, \{b\}, \{a,b\}, \{a,b,c\}\}$

$G = \{\{a\}, \{c\}, \{a,c\}, \{a,b\}, \{b,c\}, X\}$ we define a function $f:(X,\tau,G) \rightarrow (Y,\sigma)$ as follows

$f(a) = c$, $f(b) = d$, $f(c) = a$ and $f(d) = b$. Then it is easy to see that f is $G-rg-$ continuous but not continuous. Also f is $G-rg-$ continuous but not $rg-$ continuous.

Remark 4.2: Every $\theta-$ continuous function is $G-rg$ continuous.

Example 4.2: From Example 4.1, it is easy to see that f is $G-rg-$ continuous but not $\theta-$ continuous.

Remark 4.3: Every $\delta-$ continuous function is $G-rg$ continuous.

Example 4.3: From Example 4.1, it is easy to see that f is $G-rg-$ continuous but not $\delta-$ continuous.

Example 4.4: From Example 4.1, it is easy to see that f is $G-rg-$ continuous but not $\delta g-$ continuous, $\theta g-$ continuous, $Gg-$ continuous.

Remark 4.4: From the above discussions and known results we have the following implications:

$\theta-$ cont. $\qquad\qquad$ $\theta_g -$ cont.

$\downarrow$ $\qquad\qquad\qquad\qquad$ $\downarrow$

cont $\rightarrow g$ cont. $\rightarrow rg$ cont. $\rightarrow G-rg$ cont. $\leftarrow Gg-$ cont.

$\uparrow$ $\uparrow$

$\delta-$ *cont.* δ_g- *cont.*

Here cont. indicates continuous.

Theorem 4.1: For a function $f:(X,\tau,G)\to(Y,\sigma)$ the following are equivalent:

a) f is $G-rg$ continuous.

b) The inverse image of each closed set in Y is $G-rg$ closed.

c) For each $x\in X$ and each $V\in\sigma$ containing $f(x)$, there exists a $G-rg$ open set U containing x such that $f(U)\subseteq V$.

Proof: (a) $\Leftrightarrow$ (b) It follows from the definition. (a)$\Rightarrow$(c) Let $V\in\sigma$ and $f(x)\in V$. Then by (a) $f^{-1}(V)$ is $G-rg$ open set containing x. Take $f^{-1}(V)=U$, we have $x\in U$ and $f(U)\subseteq V$.

(c) $\Rightarrow$(a) Let V be any open set in Y and $x\in f^{-1}(V)$. Then $f(x)\in V\in\sigma$ and hence by (c) there exists a $G-rg$ open set U containing x such that $f(U)\subseteq V$. Now, $x\in U\subseteq\Psi(\text{int}U)\subseteq\Psi(\text{int} f^{-1}(V))$. This shows that $f^{-1}(V)\subseteq\Psi(\text{int} f^{-1}(V))$. Therefore f is $G-rg$ continuous.

Theorem 4.2: A function $f:(X,\tau,G)\to(Y,\sigma)$ is $G-rg$ continuous iff the graph function $g:X\to X\times Y$ defined by $g(x)=(x,f(x))$ for each is $G-rg$ continuous.

Proof: Suppose that $f:(X,\tau,G)\to(Y,\sigma)$ is $G-rg$ continuous. Let $x\in X$ and W be any open set in $X\times Y$ containing $g(x)$. Then there exists $U\in\tau$ and $V\in\sigma$ such that $g(x)=(x,f(x))\in U\times V\subseteq W$. Since f is $G-rg$ continuous, there exists a $G-rg$ open set G of X containing x such that $f(G)\subseteq V, G\bigcap U$ is $G-rg$ open and $g(G\bigcap U)\subseteq U\times V\subseteq W$. Therefore g is $G-rg$ continuous.

Conversely, suppose that g is $G-rg$ continuous. Let $x\in X$ and V be any open set in Y containing $f(x)$. Then $X\times V$ is open in $X\times Y$ and by $G-rg$ continuity of g, there exists a $G-rg$ open set U containing x such that $g(U)\subseteq X\times V$. Thus we have, $f(U)\subseteq V$. Therefore f is $G-rg$ continuous.

Definition 4.2: Let (X,τ) be a topological space and (Y,σ,G) be a grill topological space. A function $f:(X,\tau)\to(Y,\sigma,G)$ is said to be $G-rg$ *open* (resp. $G-rg$ closed) if for each $U\in\tau$ (resp. closed set U in (X,τ)), $f(U)$ is $G-rg$ open (resp. $G-rg$ closed) in (Y,σ,G).

Remark 4.5: Every rg closed function is $G-rg$ closed.

Example 4.5: From Example 4.1, it is easy to see that f is $G-rg$ closed function but not rg closed.

Theorem 4.3: Let $f:(X,\tau)\to(Y,\sigma,G)$ be $G-rg$ open function. If V is any subset of Y and F is a closed subset of X containing $f^{-1}(V)$, then there exists a $G-rg$ open set H in (Y,σ,G) containing V such that $f^{-1}(H)\subseteq F$.

Proof : Suppose that f is $G-rg$ open function. Let V be any subset of Y and F is a closed subset of X containing $f^{-1}(V)$. Then $X\setminus F$ is open in (X,τ) and hence by $G-rg$ openness of f, $f(X\setminus F)$ is G - open. Thus $H=Y\setminus f(X\setminus F)$ is $G-rg$ closed and consequently $f^{-1}(V)\subseteq F$ which implies that $V\subseteq H$. Further we obtain $f^{-1}(H)\subseteq F$.

Theorem 4.4: For any bijection $f:(X,\tau)\to(Y,\sigma,G)$ the following are equivalent.

(a) $f^{-1}:(Y,\sigma,G)\to(X,\tau)$ is $G-rg$ continuous.

(b) f is $G-rg$ open

(c) f is $G-rg$ closed.

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