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Comparing Different Bayes Estimator of Shape Parameter for Burr Type X

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Abstract: ***This paper deals with obtaining a new probability distribution from powering some given distribution to obtain a family that is more flexible for representing data, then we derive four Bayes estimators under exponential priors, also four Bayes estimators under Jeffery prior, then the comparison has been done through simulation, the results are compared using statistical measures (mean square error).***

Keywords: A new family Burr type X, Generalized Rayleigh, Bayes I, Bayes II, Bayes D, Bayes P under exponential prior, Jeffery prior, MSE. Risk function

1. Introduction

Many new probability distribution can be obtained from powering the CDF of any given distribution to a positive real number (λ), this gives new family which may be more flexible and fitting for the observation of random variable. The new CDF [G(z)] obtained from any given CDF [F(z)] is;

$$G(z) = [F(z)]^{\lambda} \qquad \lambda > 0 \tag{1}$$

We know that the generalized Rayleigh distribution, have the CDF;

$$F(x;\alpha,\lambda) = \left(1 - e^{-(\alpha x)^2}\right)^{\lambda} \qquad x,\alpha,\lambda > 0 \tag{2}$$

We assume the scale parameter ($\alpha = 1$), then;

$$F(x;\lambda) = \left(1 - e^{-x^2}\right)^{\lambda} \tag{3}$$

This CDF in equation (3) called (Burr type X) distribution with p.d.f;

$$f(x;\lambda) = 2\,\lambda\,x\,e^{-x^2}\left(1 - e^{-x^2}\right)^{\lambda-1} \qquad x,\lambda > 0 \tag{4}$$

2. Aim of Research

We work on comparing four Bayes estimators for parameter (λ) of the distribution (Burr type X), using different model of prior distribution for (λ), since the Bayes estimators depend on the theory that the parameter to be estimated is considered random variable and having prior distribution $\{g(\lambda)$, [Thomas Bayes (1702)], the comparison has been done through simulation procedure using different sample size.

3. Theoretical Aspect

Let $(x_1, x_2, \dots, x_3)$, be $i.d.d$ from $[f(\underline{x}|\lambda)]$ defined in equation (4), then the Bayesian estimator depend on observations represented by conditional density, which consider the parameter (λ) as a random variable, this lead to find the posterior distribution of (λ) given data $(x_1, x_2, \dots, x_3)$;

$$f(\underline{\lambda}|x) = \frac{f(\underline{x}|\lambda)g(\lambda)}{f(\underline{x})} \tag{5}$$

Where (in case of continuous);

$$f(\underline{x}) = \int_{\forall\lambda} f(x|\lambda)g(\lambda)d\lambda$$

Bayes estimator ($Bayes$) need loss function $[L(\hat{\lambda},\lambda)]$ which is the losses incurred by estimating (λ) by ($\hat{\lambda}$), then measuring the loss by difference between ($\hat{\lambda} - \lambda$), the loss function $[L(\hat{\lambda},\lambda)]$ is defined as a real valued function satisfy;

a. $L(\hat{\lambda},\lambda) \geq 0 \qquad \forall \hat{\lambda} \neq \lambda$

b. $L(\hat{\lambda},\lambda) = 0 \qquad \forall \hat{\lambda} = \lambda$

We can give some types of loss function that may use in comparing the Bayes estimator (λ);

A. Squared error loss function

This is from symmetric loss function given by Mood, Graybill, and Boes (1974), which is;

$$L(\hat{\lambda},\lambda) = (\hat{\lambda} - \lambda)^2 \tag{6}$$

The Byes estimator ($\hat{\lambda}_{Bayes\,I}$) is obtained from minimizing risk function $[R_s(\hat{\lambda},\lambda)]$.

