Analytical Model for Construction of Interaction Diagram for RC Columns Strengthened by Steel Jacket

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Abstract: ***Different methods used for repairing or strengthening of RC columns. Strengthening with steel jacket is widely used specially for rectangular and square columns. Many studies have been carried out on this technique; most of these studies did not take the effects of bending moments into consideration. This paper presents analytical models to construct the axial load-bending moment interaction diagram of a RC column strengthened with steel jacket. The models include the confinement produced by the steel jacket into consideration. The derivation of expressions was made by assuming equivalent stress block parameters for confined concrete. The proposed models show good agreements with available experimental data. In addition, the results of proposed models were compared with design proposals and illustrate good agreement.***

Keywords: RC column; strengthening; steel jacket; interaction diagram; confinement

1. Introduction

Reinforced concrete (RC) columns are the primary load-bearing structural components in the building. Overtime these columns may need to be repaired or strengthened. Common methods used for strengthening of RC columns include concrete jacketing, fiber reinforced polymer jacketing and steel jacketing. In a special case for non-ductile RC columns, steel jacketing technique is vastly used on strengthening of RC columns [1].

Many experimental and numerical studies have been conducted on RC columns strengthened with steel jacketing under axial loads [2-5]. Studies on the formulation of interaction diagram are insufficient. Mathematical models and/or experimental works on interaction diagram formulation are required because of its necessity to check the adequacy of strengthened RC columns [6].

N-M interaction diagram (Figure 1) is a graphical representation of the ultimate strength of a column subjected to axial load (N) and uniaxial bending moment (M). For any column there is a unique interaction diagram representing failure of column.

This paper presents proposed expressions for the construction of interaction diagram for RC column strengthened with steel angles and strips. In this work and due to placing of steel cage, the effects of confinement on concrete compressive strength, stress-strain response of confined concrete and the reducing in the axial resistance of steel angles will be taken into consideration.

2. Description of Analytical Models

In this section, a hand computation of N-M interaction diagram is made adopting limit state theory. Analytical equations are developed referring to four main points (A, B,C and D), which used for constructing the interaction diagram.Two models were adopted in this paper (Model I and Model II), the only difference between the two models is the equations that used to include the confinement:

- Model I: Using Badalamenti et al. [7] equations.
- Model II: Using Campione [8] equations.

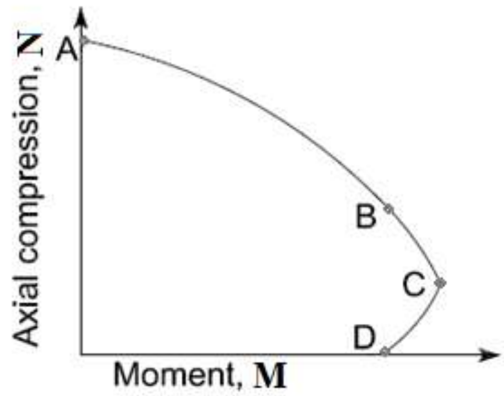

Figure 1: Main Points of N-M Interaction Diagram.

The main dimensions of the strengthened concrete column are given in Figure 2.

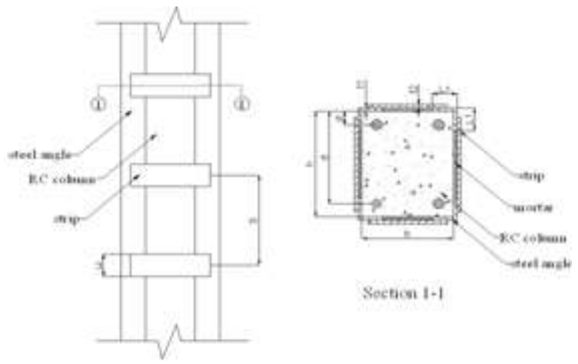

Figure 2: Details and dimensions of RC column and steel cage

For the analytical derivation of N-M interaction, direct application of basic principles of applied mechanics was adopted and as follows:

- **Compatibility**

Plain section assumed to remain plain, strain on concrete vary linearly along the section and the neutral axis located at distance c from the heaviest loading side (compression side).The strain in concrete, reinforcement bars and steel angles was assumed to be directly proportional to the distance from the neutral axis. For angles and longitudinal bars in compression and in tension it was supposed that they have to be elastic or yielded, therefore the case of over reinforced section without ductile behavior was not considered here.

