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An Analytical Study on Transverse Vibration of Both End Fixed Cable with a Moving Mass

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Abstract: ***This paper presents the theoretical study on transverse vibration of both end fixed cable with a moving mass. The non-linear equation derived using Newton's force conservation principle is non-dimensionlized and solved with the application of Galerkin's approximation method. The main excitation source of cable vibration is moving mass. The modelled equation is simulated with MATLAB, considered up to 4th mode of vibration. The study shows that when the velocity of moving mass increasing, the mid span lateral deflection of cable is increased up to certain velocity, then decreased up to zero when the mass reached to the next fixed end. Similarly, the maximum transverse deflection is seen in the first mode. Increasing the modal frequency the lateral displacement is decreased.***

Keywords: Cable Vibration, Galerkin's Approach, Transverse Vibration, Moving Mass, Dimensionless Parameters, Both End Fixed Cable

1. Introduction

Cable is the most fundamental member of the cable supported structures, e.g. suspension bridges, cable cars, rope ways, connector of posts and so on. Cable structures play an important role in many engineering field, such as mechanical, electrical, civil, ocean and so on. Cable is widely used so, its importance and applications are increasing day by day. Cable is mainly utilized on tensioned member to support structures or transmit the major loads of the structures [1, 2]. Some of the applications of cable supported structures are; cable car, cable conveyor, suspension bridge, rope way, cable-stayed bridges and so on.

The analytical solution of vibrating string was first presented by Joseph Lagrange in 1759 on Turin Academy. On the vibration suppression on long span bridge is studied by Daihai Chen et.al.2013 as Vehicle Bridge coupling vibration reduction analysis for long span highway bridge and railway Bi-purpose cable-stayed bridge [2]. The experimental study on Vortex induced vibration of marine cable using force feedback is researched by F.S. Hover et.al in 1996[3]. Hongwei Huang et.al have studied as vibration mitigation of stay cable using optimally tuned MR damper in 2012[4] and experimentally mitigation of vibration using external viscous damper is studied by Huang le in 2010[5]. Park young Myung et.al. studied on dynamic behavior of cable stayed bridge under moving vehicle and train in 2015[6]. An analytical solution for vibration of elevator cable with small bending stiffness is studied by R. Mirabdollah et.al in 2012[8]. Analysis of harmonic vibration of cable stayed footbridge under the influence of changes of the cable tension was studied by Wojciech Pakos in 2015[9], but they haven't studied considering dimensionless parameters. The investigation on nonlinear dynamics behavior of stay cables under rain wind induced vibration in consideration of restoring force and the columb force was studied by yonggang Xiao et.al.2012 [10], where chaos phenomenon is the major interpreted function.
Here in this paper, it is studied with the interest to investigate further on mechanical behavior i.e transverse vibration with moving mass on cable. This theoretical analysis on is studied using non-dimensional parameters and solved with Galerkin's approach.

2. Mathematical Modulation

2.1 Dynamic of Cable

The schematic diagram of the both end fixed cable with movi ng mass is modeled as shown in Fig. 1, where, L be the length of a tightly stretched cable, P is tensile force on the cable, θ i s the small deflection angle with *x-axis* and m is the mass per unit length of cable, $f(x,t)$ is the transverse vibrational force in duced by moving mass.