$$R_s(\hat{\lambda},\lambda) = E(loss) = E[L(\hat{\lambda},\lambda)] = \int_{\forall\lambda} L(\hat{\lambda},\lambda)f(\lambda|\underline{x})d\lambda$$

$$R_s(\hat{\lambda},\lambda) = \int_{\forall\lambda}(\hat{\lambda} - \lambda)^2 f(\lambda|\underline{x})d\lambda \tag{7}$$

$$R_s(\hat{\lambda},\lambda) = \int_{\forall\lambda}\hat{\lambda}^2 f(\lambda|\underline{x})d\lambda - 2\int_{\forall\lambda}\hat{\lambda}\lambda f(\lambda|\underline{x})d\lambda + \int_{\forall\lambda}\lambda^2 f(\lambda|\underline{x})d\lambda$$

$$R_s(\hat{\lambda},\lambda) = \hat{\lambda}^2 - 2\hat{\lambda}E(\lambda|\underline{x}) + E(\lambda^2|\underline{x}) \tag{8}$$

Put $R_s(\hat{\lambda},\lambda) = 0$, this gives the first Bayes estimator;

$$\hat{\lambda}_{Bayes\,I} = E(\lambda|\underline{x}) \tag{9}$$

Which is the conditional mean of posterior distribution $[f(\lambda|\underline{x})]$.

B. De Groot loss function

This loss function was proposed by De Groot (1970), the Bayes estimator for the positive parameter ($\lambda > 0$) by this method obtained using the loss function;

$$L(\hat{\lambda},\lambda) = \left(\frac{\lambda-\hat{\lambda}}{\hat{\lambda}}\right)^2 \qquad (10)$$

Then the risk function and Bayes estimator ($\hat{\lambda}_{Bayes\,II}$) under this loss function is;

$$R_D(\hat{\lambda},\lambda) = \hat{\lambda}^{-2}E(\lambda^2|\underline{x}) - 2\hat{\lambda}^{-1}E(\lambda|\underline{x}) + 1$$

$$\hat{\lambda}_{De\,Groot} = \hat{\lambda}_D = \frac{E(\lambda^2|\underline{x})}{E(\lambda|\underline{x})} \qquad (11)$$

To derive the formulas Bayes estimators, we need to find the posterior [$f(\lambda|\underline{x})$] from equation (5), to simplify the work, we can rewrite the p.d.f in equation (4) as follows;

$$f(x,\lambda) = 2\,\lambda\,x\,e^{-x^2+\ln\left(1-e^{-x^2}\right)^{-1}}\,e^{-\lambda\ln\left(1-e^{-x^2}\right)^{-1}} \qquad (12)$$

Then using this equation according to two types of prior [$g_1(\lambda), g_2(\lambda)$], we find posterior [$f(\lambda|\underline{x})$], then using four types of loss function to find [$\hat{\lambda}_{Bayes}$].

D. Precautionary Loss Function

This type of loss function was given by Norstrom (1996), which use the loss function given in equation (10), then the risk function and Bayes estimator for this loss and risk function is;

$$R_P(\hat{\lambda},\lambda) = \hat{\lambda} - 2E(\lambda|\underline{x}) + \hat{\lambda}^{-1}E(\lambda^2|\underline{x}) \qquad (13)$$

Therefore the third Bayes estimator is;

$$\hat{\lambda}_{3\,Bayes} = \sqrt{E(\lambda^2|\underline{x})} \qquad (14)$$

While the fourth Bayes estimator of (λ) is obtained under entropy loss function which was proposed by Calabria and Pulcini (1994), which is;

$$L(\hat{\lambda},\lambda) = \left[(\hat{\lambda}|\lambda)^t - t\,\ln(\hat{\lambda}|\lambda) - 1\right] \qquad (15)$$

Risk function according to this loss function is;

$$R_E(\hat{\lambda},\lambda) = \hat{\lambda}^t E(\lambda^{-t}|\underline{x}) - t\ln\hat{\lambda} + tE(\lambda^2|\underline{x}) - 1 \qquad (16)$$