The strain corresponding to the maximum concrete stress is defined according to Mander et al. [9] and given by (1):

$$\varepsilon_{cc} = 0.002 \times \left[1 + 5\left(\frac{f_{cc}}{f_c} - 1\right)\right] \qquad (1)$$

Wheref_{cc} is the compressive strength of confined concrete and $f_c$is the unconfined concrete compressive strength (Figure 1). Paulay and Priesltey [10] limited the crushing of concrete to a maximum axial strain of confined concrete (ε_{cu}):

$$\varepsilon_{cu} = 0.004 + \frac{2.8\, \varepsilon_{sh} t_2\, L_2\, f_{yh}}{f_{cc}\; S\; b} \qquad (2)$$

Where L_2 is the height of steel strip, t_2 is the thickness of steel strip, f_{yh} is the yield stress of steel strip, S is center to center distance between strips, and b is the width of the column.

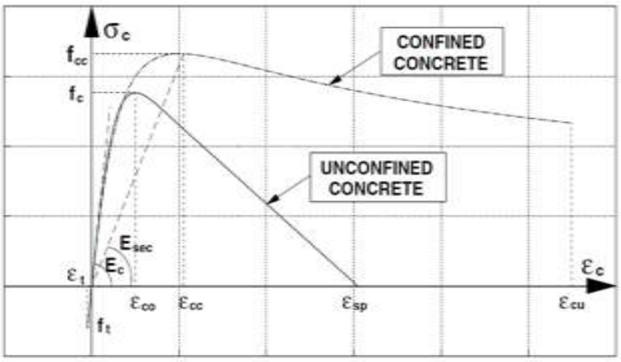

Figure 3: Stress-Strain Diagram of Concrete [11].

- **Constitutive relationships:**

Using the model proposed by Karthik and Mander [12], the actual concrete compressive stress distribution is replaced by an equivalent rectangular distribution having a width equal to $\alpha_1 f_{cc}$ and a height equal to βc as follows:

$$\beta = 0.8815 - 0.0884\,\frac{S}{b} \qquad (3)$$

$$\alpha_1 \cong 0.93$$

- **Equilibrium equations:**

Two basic equilibrium equations are used:

1) Force equilibrium between internal (concrete, reinforcing bars and steel angles) axial forces and external axial forces, by applying $\sum F = 0$.
2) Moment equilibrium by applying $\sum M = 0$ about the center line of the section between internal and external moments.

3. Main Points

Four points were used to obtain the N-M interaction diagram and as follows:

1- Point A is defined with the design value of the resistance of the composite section to compressive axial forceN_A while the bending moment is zero (eccentricity e = 0). Many investigators tried to obtain an accurate equation for the load-carrying capacity of a reinforced concrete column strengthened by steel angles and horizontal strips. The main factor implemented in these investigations was the confining effect of both vertical angles placed in the corner of the concrete column as well as the horizontal strips. In this work, the analytical model presented by Campione [8] has been used for predicting N_A using (4).

$$N_A = f_{cc}\; b\; h + n_a A_s f_{ya} + A_{sr}\, f_{yr} \qquad (4)$$

Where$n_a$isa dimensionless ratio of the maximum axial force available in the vertical angles, h is the height of the column section, A_s is the total area of steel angles, f_{ya} is the yield stress for steel angle, A_{sr} is the total area for reinforcing bars, and f_{yr} is the yield stress for reinforcing bars.

The calculus of the strength contribution of steel anglesin strengthened columns adopted a loading scheme ofcontinues beams supported by steel strips. These angels areaxially loaded and laterally loaded by the confinementpressure of concrete core. The angles on the four sides ofthe column core are in equilibrium. In this assumption, thesteel strips are acting as tension ties with small axial deformation [8].

Two approaches were used for finding the value of f_{cc} and the factor n_a and as illustrated in Table 1.

Table 1: Constitutive models for confined concrete and steel angles

Model I	$f_l = \frac{2t_2 L_2 f_{yh}}{S\; b} e^{-1.5\frac{S}{b}}$ $f_{cc} = f_c\left[1 + 3.7\left(\frac{f_l}{f_c}\right)^{0.87}\right]$ $n_a = \frac{\sqrt{t_1 f_{ya}\left(t_1 f_{ya} {L_1}^2 - 0.155\frac{4t_2 L_2 f_{yh}}{Sb} e^{-1.5\frac{S}{b}} L_1 S^2\right)}}{2L_1 t_1 f_{ya}}$ ≤ 1
Model II	$f_{cc} = f_c\left[1 + 1.42\left(\frac{4t_2 L_2 f_{yh}}{S\, b f_c} e^{-1.5\frac{S}{b}}\right)^{0.87}\right]$ $n_a = \sqrt{1 - \frac{0.63\; S e^{-1.5\frac{S}{b}}}{t_1 b\left(\frac{L_1}{S t_1} + \frac{0.5b - L_1}{t_2 L_2}\right)}}$

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Where L_1 is the length of steel angle's leg, and t_1 is the thickness of steel angle.

