An Inventory Model with Price Dependent Demand Rate, Quadratic Holding Cost and Deteriorating Rate with Shortages

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Abstract: ***In this paper, a deterministic inventory model has been developed with a selling price dependent demand rate. A quadratic holding cost is being considered of time, in which shortages are allowed and is completely backlogged. The objective of this model is to minimize the total inventory cost. At the end of the paper, numerical examples are provided to illustrate the problem and sensitivity analysis they have been carried out to show the effect of variation in the parameters with the graphs.***

Keywords: Inventory, demand, Selling Price, Deteriorating items, Holding Cost Optimum cost, Inflation

1. Introduction

Normally in the economic order quantity model, the demand is either constant or time dependent, but independent of stock status. However, in the market situation customers are attracted by display of units in the market. The presence of inventory has a motivational effect on the people around it and attracts the people to buy them. In classical inventory models the demand rate is assumed to be constant but in real life, demand may be price dependent. Hence the selling price demand rate is important factor in every inventory model.

Most of the classical inventory model would not take into account the effect of inflation and time value of money. Symbolically it was being considered that inflation will not influence the cost and price components to any of the significant degree. However, in the last several years most of the countries have suffered from large-scale inflation and shape decline in the purchasing power of money. As a result while determining the optimal inventory policy, the effects of inflation and time value of money cannot be ignored.

The recent research works has been done for inventory model in determining the deteriorating selling price. Further study has been made with quadratic holding cost including time variation and dependent demand rate with deterministic inventory model with shortages. For examples, Burewell [2] has developed economic lot size model for price dependent demand under quantity and freight discounts. Further studies have been developed for the procedures which determine quantum lot size and selling price when demand depends on price and are unit quantity discounts are being offered.

Bhunia and Maiti [1] have found the inventory model for selling price demand and linearly time dependent demand. Furthermore investigation has been made for deterministic inventory model in deteriorating item which is being analyzed by demand rate with instant of time being the function of selling price and for that, shortages are allowed and being backlogged. Teng and Chang [14] have considered the economic production quantity model for deteriorating items with stock level and selling price dependent demand. Further investigative study has been made on a lot of goods being used by consumer displayed in supermarkets are occasionally being associated with regular sale items for inducing maximum. Dr. Kapil Kumar Bansal [6] has developed an inventory model for deteriorating items with the effect of inflation. Further investigation has marked on the effect of inflation, for which deterioration rate has been considered time dependent. Mohan [9] has developed a quadratic demand variable holding cost with time dependent deterioration without shortages and salvage value. Furthermore for the conditioned paper recognized an attempt for studying deterministic inventory models for deteriorating item with the inclusion of variable holding cost.

Varsha Sharma and Anil Kumar Sharma [15] have found a deterministic inventory model with selling price dependent demand rate, quadratic holding cost and quadratic time varying deteriorating rate. Further investigation has been made for deterministic inventory model for deteriorating products having demand which is function of selling price, holding cost is considered as a quadratic function of time. Since most of the authors have put their work on inventory model of analysis following shortages of dependent demand, hence with this preview of work, the present study deals with inventory model in which the deterioration rate and holding cost are quadratic, shortages are allowed and is completely backlogged. The model is solved for minimizing the total cost.

2. Paper Structure

- The demand rate is the function of selling price $D\{p(t)\} = a - bf(t)$ where 'a ' is fixed demand; a, b > 0 and a>>b
- P(t) is the selling price of the item at the time 't' and consider as P(t) = pe^{rt} is the selling price per unit at time t

- There is no repair or replacement of the deteriorated unit during the cycle time under consideration.
- Shortages are allowed and completely backlogged.
- Lead time is zero.
- r is the inflation rate.
- I(t) is the level of inventory at any instant of time ‘t’.
- C is the unit purchase cost & K is the ordering cost per order.
- C_1 is the shortage cost per unit time.
- T is the length of the cycle.
- t_1 is the length of the of period with positive stock of the time.
- The holding cost is quadratic with time dependent $H(t)=h+\delta t+\Delta t^2$ h >0, δ >0, Δ >0.
- The deterioration rate is time varying $\theta(t)=\alpha+\beta t+\gamma t^2$ β >0, t >0, α ≥ 0, ϒ > 0