Therefore the fourth Bayes estimator for (λ) is;

$$\hat{\lambda}_{4\,Bayes} = E(\lambda^{-t}|\underline{x})^{-\frac{1}{t}} \qquad (17)$$

4. Bayesian Analysis

From the studied $p.d.f$ as defined in equation (4) and rewritten as equation (12) we find;

$$L(x|\lambda) = \prod_{i=1}^{n} f(x_i,\lambda)$$

$$L(\underline{x}|\lambda) = 2^n\lambda^n exp\left[\sum_{i=1}^{n}\ln x_i - \sum_{i=1}^{n}x_i^2 + \sum_{i=1}^{n}\ln\left(1-e^{-x_i^2}\right)^{-1}\right] exp\left[-\lambda\sum_{i=1}^{n}\ln\left(1-e^{-x_i^2}\right)^{-1}\right]$$

Let;

$$K = 2^n exp\left[\sum_{i=1}^{n}\ln x_i - \sum_{i=1}^{n}x_i^2 + \sum_{i=1}^{n}\ln\left(1-e^{-x_i^2}\right)^{-1}\right] \qquad (19)$$

$$g_1(x) = \sum_{i=1}^{n}\ln\left(1-e^{-x_i^2}\right)^{-1} \qquad (20)$$

First of all we find the posterior distribution [$h(\lambda|\underline{x})$];

$$h_1(\lambda|\underline{x}) = \frac{\prod_{i=1}^{n} f(x_i,\lambda)g(\lambda)}{\int_{\forall\lambda}\prod_{i=1}^{n} f(x_i,\lambda)g(\lambda)d\lambda} \qquad (21)$$

Assuming that (λ) is random variable have exponential distribution with parameter (b);

$$g(\lambda) = \begin{cases} b\,e^{-\lambda b} & \lambda>0 \\ 0 & b>0 \\ 0 & o/w \end{cases} \qquad (22)$$

Then using $p.d.f$ in equation (4), the prior in (22), applying (21) we find the equation of posterior distribution [$h(\lambda|\underline{x})$] under this assumed prior (22), i.e;

$$h_1(\lambda|\underline{x}) = \frac{[g_1(x)+b]^{n+1}}{\Gamma(n+1)}\lambda^n e^{-\lambda[g_1(x)+b]} \qquad (23)$$

According to the posterior [$h_1(\lambda|\underline{x})$] and types of loss function selected, we find that [$\hat{\lambda}_{Bayes}$] formulas are;

$\hat{\lambda}_{Bayes\,I}$ under squared error loss function is;

$$\hat{\lambda}_{Bayes\,I} = \frac{n+1}{g_1(x)+b} \qquad (24)$$

Risk function under this estimator;

$$\hat{R}_{Bayes\,I} = \frac{n+1}{[g_1(x)+b]^2} \qquad (25)$$

While the second type of Bayes estimator ($\hat{\lambda}_{2D}$) under De Groot loss function is;

$$\hat{\lambda}_{2D} = \frac{n+2}{g_1(x)+b} \qquad (26)$$

$$\hat{R}_{2D} = \frac{1}{n+2} \qquad (27)$$

While the third Bayes estimator of ($\lambda \rightarrow \hat{\lambda}_{Bayes\,3}$) according to precautionary loss function is;

$$\hat{\lambda}_{3P} = \sqrt{\frac{(n+1)(n+2)}{(g_1(x)+b)^2}} \qquad (28)$$

$$\hat{R}_{3P} = \frac{2\sqrt{(n+1)(n+2)}-(n+1)}{(g_1(x)+b)} \qquad (29)$$

The fourth Bayes estimator according to exponential prior and entropy function is;

$$\hat{\lambda}_{4E} = \sqrt[t]{\left[\frac{\Gamma(n+1-t)}{\Gamma(n+1)}(g_1(x)+b)^t\right]^{-1}} \qquad (30)$$