2- Point C is defined with the maximum design value of the resistance moment M_C in the presence of a compressive normal force N_C. Both compression and tension reinforcement are yielded. Strain at end of vertical angle's leg in both of compression and tension sides must be checked and compared with the yielding value. Most cases both of compression and tension angles are yielded and this assumption has been used in this paper.

$$N_C = \alpha f_{cc}\ a\ b \tag{5}$$

$$M_C = M_{max} = \alpha f_{cc}\left[\frac{bh^2}{8}\right] + Z_a f_{ya} + f_{yr} A_{sr}\left[d - \frac{h}{2}\right] \tag{6}$$

Where a is the height of equivalent stress block ($a = \beta c = h/2$), d is the effective depth for reinforcing bars and Z_a is the plastic section modulus for steel angles.

$$Z_a = 4\left[\left[(L_1 - t_1)t_1\left(\frac{h+t_1}{2}\right)\right] + \left[L_1 t_1\left(\frac{h}{2} + t_1 - \frac{L_1}{2}\right)\right]\right] \tag{7}$$

3- Point D is defined with the design value of the bending moment resistance of the composite section M_D while the axial force is zero (eccentricity e = ∞). The position of neutral axis must be assumed and to be checked later. By applying $\sum F = N_D = 0$, the position of neutral axis can be found. Two possible assumptions have been considered in this paper and as follows:

1) $(L_1 - t_1) \le c < h/2$

In this case (Figure 4), strain in reinforcement bars and steel angles in compression side must be checked with yielding value and taking into consideration yielding of reinforcement bars and steel angles in tension side.When $\varepsilon_1 < \varepsilon_{ya}$, a point located at distance y_1, the part of steel angle before this point is on the elastic range and having a stress f_1 equal to $\varepsilon_1 \times E_s$, while the stress at this point and all points after is f_{ya}. Distance y_1 can be found by using strain compatibility and using $\varepsilon_2 = \varepsilon_{ya}$.

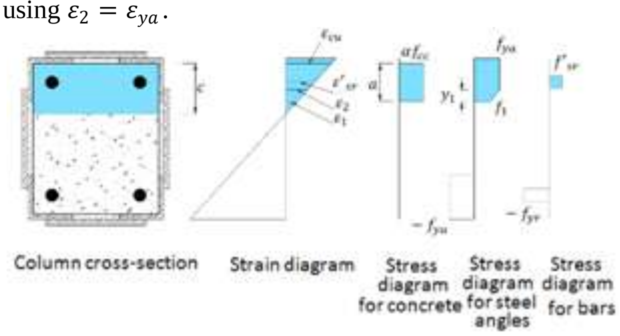

Figure 4: Stress-strain diagram for case 1.

$$\varepsilon'_{sr} = \varepsilon_{cu}\frac{c - d'}{c}$$

$$f'_{sr} = E_s\varepsilon'_{sr} = E_s\varepsilon_{cu}\frac{c - d'}{c}$$

$$\varepsilon_1 = \varepsilon_{cu}\frac{c - L_1 + t_1}{c}$$

$$f_1 = E_s\varepsilon_1 = E_s\varepsilon_{cu}\frac{c - L_1 + t_1}{c}$$

$$y_1 = \frac{\varepsilon_{ya}\ c}{\varepsilon_{cu}} - c + L_1 - t_1$$

$$N_D = 0 = \alpha f_{cc}\ a\ b + A'_{sr} f'_{sr} - A_{sr} f_{yr} + \left[2\left(\frac{f_1 + f_{ya}}{2} y_1 t_1\right) + 2\left((L_1 - t_1 - y_1)\ t\ f_{ya}\right) + 2L_1 t_1 f_{ya}\right] - \frac{A_s}{2} f_{ya} \tag{8}$$

By using trial and error, the value of c can be found form (8) and checked with the assumption above.Then by applying $\sum M$ about the centerline of the column, the bending moment resistance M_D can be found.

2) $0 < c < (L_1 - t_1)$

In this case, the neutral axis will divide the steel angles into two parts as shown in Figure5. Strain in reinforcement bars and steel angles in compression side must be checked with yielding value and taking into consideration yielding of reinforcement bars and steel angles in tension side.