3. Model Development

A variable function $\theta(t)$ of on hand inventory deteriorates per unit time. In the present model, the function $\theta(t)$ is assumed quadratic of the form $\theta(t)=\alpha+\beta t+\gamma t^2$ β > 0, t >0, α ≥ 0, ϒ > 0

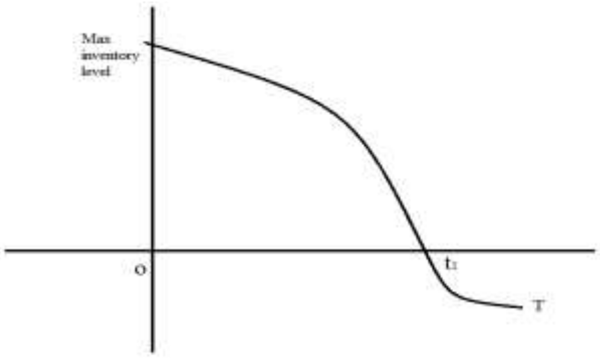


Figure 1

H(t) holding cost of the item at time t

$H(t)=h+\delta t+\Delta t^2$ h > 0, δ >0, Δ > 0 Generally the inventory level decreases mainly due to demand and due to deterioration of units, the differential equation governing the system in the interval (0 , T) is given by

$$\frac{dI(t)}{dt}=-\theta(t)I(t)-D\{p(t)\},0\le t<T \quad (1)$$

$$=-\left(\alpha+\beta t+\gamma t^2\right)I(t)-\left(a-bpe^{rt}\right) \quad (2)$$

Solution of the differential equation after adjusting constant of integration and initial condition, t = 0, $I(t)=I(0)$

$$I(t)=\exp\left\{-\left(\alpha t+\beta\frac{t^2}{2}+\gamma\frac{t^3}{3}\right)\right\}\left[-a\left(t+\alpha\frac{t^2}{2}+\beta\frac{t^3}{6}+\gamma\frac{t^4}{12}\right)\right]$$

$$+bp$$

$$\left\{t+\left(\frac{\alpha+r}{2}\right)t^2+\left(\frac{\beta+2rd+r^2}{6}\right)t^3\right.$$

$$+\left(\frac{\gamma}{12}+\frac{r\beta}{8}+\frac{r^2\beta}{8}+\frac{r^3}{24}\right)t^4$$

$$+\left(\frac{\gamma r}{15}+\frac{r^2\beta}{20}+\frac{r^2\alpha}{30}\right)t^5+\left(\frac{r^2\gamma}{36}+\frac{\beta r^3}{72}\right)t^6+\frac{\gamma r^3}{126}t^7$$

$$\left.+\left(\frac{\gamma r}{15}+\frac{r^2\beta}{20}+\frac{r^2\alpha}{30}\right)t^5+\left(\frac{r^2\gamma}{36}+\frac{\beta r^3}{72}\right)t^6+\frac{\gamma r^3}{126}t^7\right\}+I(0) \quad (3)$$

Inventory without decay $I_W(t)$ at time ‘t’ is given by

$$\frac{d}{dt}I_W(t)=-\left(a-bpe^{rt}\right)$$

$$\Rightarrow I_W(t)=-at+\frac{bpe^{rt}}{r}+I_0-\frac{bp}{r} \quad (4)$$

[Using initial condition at $t=0$, $I(t)=I(0)$]