Which can be simplified to;

$$\hat{\lambda}_{4E} = \frac{\sqrt[t]{\frac{\Gamma(n+1)}{\Gamma(n+1-t)}}}{(g_1(x)+b)} \qquad (31)$$

$$\hat{R}_{4E} = \ln\left[\frac{\Gamma(n+1-t)}{\Gamma(n+1)}(g_1(x)+b)^t\right] + t\left[\frac{(g_1(x)+b)^{n+1}}{\Gamma(n+1)}\sum_{i=1}^{n}\frac{(-g_1}{k!\,(n-}\right.$$

Now we assume second prior which is Jeffrey's prior defined by;

$$g_2(\lambda) = \frac{k}{\lambda^3} \quad (33)$$

$$h_2(\lambda|\underline{x}) = \frac{\prod_{i=1}^{n} f(x_i,\lambda) g_2(\lambda)}{\int_{\forall\lambda} \prod_{i=1}^{n} f(x_i,\lambda) g_2(\lambda) d\lambda} = \frac{k\lambda^{n-3} e^{-\lambda g_1(\lambda)}}{\int_0^{\infty} k\lambda^{n-3} e^{-\lambda g_1(\lambda)} d\lambda} \quad (34)$$

The second posterior distribution of $(\lambda|\underline{x})$ is;

$$h_2(\lambda|\underline{x}) = \frac{[g_1(x)]^{n-3}}{\Gamma(n-3)} \lambda^{n-3} e^{-\lambda g_1(x)} \qquad \lambda > 0 \quad (35)$$

Which is also Gamma $(n-3, \lambda)$

The fourth Bayesian estimators under Jeffrey's prior (using squared error loss function) are;

a. $$\hat{\lambda}_{Bayes1J} = \frac{n-3}{g_1(x)}, \qquad \hat{R}_{Bayes1J} = \frac{n-3}{[g_1(x)]^2} \quad (36)$$

b. The second Bayes estimator under Jeffrey's using De Groot loss function is;

$$\hat{\lambda}_{BayesDJ} = \frac{n-2}{g_1(x)}, \qquad \hat{R}_{BayesDJ} = \frac{1}{n-2} \qquad n > 2 \quad (37)$$

c. The third Bayes estimator under Jeffrey's and precautionary loss function is;

$$\hat{\lambda}_{3BayesJ} = \sqrt{\frac{(n-3)(n-2)}{[g_1(x)]^2}}, \qquad \hat{R}_{3BayesJ} = \frac{2\sqrt{(n-3)(n-2)} - n}{[g_1(x)]} \qquad n > 2 \quad (38)$$

Finally the fourth Bayes estimator for (λ) using entropy loss function and Jeffery's prior $[h(\lambda) = \frac{k}{\lambda^3}]$ is;

$$\hat{\lambda}_{4BayesE} = \frac{\sqrt[t]{\frac{\Gamma(n+1)}{\Gamma(n+1-t)}}}{g_1(x)}$$

$$\hat{R}_{4BayesE} = \ln\left[\frac{\Gamma(n-t)}{\Gamma(n)}(g_1(x))^t\right] + t\left[\frac{(g_1(x))^n}{\Gamma(n)} \sum_{k=0}^{n} \frac{(-g_1(x))^k}{k!\,(n+1+k)^2}\right] \quad (39)$$

5. Simulation Procedures

To find the estimator's $(\hat{\lambda}_{Bayes1}, \hat{\lambda}_{Bayes2}, \hat{\lambda}_{Bayes3}, \hat{\lambda}_{Bayes4})$ we perform simulation experiments using Monte Carlo assuming that;

b	0.8	1	1.5
λ	0.9	1.7	3

Table 1: MSE Four Bayes ($b = 0.8, \lambda = 0.9$).