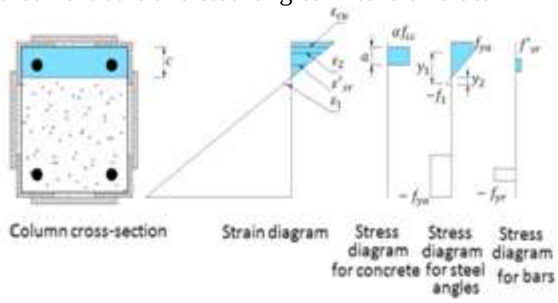

Figure 5: Stress-strain diagram for case 2.

$$\varepsilon'_{sr} = \varepsilon_{cu}\frac{c - d'}{c}$$

$$f'_{sr} = E_s\varepsilon'_{sr} = E_s\varepsilon_{cu}\frac{c - d'}{c}$$

$$\varepsilon_1 = \varepsilon_{cu}\left(\frac{L_1 - t_1}{c} - 1\right)$$

$$f_1 = E_s\varepsilon_1 = E_s\varepsilon_{cu}\left(\frac{L_1 - t_1}{c} - 1\right)$$

$$y_1 = \frac{\varepsilon_{ya}\ c}{\varepsilon_{cu}} + L_1 - t_1 - c$$

$$y_2 = L_1 - t_1 - c$$

$N_D = 0 = \alpha f_{cc}\ a\ b + A'_{sr} f'_{sr} - A_{sr} f_{yr} + [2L_1 t_1 f_{ya} +$
2fya2(y1−y2)t1−2f12y2t1−As2fya (9)

By using trial and error, the value of c can be found from (9) and checked with the assumption above. Then by applying $\sum M$ about the centerline of the column, the bending moment resistance M_D can be found.

4- Point B corresponds to a neutral axis location that results in the same flexural capacity as point D and twice the axial load of point C.

$$N_B = 2N_C$$
$$M_B = M_D$$

4. Design Proposals

ANSI/AISC 360-16 [13] and EN 1994-1-1 (2004) [14] presented an analytical model for construction of an interaction diagram for a composite column using the plastic stress distribution method. These models did not include any direct application to RC columns strengthened by steel caging nor the effect of the confinement. The four points identified in Figure 1 are defined by the plastic stress distribution used in their determination. Point A is determined from (10) or (11).

$$N_{AISC} = N_A = 0.85A_c f_c + A_{sr} f_{yr} + A_s f_{ya} \quad (10)$$

$$N_{EC4} = N_A = A_c f_c + A_{sr} f_{yr} + A_s f_{ya} \quad (11)$$

Point D is determined as the flexural strength of the section. Point C corresponds to maximum flexural strength with an axial strength.Point B corresponds to a plastic neutral axis location that results in the same flexural capacity as point D but with twice the axial load of point C.

5. Validation of Proposed Models

Sets of experimental columns strength results presented by Ezz-Eldeen [15] (CS22e1, CS22e2, CS22e3 and CS22e4), Montuori et al. [11] (E-R1, D-R1, A-R1 and B-R1a), Belal et al. [16] (Col.01.L.3P) and Tarabia and Albakry [17] (SCN1 and SCN2) were used to validate the presented models.The Details of thesevalidationsare illustrated in Table 2. For each set an N-M interaction diagram has been drawn (Figures 6-8).

The comparisons show that the interaction diagramsobtained by the proposed modelsgive underestimate values comparative with experimental results by 1.0% to 37.0%, also it can be noticed that the increased in load eccentricity (increasing of bending moment) will decrease the difference between experimental results and the results obtained from proposed models.Figures 6-8 contain N-M interaction diagramsobtained from the design proposals of ANSI/AISC 360-16 and EN 1994-1-1. N-M interaction diagrams according to EN 1994-1-1 were overestimated in some cases compared with the experimental results.