The stock loss $Z(t)$ due to decay in [0, T] is given by

$$Z(t)=I_W(t)-I(t)$$

$$=-at-\frac{bp\left(1-e^{rt}\right)}{r}+I(0)-I(t) \quad (5)$$

Equation (3) gives

$$I(0)$$

$$=I(t)\exp\left(\alpha t+\beta\frac{t^2}{2}+\gamma\frac{t^3}{3}\right)+a\left(t+\alpha\frac{t^2}{2}+\beta\frac{t^3}{6}+\gamma\frac{t^4}{12}\right)$$

$$-bp$$

$$\left\{t+\left(\frac{\alpha+r}{2}\right)t^2+\left(\frac{\beta+2rd+r^2}{6}\right)t^3+\left(\frac{\gamma}{12}+\frac{r\beta}{8}+\frac{r^2\beta}{8}+\frac{r^3}{24}\right)t^4\right.$$

$$\left.+\left(\frac{\gamma r}{15}+\frac{r^2\beta}{20}+\frac{r^2\alpha}{30}\right)t^5+\left(\frac{r^2\gamma}{36}+\frac{\beta r^3}{72}\right)t^6+\frac{\gamma r^3}{126}t^7\right\} \quad (6)$$

Substituting value of $I(0)$ from (6) in equation (5), we get

$$Z(t)=-at-\frac{bp\square\left(1-e^{\square rt}\right)}{r}$$

$$+I(t)\left[\exp\left(\alpha t+\beta\frac{t^2}{2}+\gamma\frac{t^3}{3}\right)-1\right]$$

$$+a\left(t+\alpha\frac{t^2}{2}+\beta\frac{t^3}{6}+\gamma\frac{t^4}{12}\right)$$

$$-bp\left\{t+\left(\frac{\alpha+r}{2}\right)t^2+\left(\frac{\beta+2rd+r^2}{6}\right)t^3+\left(\frac{\gamma}{12}+\frac{r\beta}{8}+\frac{r^2\beta}{8}+\frac{r^3}{24}\right)t^4\right.$$

$$\left.+\left(\frac{\gamma r}{15}+\frac{r^2\beta}{20}+\frac{r^2\alpha}{30}\right)t^5+\left(\frac{r^2\gamma}{36}+\frac{\beta r^3}{72}\right)t^6+\frac{\gamma r^3}{126}t^7\right\} \quad (7)$$

At t = T, we get

$$Z(t) = -aT - \frac{bp(1-e^{rT})}{r} + a\left(T + \alpha\frac{T^2}{2} + \beta\frac{T^3}{6} + \gamma\frac{T^4}{12}\right) - bp\left\{T + \left(\frac{\alpha+r}{2}\right)T^2 + \left(\frac{\beta+2rd+r^2}{6}\right)T^3 + \left(\frac{\gamma}{12}+\frac{r\beta}{8}+\frac{r^2\beta}{8}+\frac{r^3}{24}\right)T^4 + \left(\frac{\gamma r}{15}+\frac{r^2\beta}{20}+\frac{r^2\alpha}{30}\right)T^5 + \left(\frac{r^2\gamma}{36}+\frac{\beta r^3}{72}\right)T^6 + \frac{\gamma r^3}{126}T^7\right\} \quad (8)$$

Note that $I(t) = 0$

Order quantity is given by

$$Q_T = Z(T) + \int_0^T (a - bpe^{rt})dt = a\left(T + \alpha\frac{T^2}{2} + \beta\frac{T^3}{6} + \gamma\frac{T^4}{12}\right) - bp\left\{T + \left(\frac{\alpha+r}{2}\right)T^2 + \left(\frac{\beta+2rd+r^2}{6}\right)T^3 + \left(\frac{\gamma}{12}+\frac{r\beta}{8}+\frac{r^2\beta}{8}+\frac{r^3}{24}\right)T^4 + \left(\frac{\gamma r}{15}+\frac{r^2\beta}{20}+\frac{r^2\alpha}{30}\right)T^5 + \left(\frac{r^2\gamma}{36}+\frac{\beta r^3}{72}\right)T^6 + \frac{\gamma r^3}{126}T^7\right\} \quad (9)$$