n	$\hat{\lambda}_{Bayes1}$	$\hat{\lambda}_{Bayes2}$	$\hat{\lambda}_{Bayes3}$	$\hat{\lambda}_{Bayes4}$	Best
20	0.0724	0.0251	0.0246	0.0307	$\hat{\lambda}_{Bayes3}$
40	0.0281	0.0162	0.0168	0.0211	$\hat{\lambda}_{Bayes3}$
60	0.0186	0.0101	0.0102	0.0112	$\hat{\lambda}_{Bayes2}$
80	0.0107	0.0054	0.0055	0.0061	$\hat{\lambda}_{Bayes2}$
100	0.0052	0.0049	0.0050	0.0053	$\hat{\lambda}_{Bayes2}$

Table 2: MSE Four Bayes ($b = 1, \lambda = 0.9$).

n	$\hat{\lambda}_{Bayes1}$	$\hat{\lambda}_{Bayes2}$	$\hat{\lambda}_{Bayes3}$	$\hat{\lambda}_{Bayes4}$	Best
20	0.0341	0.0250	0.0249	0.0279	$\hat{\lambda}_{Bayes3}$
40	0.0216	0.0173	0.0163	0.0181	$\hat{\lambda}_{Bayes3}$
60	0.0107	0.0103	0.0101	0.0106	$\hat{\lambda}_{Bayes2}$
80	0.0063	0.0056	0.0055	0.0057	$\hat{\lambda}_{Bayes2}$
100	0.0045	0.0042	0.0042	0.0041	$\hat{\lambda}_{Bayes4}$

Table 3: MSE Four Bayes ($b = 1.5, \lambda = 0.9$).

n	$\hat{\lambda}_{Bayes1}$	$\hat{\lambda}_{Bayes2}$	$\hat{\lambda}_{Bayes3}$	$\hat{\lambda}_{Bayes4}$	Best
20	0.1224	0.1040	0.1068	0.1348	$\hat{\lambda}_{Bayes2}$
40	0.0802	0.0682	0.0710	0.0853	$\hat{\lambda}_{Bayes2}$
60	0.0387	0.0354	0.0361	0.0407	$\hat{\lambda}_{Bayes2}$
80	0.0206	0.0200	0.0203	0.0216	$\hat{\lambda}_{Bayes2}$
100	0.0132	0.0121	0.0124	0.0138	$\hat{\lambda}_{Bayes2}$

Table 4: MSE Four Bayes ($b = 0.8, \lambda = 1.7$).

n	$\hat{\lambda}_{Bayes1}$	$\hat{\lambda}_{Bayes2}$	$\hat{\lambda}_{Bayes3}$	$\hat{\lambda}_{Bayes4}$	Best
20	0.1507	0.1071	0.1038	0.1130	$\hat{\lambda}_{Bayes3}$
40	0.1030	0.0678	0.0688	0.0750	$\hat{\lambda}_{Bayes2}$
60	0.0432	0.0350	0.0355	0.0383	$\hat{\lambda}_{Bayes2}$
80	0.0228	0.0202	0.0201	0.0211	$\hat{\lambda}_{Bayes3}$
100	0.0138	0.0122	0.0123	0.0128	$\hat{\lambda}_{Bayes2}$

Table 5: MSE Four Bayes ($b = 1, \lambda = 1.7$).

n	$\hat{\lambda}_{Bayes1}$	$\hat{\lambda}_{Bayes2}$	$\hat{\lambda}_{Bayes3}$	$\hat{\lambda}_{Bayes4}$	Best
20	0.5228	0.4503	0.4605	0.5725	$\hat{\lambda}_{Bayes2}$
40	0.3007	0.2720	0.2754	0.3213	$\hat{\lambda}_{Bayes2}$
60	0.2010	0.1741	0.1782	0.2086	$\hat{\lambda}_{Bayes2}$
80	0.1201	0.1112	0.1134	0.1231	$\hat{\lambda}_{Bayes2}$
100	0.0833	0.0804	0.0803	0.0865	$\hat{\lambda}_{Bayes3}$