Table 2: Details of selected columns and the validations results

Model	Cross-section (mm)	Steel angle (mm)	Longitudin-al bars	f'_c (MPa)	f_{ya} (MPa)	f_{yr} (MPa)	e (mm)	$N_{exp.}$ (kN)	Analytical model		Design proposal		Comparison	
									N_I	N_{II}	N_{AISC}	N_{EC4}	$\frac{N_I}{N_{exp.}}$	$\frac{N_{II}}{N_{exp.}}$
CS22e1	120×160	4 L 20×2	4 φ 8 mm	28	380	260	10	643	555	580	558	630	0.86	0.90
CS22e2	120×160	4 L 20×2	4 φ 8mm	28	380	260	20	552	500	510	500	560	0.90	0.92
CS22e3	120×160	4 L 20×2	4 φ 8 mm	28	380	260	30	474	455	455	447	500	0.96	0.96
CS22e4	120×160	4 L 20×2	4 φ 8mm	28	380	260	40	420	410	417	400	448	0.97	0.99
E-R1	150×150	4 L 30×2	4 φ 16 mm	26.4	353	539	50	745.4	580	570	620	650	0.77	0.76
D-R1	150×150	4 L 30×2	4 φ 16 mm	26.4	353	539	75	555.7	470	460	500	520	0.85	0.83
A-R1	150×150	4 L 30×2	8 φ 10 mm	26.4	353	539	50	716.8	455	455	560	570	0.63	0.63
B-R1a	150×150	4 L 30×2	8 φ 10 mm	26.4	353	539	75	523.9	445	440	440	450	0.85	0.84
Col.01.L.3P	200×200	4 L 50×5	4 φ 12 mm	27.2	240	360	0	1821	1405	1624	1543	1706	0.77	0.89
SCN1	150×150	4 L 50×4.5	4 φ 10 mm	46.25	415	420	0	1990	1722	1888	1730	1886	0.87	0.95
SCN2	150×150	4 L 30×3	4 φ 10 mm	38	485	420	0	2000	1540	1719	1572	1700	0.77	0.86

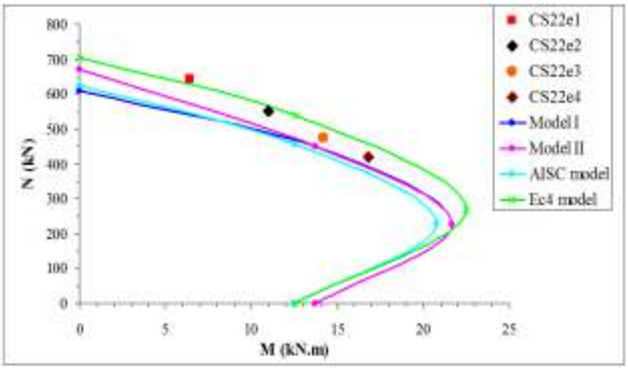

Figure 6: Comparison of experimental results of Ezz-Eldeen (2016) with proposed N–M interaction diagram models.

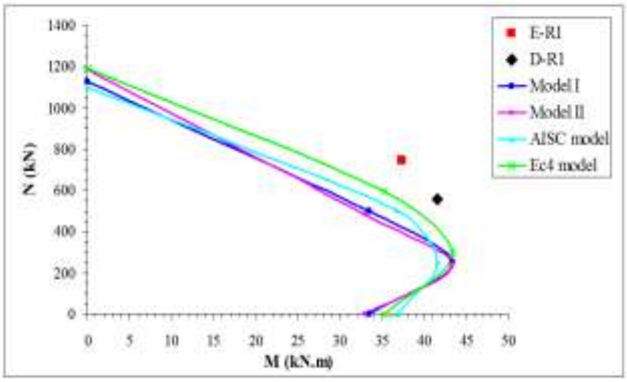

Figure 7: Comparison of experimental results of Montuori et al. (2004) with proposed N–M interaction diagram models.

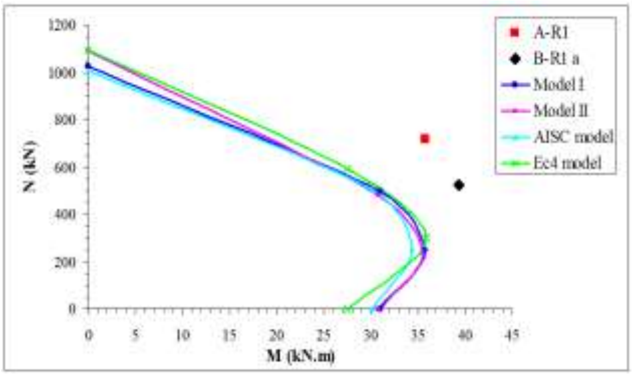

Figure 8: Comparison of experimental results of Montuori et al. (2004) with proposed N–M interaction diagram models.

6. Conclusions

In the present work, analytical modelswere derived for the hand computation to construct the N-M interaction diagram for RC column strengthened with steel jacket. In this work, the analytical expressions adopted for the stress-strain responses of confined concrete and steel reinforcement (steel bars and steel angles in compression) are able to include confined effects induced by steel strips and angles subjected to bending moment and axial forces. The results obtained were comparedwith the experimental columns strength results presented by some researchers and design proposals contained in ANSI/AISC 360-16 and EN 1994-1-1. The results obtained by the analytical models showedfairly good agreement with the design proposal results.

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