Also $I(0) = Q_T$ implies

$$I(t) = \exp\left[-\left(\alpha t + \beta\frac{t^2}{2} + \gamma\frac{t^3}{3}\right)\right] [a\left((T-t) + \frac{\alpha(T^2-t^2)}{2} + \beta\frac{(T^3-t^3)}{6} + \frac{\gamma}{12}(T^4-t^4)\right) - bp\left\{(T-t) + \left(\frac{\alpha+r}{2}\right)(T^2-t^2) + \left(\frac{\beta+2rd+r^2}{6}\right)(T^3-t^3) + \left(\frac{\gamma}{12}+\frac{r\beta}{8}+\frac{r^2\beta}{8}+\frac{r^3}{24}\right)(T^4-t^4) + \left(\frac{\gamma r}{15}+\frac{r^2\beta}{20}+\frac{r^2\alpha}{30}\right)(T^5-t^5) + \left(\frac{r^2\gamma}{36}+\frac{\beta r^3}{72}\right)(T^6-t^6) + \frac{\gamma r^3}{126}(T^7-t^7)\right\} \quad (10)$$

As stated earlier the holding cost is assumed to be a quadratic function of time i.e.

$$H(t) = h + \delta t + \Delta t^2$$

Shortage cost

$$\frac{dI(t)}{dt} = -\theta(t)I(t) - D\{p(t)\}, 0 \le t < T$$

$$\frac{dI(t)}{dt} = -\theta(t)I(t) - D\{p(t)\}, 0 \le t < t_1$$

$$\frac{dI(t)}{dt} = -(a - bpe^{rt}), t_1 \le t \le T$$

With boundary condition $I(t) = 0$ at t = t_1

$$I(t) = -at + \frac{bpe^{rt}}{r} + I_0 - \frac{bp}{r}$$

The total amount of shortages cost during period [0, t_1] is given by

$$SC = -C_1\int_{t_1}^{T} I(t)dt$$

$$SC = -C_1\left[\frac{a}{2}(t_1^2 - T^2) - \frac{bp}{r^2}(e^{rt_1} - e^{rT}) - I_0(t_1 - T) + \frac{bp}{r}(t_1 - T)\right] \quad (11)$$

In this case

$$C(T,P) = \frac{K}{T} + \frac{CQ_T}{T} + \frac{1}{T}\int_0^T h(t)I(t)dt = \frac{K}{T} + C\left[a\left(1 + \frac{\alpha T}{2} + \frac{\beta T^2}{6} + \gamma\frac{T^3}{12}\right) - bp\left\{1 + \left(\frac{\alpha+r}{2}\right)T + \left(\frac{\beta+2rd+r^2}{6}\right)T^2 + \left(\frac{2\gamma+3r\beta+3r^2\alpha+r^3}{24}\right)T^3\right\}\right]$$

$$+h(a-bp)\frac{T}{2} + \left(\frac{a-bp}{6}\right)(h\alpha+\delta)T^2 - \frac{bprh}{3}T^2 + \left(\frac{2h\beta+a\delta}{24}\right)(a-bp)T^3 - \frac{bpr}{8}h\alpha T^3 - \frac{bpr^2T^3}{8} - \frac{bprT^3}{8} + \left(\frac{\Delta}{12} - \frac{h\alpha^2}{8}\right)(a-bp)T^3 - C_1\left[\frac{a}{2}(t_1^2 - T^2) - \frac{bp}{r}(e^{rt_1} - e^{rT}) - I_0(t_1 - T) + \frac{bp}{r}(t_1 - T)\right] \quad (12)$$

For minimum total average cost the necessary criterion is

$$\frac{d}{dt}\{C(T,P)\} = 0$$

$$\Rightarrow -K + T^2C\left[a\left(\frac{\alpha}{2} + \frac{\beta T}{3} + \gamma\frac{T^2}{4}\right) - bp\left\{\left(\frac{\alpha+r}{2}\right) + \left(\frac{\beta+2rd+r^2}{6}\right)T + \left(\frac{2\gamma+3r\beta+3r^2\alpha+r^3}{24}\right)T^2\right\}\right]$$

$$+h(a-bp)\frac{T^2}{2}+\left(\frac{a-bp}{6}\right)(h\alpha+\delta)T^2-\frac{bprh}{3}T^2$$
$$+\left(\frac{2h\beta+a\delta}{24}\right)(a-bp)T^3-\frac{bpr}{8}h\alpha T^3-\frac{bpr^2T^3}{8}-\frac{bprT^3}{8}$$
$$+\left(\frac{\Delta}{12}-\frac{h\alpha^2}{8}\right)(a-bp)T^4-C_1\left[-aT+\frac{bp}{r}\left(e^{rt}-1\right)+I_0\right]$$

