Gravitational Curvature and Its Asymptote

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Abstract: As we know the whole universe is covered by curved space, so everything works by disturbing curve in space, even a smallest part of the universe cannot perform its work without creating curvature in space and so gravitational pull also construct wide curvature in space. If gravity can be defined as the curvature of space rather than a force of attraction, why does not a bullet shot out of a gun, say perpendicular to the Earth’s crust, and a ball thrown on the same trajectory (but obviously at a much slower speed) follow the same curve? If gravity is actually curved space and if falling objects are following simply the natural curves of space why does each object have its own curve?

Keywords: Curvature, Gravity, Space curve, Gravitational Pull, Forces of attraction, Earth’s Crust

1. Introduction

The whole space of universe is surrounded with dust particles, planets, stars, million- billion galaxies, the curve of space always be affected and be disturbed with the presence or the interference by other cosmological effect. Whenever we walk or work, there must create a curvature. If gravity is actually curved space and if falling objects are following simply the natural curves of space why does each object have its own curve? Here it is obvious that each effects create its own curve, and always new curves come in to the existence.

The gravitational deflection of light deduced from applying the equivalence principle is stated to be only half the general relativistic result. It is seen that the correct result is obtained entirely from applying the equivalence principle if the wave nature of light and particles is taken into account. This is equivalent to incorporating the gravitational red shift in the calculation. In Einstein’s 1911 derivation, he used the wave aspect to get half the general relativistic value, but perhaps presumed that it was physically identical to the part that came from the application of the equivalence principle to the ray trajectory. Here, it has been pointed out that the two contributions are independent and have different physical origin and characteristics. Adding the two contributions together gives the correct general relativistic value for light and particles. For material particles, the ‘particle’ part dominates, and for relativistic quantum particles, the deflection approaches that for light. Total deflection is the sum of the Newtonian deflection of the mean trajectory and the gradual bending of the wave fronts due to the gravitational red shift, and both these follow from the equivalence principle. The result stresses the fact that the physical results of general relativity are potentially fully contained in the physics of Lorentz transformations combined with the equivalence principle.

For example, we might fire two different bullets from the same gun, with each bullet made of a different material. Or we might fire a bullet and a cannonball from the same point in space, with each starting out at the same speed. It turns out that in these cases, if no other forces such as air resistance act on the objects being fired, they will follow the exact same paths and hit the ground at the exact same time and exact same place. This is completely consistent with Einstein’s theory -- the path that an object takes through space-time doesn’t depend on the mass of the object or on the material it is made of; it only depends on the initial direction that the object starts off in.

In some sense, therefore, what Einstein's theory tells us is that we really need to consider space-time, rather than space, as the fundamental "playing field" upon which the events of the universe occur. It is a profound realization to understand this fact -- all the objects around us actually exist in a realm of "space-time" that is much more complex than the simple realm of space in which we perceive them.

Further, the identification of the gravitational deflection of light as the crucial test of general relativity, in contrast to the gravitational red shift, does not seem defendable. The gravitational deflection of light near a massive body of mass M was derived by Einstein in his 1911 precursor paper to General Theory of Relativity as

\[ a = \frac{2GM}{R^2} \]  \hspace{1cm} (1.1)

R is the impact parameter, and it is equal to the radius of the body for grazing incidence. This is half the correct general relativistic value. The full general relativistic result for the deflection angle is

\[ a = \frac{4GM}{c^2R} \]  \hspace{1cm} (1.2)

In some sense, therefore, what Einstein's theory tells us is that we really need to consider space upon which the events -time, rather than space, as the fundamental
"playing field" of the universe occur. It is a profound realization to understand this fact -- all the objects around us actually exist in a realm of "space-time" that is much more complex than the simple realm of space in which we perceive them.

For deriving the expression for the deflection of quantum particles, consider the matter wave propagating in a gravitational field (the derivation is applicable to any wave). The matter wave obeys the same equation (eq. 10) of frequency redshift as for light,

\[ n' = n(1 + gh/c^2) \] .......................... 1.3

The angle of deflection can be calculated by integrating the phase over the entire trajectory to find the path difference between two points on the wave front separated by a distance h and then dividing by h. Noting that h or [r1-r2] ≪ R and that the angle q is essentially the same for both the vectors r1 and r2 and phase difference is

\[ \Delta = \int_{-\infty}^{\infty} w(r1) dt - \int_{-\infty}^{\infty} w(r2) dt \]

\[ = \frac{-2G M_{w0}}{c^2} \int_{-\infty}^{\infty} \frac{dr}{r^2} \left[ 1 - \frac{1}{r^2} \right] dx \]

\[ = \frac{-2G M_{w0}}{c^2} \int_{-\infty}^{\infty} \frac{h}{c \sinh dx} \]

\[ = \frac{-2G M_{h.w}}{yc^2} \] .......................... 1.4

where \( r^2 = x^2 + y^2 \) and h = [r1-r2]

2. Asymptotic Curvature

As we know an asymptote of a curve decides the variation, nature and limits of a curve. The crest and trough of a curve must be followed by a tangent line towards the crest with point of contact on it and a straight line towards the trough meeting the curve at infinity. Such a line parallel to a curve fixes the area and volume of the curve in three dimension and with the increasing in dimension the curve nature in space may have wide diversities. So, after detecting the curve with asymptotes and tangent, the whole may describe the physical identity of the objects, such curves create the virtual objects in the space-time.

If \( y = f(x,t) \) be a curve in the space time , the graph in 2.1 plotted below has several crests and troughs, the tangents at crest determine the nature of fluctuation of the curve parallel to x-axis and the troughs play the vital role on interfering the another types of curves given below as series 1 and 2.

The curve shown in series 1 propagates in space with time and it provides the idea of shape, size and nature of asymptotic curvature parallel to x-axis and the crests.

3. Conclusions

Thus, we can observe several types of changing curve with the change of time. Same objects throwing in space with different speed in different time will create the different trajectories. If a cricketer throws a ball construct a projectile. If a boy slip in inclined makes a cycloid. If we throw a small stone in a pond or river create concentric circles. In case of stars, planets, galaxies etc all are moving in elliptical orbit always occupying its place on the foci of the path.

References


Author Profile


Dr Jitrendra Kumar, received Ph.D in 2004 from BRA Bihar University, Muzaffarpur, Bihar, under the supervision of Dr Pradeep kumar, University Professor and head of the deptt Mathematics. And now working as Headmaster in SSHS plus 2 areraij, Bihar.