Table 6: MSE Four Bayes ($b = 1.5, \lambda = 1.7$).

n	$\hat{\lambda}_{Bayes1}$	$\hat{\lambda}_{Bayes2}$	$\hat{\lambda}_{Bayes3}$	$\hat{\lambda}_{Bayes4}$	Best
20	0.6363	0.3703	0.04484	0.4567	$\hat{\lambda}_{Bayes3}$
40	0.3478	0.2436	0.2710	0.2760	$\hat{\lambda}_{Bayes3}$

60	0.2103	0.1640	0.1752	0.1802	$\hat{\lambda}_{Bayes2}$
80	0.1281	0.1070	0.1118	0.1141	$\hat{\lambda}_{Bayes2}$
100	0.0884	0.0767	0.0805	0.0806	$\hat{\lambda}_{Bayes3}$

Table 7: MSE Four Bayes ($b = 0.8, \lambda = 3$).

n	$\hat{\lambda}_{Bayes1}$	$\hat{\lambda}_{Bayes2}$	$\hat{\lambda}_{Bayes3}$	$\hat{\lambda}_{Bayes4}$	Best
20	0.1627	0.1651	0.1603	0.1682	$\hat{\lambda}_{Bayes3}$
40	0.1345	0.1330	0.1314	0.1377	$\hat{\lambda}_{Bayes3}$
60	0.1055	0.1041	0.1037	0.1072	$\hat{\lambda}_{Bayes3}$
80	0.0827	0.0818	0.0816	0.0837	$\hat{\lambda}_{Bayes3}$
100	0.0687	0.0680	0.0678	0.0704	$\hat{\lambda}_{Bayes3}$

Table 8: MSE Four Bayes ($b = 1, \lambda = 3$).

n	$\hat{\lambda}_{Bayes1}$	$\hat{\lambda}_{Bayes2}$	$\hat{\lambda}_{Bayes3}$	$\hat{\lambda}_{Bayes4}$	Best
20	0.1764	0.1485	0.1610	0.1557	$\hat{\lambda}_{Bayes2}$
40	0.1418	0.1246	0.1315	0.1302	$\hat{\lambda}_{Bayes2}$
60	0.1105	0.1002	0.1036	0.1032	$\hat{\lambda}_{Bayes2}$
80	0.0851	0.0807	0.0816	0.0814	$\hat{\lambda}_{Bayes2}$
100	0.0703	0.0666	0.0678	0.0678	$\hat{\lambda}_{Bayes2}$

Table 9: MSE Four Bayes ($b = 1.5, \lambda = 3$)

n	$\hat{\lambda}_{Bayes1}$	$\hat{\lambda}_{Bayes2}$	$\hat{\lambda}_{Bayes3}$	$\hat{\lambda}_{Bayes4}$	Best
20	0.6023	0.5116	0.5235	0.6770	$\hat{\lambda}_{Bayes2}$
40	0.4080	0.3483	0.3631	0.4417	$\hat{\lambda}_{Bayes2}$
60	0.1877	0.1745	0.1773	0.2057	$\hat{\lambda}_{Bayes2}$
80	0.1160	0.1087	0.1107	0.1207	$\hat{\lambda}_{Bayes2}$
100	0.0863	0.0815	0.0830	0.0886	$\hat{\lambda}_{Bayes2}$

6. Conclusion

1) According to different sets of initial values and different sets of sample sizes we find the best estimators are Bayes 2 and Bayes 3, where the second is called ($\hat{\lambda}_{2D}$) under De Groot loss function, the third Bayes is obtained under precautionary loss function.
2) The comparison between four estimators depends on statistical measure mean square error, $\left[MSE(\hat{\lambda}) = \frac{\sum_{i=1}^{n}(\hat{\lambda}-\lambda)^2}{R}\right]$, where (R) is the replicate of each simulation experiment.

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