(13)

For fixed p which can be solved for $C(T,P)$ numerically by using theory of equations.

$$\frac{d^2\left[C(T,P)\right]}{dT^2}$$
$$=C\left[a+bp\left\{(\alpha+r)T+T^2(\beta+2r\alpha+r)+\left(\frac{2\gamma+3r\beta+3r^2\alpha+r^3}{2}\right)T^3\right\}\right.$$
$$+h(a-bp)T+(a-bp)(h\alpha+\delta)T^2-2bprhT^2$$
$$+\frac{3}{8}(2h\beta+a\delta)(a-bp)T^2+\frac{3}{2}bprh\alpha T^3+\frac{3}{2}bpr^2T^3$$
$$\left.+\frac{3}{2}bpr\delta T^3+12\left(\frac{\Delta-3h\alpha^2}{24}\right)(a-bp)T^3\right]-C_1\left[-a+bpe^{rt}\right]$$

(14)

Furthermore, Equation (14) shows that C (T, P) is convex with respect to T, so for a given positive integer T, the optimal value of P can be obtained from equation (13).

$$\frac{d^2\left[C(T,P)\right]}{dT^2}>0$$

4. Numerical Example

To illustrate the results obtained for the suggested model, a numerical example with the following parameter value is considered using Matlab.
Let α = 0.002, β = 1, ϒ = 2, a = 1.5, b = 1.9, p = 2, r = 0.52, T = 0.75, C = 2.5, δ = 0.7, Δ = 2, h = 0.52, C_1 = 0.50, I_0 = 1, t_1 = 0.75, K = 249.086, substituting these values in equation (12), the optimal solution (total inventory cost) is obtained as 322.0961

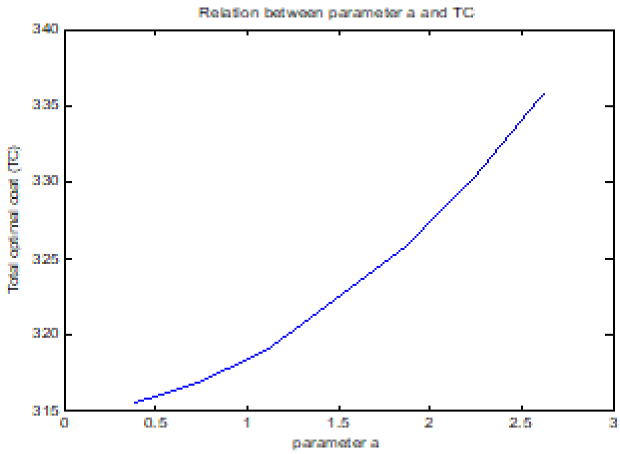
5. Sensitivity Analysis

Now we will test the sensitivity of the optimal with respect to demand parameter a & b and α, β, ϒ in deterministic model.

Table 1: Relation between demand parameter a and optimal cost (TC)

Parameter a	% change	Value of the parameter a	Optimal cost or total cost for deterministic model (TC)
1.5	75%	2.625	335.7639
1.5	50%	2.25	330.4221
1.5	25%	1.875	325.8661
1.5	-25%	1.125	319.1119
1.5	-50%	0.75	316.9136
1.5	-75%	0.375	315.5012

From the table-1, we conclude that, when the value of the parameter a decreases, the total cost of the deterministic model decreases.