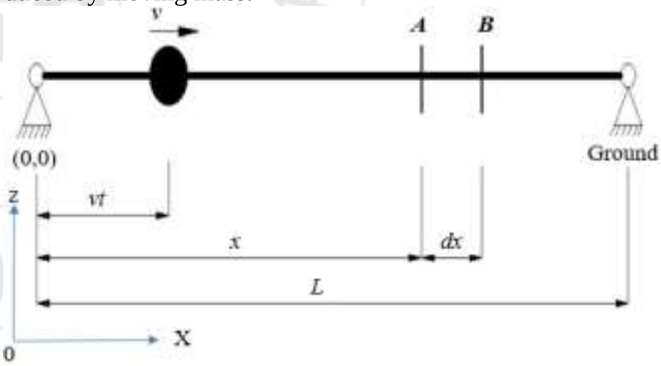

Figure 1: Schematic Diagram of Both end Fixe Cable with Moving Mass

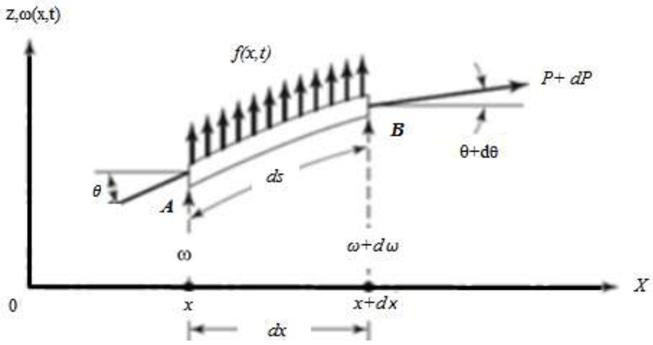

Figure 2: Free body Representation of Cable Forces

In an elementary length *dx,* the transverse displacement *w(x,t)* is assumed to be small, then equilibrium force equation i.e the modeled dynamic equation of cable with moving mass is fou nd as equation 1. This equilibrium force equation is derived without considering the Coriolis force, internal strain rate (Ke lvin Voigt) and other air resistivity with using Newton's secon d law of motion with equilibrium approach, where net force a cting on an element is equal to the inertial force acting on the element[7]. The free body diagram of the system is as in Fig. 2.

$$(P+dP)\sin(\theta+d\theta)+f(x,t)dx - P\sin\theta = mdx\frac{\partial^2\omega}{\partial t^2} \tag{1}$$

For an elementary length *dx*,

$$dP=\frac{\partial P}{\partial x}dx \tag{2}$$

3.
4.

$$\sin\theta \cong \tan\theta = \frac{\partial\omega(x,t)}{\partial x} \quad \& \quad \sin(\theta+d\theta)\cong\tan(\theta+d\theta) = \frac{\partial\omega(x,t)}{\partial x}+\frac{\partial\omega(x,t)}{\partial x^2}dx \tag{3}$$

The resultant force due to the external mass on vertical direction is assumed as;

$$f(x,t)=Mg-M\frac{\partial\omega(x,t)}{\partial t^2} \tag{4}$$

Here, *f(x,t)* is the resultant force due to external mass, *g* is the acceleration due to gravity. Since transverse vibrating force is the force due to moving mass *M*, which moves with longitudinal velocity *v* and transverse displacement is same as cable vibration, then applying the direc delta function to determine the effective force applied on cable it becomes,

$$f(x,t)\begin{cases}\delta(x-vt)Mg-M\frac{\partial\omega(x,t)}{\partial t^2} & 0\le vt\le L\\ 0 & vt>L\end{cases} \tag{5}$$

Rearranging the equation (1) using equation (2), (3),(4) and (5) we have,

$$\frac{\partial}{\partial x}\left[P\frac{\partial\omega}{\partial x}\right]-\delta(x-vt)M\frac{\partial^2\omega}{\partial t^2} = m\frac{\partial^2\omega}{\partial t^2}-\delta(x-vt)Mg \tag{6}$$

Let the cable is simply supported at both ends, the boundary conditions are;

$$\omega(0,t)=0,\ \text{and}\ \omega(L,t)=0 \tag{7}$$

2.2 Non-dimensionalization of Equation

Some of the dimensionless quantities and the boundary conditions are as below;

$$n=\frac{\omega}{L},\zeta=\frac{x}{L},\tau=\frac{tv}{L} \tag{8}$$

$$n(0,\tau)=0\ \text{And}\ n(\zeta,\tau)=0 \tag{9}$$

Substituting the equation (8) on equation (6) we get;

$$\left(\delta(\zeta-\tau)\frac{M}{L}+m\right)\frac{\partial^2 n(\zeta,\tau)}{\partial\tau^2}=\frac{P}{v^2}\frac{\partial^2 n(\zeta,\tau)}{\partial\zeta^2} +\delta(\zeta-\tau)\frac{Mg}{v^2} \tag{10}$$