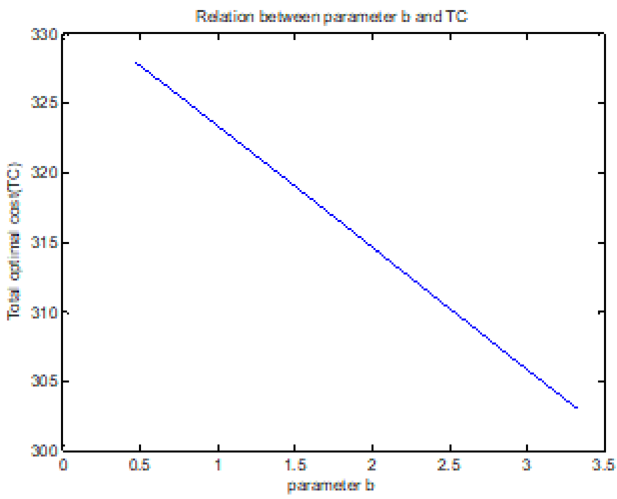
Graph 1

Graph1: Shows that if parameter a increases then the total cost increases.

Table 2: Relation between b and (TC)

Parameter b	% change	Value of the parameter b	Optimal cost or total cost for deterministic model (TC)
1.9	+75%	3.325	303.0268
1.9	+50%	2.85	307.1850
1.9	+25%	2.375	311.3431
1.9	-25%	1.425	319.6593
1.9	-50%	0.95	323.8175
1.9	-75%	0.475	327.9756

From the table- 2, we conclude that, when the value of the parameter b decreases, the total cost of the deterministic model increases.



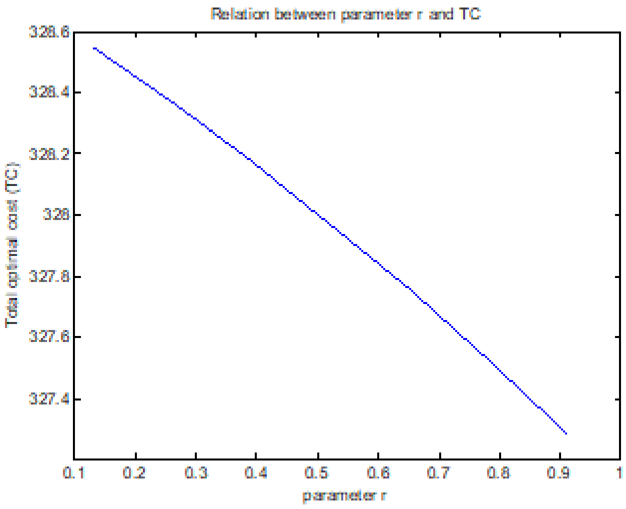
Graph 2

Graph 2: Shows that if parameter b increases then the total cost decreases.

Table 3: Relation between parameter r(i. e. Inflation rate) and (TC)

Parameter r	% change	Value of the parameter r	Optimal cost or total cost for deterministic model (TC)
0.52	+75%	0.91	327.2876
0.52	+50%	0.78	327.5305
0.52	+25%	0.65	327.7596
0.52	-25%	0.39	328.1790
0.52	-50%	0.26	328.3706
0.52	-75%	0.13	328.5508

From the table -3, we conclude that when the value of the parameter r decreases, the total cost of the deterministic model increases.



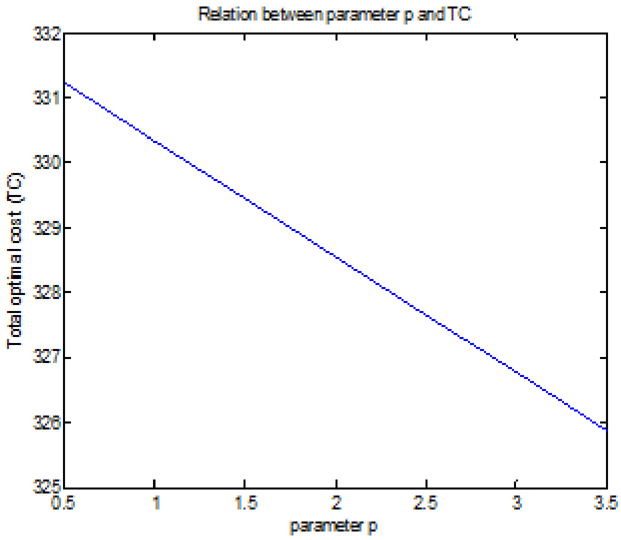
Graph 3

Graph 3: Shows that if parameter r increases then the total cost decreases.