Using the Galerkin's approach to solve the non-homogeneous equation (10), which satisfies the same boundary conditions (9) and neglecting the static deflection of the cable the dynamic deflection is become as equation 11. The mass of the cable in comparison to the external mass is too small, so, static deflection here in this paper is not considered.

$$n(\zeta,\tau)=\sum_{i=1}^{k}\phi_i(\zeta)q_i(\tau) \tag{11}$$

where, *i* is number of mode, $\phi_i(\zeta)$ is spaital mode shape function (the comparison function) which have same boundary condition and characteristics and $q(\tau)$ is the generalized time dependent co-ordinate. Using the shape function $\phi(\zeta)=\sin(i\pi\zeta)$, the desired roots become as;

$$n(\zeta,\tau)=\sum_{i=1}^{k}\sin(i\pi\zeta)q_i(\tau) \tag{12}$$

Substituting equation (12) on equation (10), and normalized by multiplying and integrating with orthogonal function $\sin j\pi\zeta(j=1....k)$, from 0 to L, then using some more parameters as (13)

$$\gamma=\frac{M}{mL},c^2=\frac{P}{m} \tag{13}$$

where, γ is the mass ratio and *c* is the wave velocity of the cable system, the governing equation becomes;

$$\left(\frac{1}{2}+\gamma\sum_{i,j=1}^{k}\sin(i\pi\tau)\sin(j\pi\tau)\right)\ddot{q}(\tau) +\left(\frac{i^2\pi^2c^2L}{2v^2}\right)q(\tau)=\frac{\gamma gL}{v^2}\sin(j\pi\tau) \tag{14}$$

The linearlized dimensionless governing equation of motion of cable becomes;

$$\left|\overline{M}\right|\ddot{q}+\left|\overline{K}\right|q=F_m \tag{15}$$

where, $\left|\overline{M}\right|$ is mass matrix and $\left|\overline{K}\right|$ is stiffness matrix of size (*n x n*)

5. Results and Discussion

In this study, 2[m] length of cable having mass ratio, i.e ratio of external moving mass to cable mass is 0.3, is excited with the moving mass velocity 0.3[m/s] in parallel to the cable length is considered. When the cable vibrates up to 4^{th} mode, the multimodal vibration nature in time domain is as shown in Fig. 3 below.

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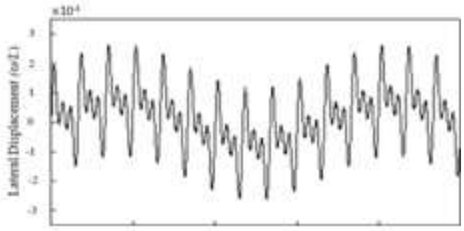

Figure 3: Multimodal Vibration on Time Domain

When the moving mass velocity on cable is 0.3[m/s], the acceleration calculated at mid-point of the cable in time domain in different mode is as presented in Fig. 4. The maximum acceleration in first mode is about $2e^{-3}$ [m/s^2], $4.89e^{-4}$ [m/s^2] on 2nd mode, $8e^{-5}$ [m/s^2] on 3rd mode and $8e^{-5}$ [m/s^2] and $1e10^{-5}$ [m/s^2] on 4th mode measured at the midpoint of the cable. Increasing the mode of the vibration, the lateral vibrational acceleration decreased. The initial and final excitation acceleration are zero and it fluctuated with different modes.