Table 4: Relation between parameter p and (TC)

Parameter p	% change	Value of the parameter p	Optimal cost or total cost for deterministic model (TC)
2	+75%	3.5	325.8672
2	+50%	3	326.7617
2	+25%	2.5	327.6563
2	-25%	1.5	329.4453
2	-50%	1	330.3398
2	-75%	0.5	331.2344

From the table -4, we conclude that when the value of the parameter p decreases, the total cost of the deterministic model increases.



Graph 4

Graph 4: Shows that if parameter p increases then the total cost decreases

6. Conclusion

The prime objective of this study is the formulation of a deterministic inventory model for the deteriorating items under price dependent demand rate and time dependent holding cost , when the supplier offers a trade credit for a specified period. Deterioration is taken as quadratic function of time. The model is solved for cost minimization.

References

[1] Bhunia, A.K. and Maiti, M. (1997): An inventory model for decaying items with selling price, frequency of advertisement and linearly time dependent demand with shortages, IAPQR Transactions, Vol. 22, pp. 41-49.
[2] Burewell, T.H. (1997): Economic lot size model for price dependent demand under quantity and freight discounts; International J. of Prod. Eco., Vol. 48, pp. 141-155.
[3] Deb, M.M. and Chaudhari, K.S. (1986): An EOQ model for items with finite rate of production and variable rate of deterioration, Opsearch, Vol. 23, pp. 175-181.
[4] Goh, M. (1994): EOQ Models with general demand and holding cost functions, European journal of Operational Research, Vol. 73, pp. 50-54.
[5] Ghare, P.M. and Schrader, G.H. (1963): A model for exponentially decaying inventory systems, Int. Prod. Res., Vol. 21, pp. 449-460.
[6] Dr. Kapil Kumar Bansal, (2013): Inventory model for deteriorating items with the effect of inflation. IJAIEM Vol. 2, pp. 143-150.
[7] Kumar Vipin, and Pathak Gopal, and Gupta, C.B. (2013): A deterministic inventory model for deteriorating items with selling price dependent demand and parabolic time varying holding cost under Trade Credit. IJSCE, Vol. 3, ISSN: 2231-2307.
[8] Mandal, M. and De, S.K. (2012): An EOQ model with parabolic demand rate and time varying selling price.

Analysis of pure and applied Mathematics Vol.1, pp. 32-43.
[9] Mohan, R. (2017): Quadratic demand, variable holding cost with time dependent deterioration without shortages and salvage value. IOSR-JM Vol.13, pp. 59-66.
[10] Philip, G.C. (1974): A generalized EOQ model for items with weibull distribution, AIIE Trans., Vol. 6, pp. 159-162.
[11] Roy, A. (2008): An inventory Model for deteriorating items with price dependent demand and time varying holding cost, AMO, Vol. 10, pp. 25-37.
[12] Sahoo, N.K. and Sahoo, C.K. and Sahoo, S.K. (2010): An inventory model for constant deteriorating items with price dependent demand and time varying holding cost; International J. Comp. Sci., Commun., Vol. 1(1), pp. 267-271.
[13] Sharma, A.K. and Sharma, M.K. (2012): An inventory model with weibull distribution deteriorating items time dependent demand with salvage value and time dependent holding cost. Vijanana parishad Anusandhan patrika, Vol. 55, pp. 55-64.
[14] Teng, J.T. and Chang, H.T. (2005): Economic production quantity model for deteriorating items with price and stock dependent demand; Computers and Oper. Res., Vol. 32, pp. 279-308.
[15] Varsha Sharma, Anil Kumar Sharma, (2017): A deterministic inventory model with selling price dependent demand rate, quadratic holding cost and quadratic time varying deteriorating rate, IISR vol.6, pp.193-198.
[16] Weiss, H.J., (1982): Economic order Quantity models with nonlinear holding cost, European journal of operational Research, Vol. 9, pp. 56-60.
[17] Yadav, Smita, and Aggarwal, N.K. (2014): A deterministic inventory model for deteriorating items with selling price dependent demand and the time dependent deteriorating under inflation with variable holding cost. IJMCR Vol.2, pp. 484-489.