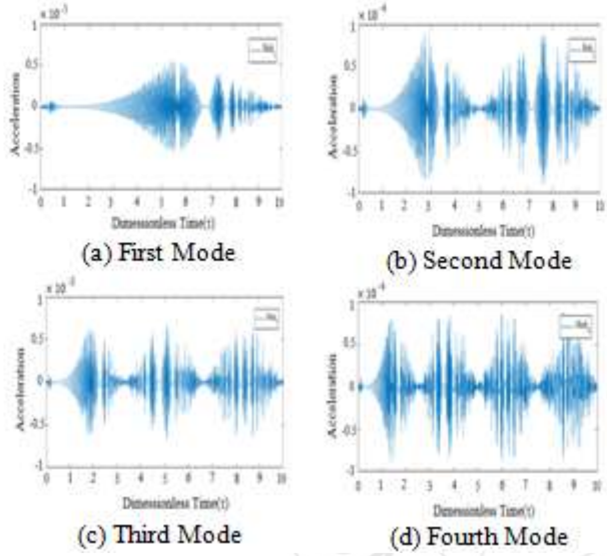

Figure 4: Lateral Acceleration in Different Modes

Similarly, when cable vibrates due to moving mass velocity of 0.3[m/s] and the lateral displacement analyzed at mid-point on different modes, the displacement vs moving mass position plot can be formed as Fig. 5 below. It is found that the maximum displacement (0.00358) appears on first mode at frequency of 45.48 Hz. This maximum value can be observed when the moving mass reached at L/2 and for the higher modes; its maximum value can be seen after half of length. It is happened due to inertial force of the moving mass and tensile force acting on the cable. When the cable vibrates in higher modes, it shows that the amplitude is decreasing.

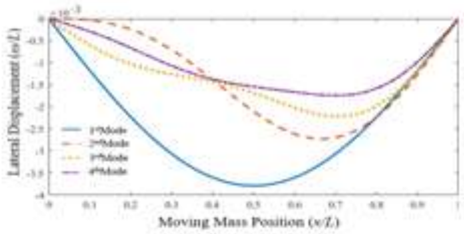

Figure 5: Lateral Displacement under Moving Mass on Diff. Modes

The lateral displacement with moving mass position observed at the midpoint of the cable while vibrate on 1st mode, with different velocity at mass ratio γ =0.3 can be seen in Fig. 7. It shows that, when moving mass speed is reached up to 0.5[m/s], lateral displacement is increased then decreased on increasing the velocity. In this case the maximum amplitude of cable can be seen about 0.4*L* of the cable. As per the boundary condition we applied during simulation increasing the velocity the position of moving mass reached near to the end point, so, the amplitude is decreased with increasing velocity.

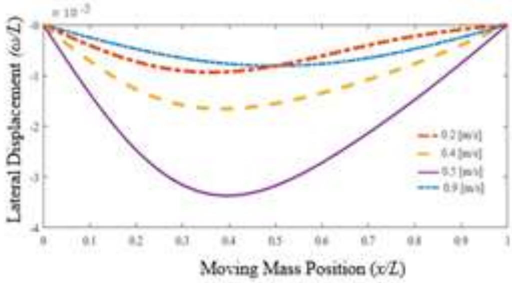

Figure 6: Lateral Deflection of Cable at Various Speed at γ =0.3

On the same way, when the mass ratio of the cable be changed to 1 the lateral displacement vs moving mass position plot can be seen in Fig. 8. It shows that the maximum amplitude is found when the moving mass reached about 0.7L, while the velocity is 0.5[m/s].

After increasing the velocity then 0.5[m/s], its amplitude is decreasing at γ =1. So, through its inertial force and the velocity effects, the maximum deflection appears slightly right side of the cable.

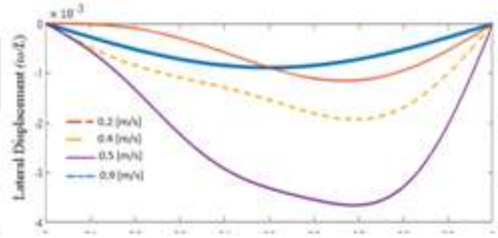

Figure 7: Lateral Deflection of Cable at Various Speed at γ =1.0

For the same system, when mass ratio of the system is changed, the lateral displacement with moving mass position can be seen in Fig. 8. The maximum lateral displacement (about 0.0045) can be seen, when the moving mass weight is equal to the total weight of cable i.e γ =1. We can see the maximum displacement, when the mass reached about 0.6L of the cable. It is found out that the lower mass ratio, lower the lateral displacement.

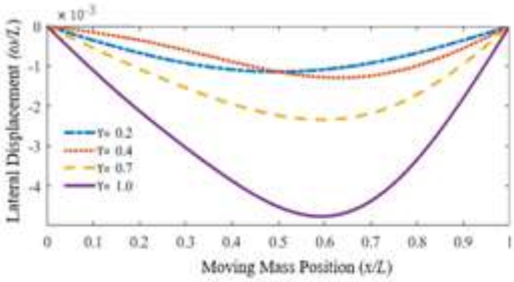

Figure 8: Lateral Def. of Cable at Various Mass Ratio at

$v = 3$ [m/s]

Similarly, while the velocity of moving mass on cable is changed, the modal frequency and the lateral displacement on midpoint of the cable is also changed. The cable of 2[m] length is while taken for study on 0.3[m/s] and 0.4[m/s] and 0.5 [m/s], the lateral displacement vs frequency plot can be seen in Fig. 10. Same as previous simulation the mass ratio γ is 0.3, cable wave velocity is 14.14[m/s] and the measurement is done at the midpoint of cable. The figure shows that the vibrating frequency and the amplitude is changed on changing the velocity of moving mass.

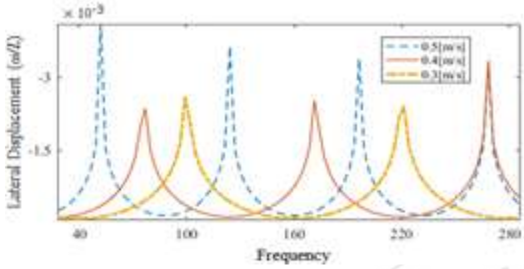

Figure 9: Lateral Displacement on Different Frequencies at γ =0.3

While the mass ratio is changed, modal frequency is not changed. The Fig 11 below shows that the frequency vs lateral displacement graph at moving mass velocity 0.3[m/s], cable wave velocity 14.14[m/s] on changing the mass ratio. The frequency on different mode is not changed but it effects on amplitude with changing the mass ratio. In comparison with Fig.10 and 11 it is found that modal frequency is not changed with changing its mass ratio but changed with moving mass velocity.

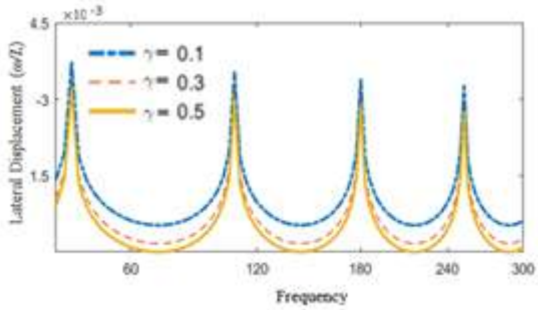

Figure 10: Frequency vs Lateral Displacement on Different Mass Ratio (γ) at $v = 0.3$[m/s]

6. Conclusion

When the external mass is not applied on the cable, there is no vibration occurred at its zero initial condition. From the vibration response plot, some key things on cable Vibration are:

- The amplitude of vibration is maximum in 1st mode and increasing the frequency the mode shape of the cable is increased. Similarly, the amplitude of vibration is decreased in higher mode.
- While the position of moving mass shifted, the shape and the vibration nature of cable is also changed.
- Similarly, when velocity of moving mass is changed, the frequency on cable and the amplitude of vibration is also changed. Study shows that the maximum amplitude can be seen while the velocity of moving mass be 0.5[m/s], after increasing from this value amplitude is decreased.

7. Future Study

Design and analyze the cable structure, and apply to suppress the sound and vibration of supporting structures with changing the position of moving mass acting as counter load.

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