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On Shrinkage Estimation for Stress-Strength Reliability in Case of Generalized Exponential Distribution

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Abstract: ***The present paper deal with estimation the "stress – strength reliability" of Generalized Exponential Distribution, using different estimation methods like, maximum likelihood, moment and shrinkage methods (Three types). Comparisons between the proposed estimators were made using simulation based on statistical indicter mean squared error (MSE).***

Keywords: Generalized Exponential Distribution, Stress – Strength Reliability, Maximum Likelihood Estimator, Moment Estimator, Shrinkage Estimator and Mean Squared Error.

1. Introduction

"The two-parameters Generalized Exponential Distribution (GED) has been introduced and studied quite extensively by Gupta & Kundu (1999,2001,2002, 2003, 2004), Raqab (2002), Raqaband Ahsanullah (2001), Zheng (2002) and Kundu, Gupta and Manglick (2004).The two-parameter GED is an alternative to the well-known two-parameter Gamma, two-parameter Weibull"[8].

The probability density and the distribution functions of two-parameter GED are respectively given by:

$$f(x;\alpha,\theta)=\alpha\theta e^{-x\theta}(1-e^{-x\theta})^{\alpha-1};\quad \text{for } x>0 \qquad (1)$$

$$F(x;\alpha,\theta)=(1-e^{-x\theta})^{\alpha-1};\quad \text{for } x>0 \qquad (2)$$

Where, α is unknown shape parameter with known scale parameter (θ=1). Now onwards GE distribution with the shape parameter α and scale parameter θ will be denoted by GED(α, θ).

The reliability of Stress – Strength (S-S) model was used in many application in engineering and physics. "The problem of estimation the (S –S) reliability (R =P(Y<X)) arises in the situation of mechanical reliability of component with strength X and stress Y"; [10]. The component fails if and only if at any time the stress exceeds the strength.

The target of this work involve estimation the reliability of stress – strength (S-S) model for the two parameters Exponential distribution using different estimation methods and make a comparison between the proposed methods using simulation depends on the statistical indicator MSE.

Our hypothesis in(S -S) model, the stress (Y) and the strength (X) are independent variables, where X~GED(α_1,1) and Y~GED(α_2,1).

The (S-S) reliability of X and Y is defined as follows: [5],[6] and [8].

$$R = P(Y < X) = \iint_{y<x} f(x)f(y)$$

$$R = \int_0^\infty \alpha_1 e^{-x}\ (1-e^{-x}\ )^{\alpha 1-1}dx \int_0^x \alpha_2\, e^{-y}\ (1-e^{-y}\ )^{\alpha 2-1}dy$$

$$= \int_0^\infty \alpha_1 e^{-x}\ \ (1-e^{-x}\ \ )^{\alpha_2+\alpha_1-1}dx$$

$$= \frac{\alpha_1}{\alpha_1+\alpha_2} \quad (3)$$

2. Estimation Methods of R =P(Y<X)

2.1Maximum Likelihood Estimator (MLE)

Let $x_1, x_2, \ldots, x_n$ be a random sample of GED (α_1,1) and $y_1, y_2, \ldots, y_m$ be a random samples of GED (α_2,1) then , the likelihood function of the observed sample is given as:

$$l = L(\alpha_1,\alpha_2;x,y) = \prod_{i=1}^{n} f(x_i) \prod_{j=1}^{m} g(y_j)$$

$$= \prod_{i=1}^{n} \alpha_1 e^{-x_i}\ (1-e^{-x_i}\ )^{\alpha_1-1} \prod_{j=1}^{m} \alpha_2 e^{-y_j}\ (1-e^{-y_j}\ )^{\alpha_2-1}$$

$$= \alpha_1^n \alpha_2^m e^{-\sum_{i=1}^n x_i\ -\sum_{j=1}^m y_j} \prod_{i=1}^{n}\left(1-\ e^{-x_i}\ \right)^{\alpha_1-1} \prod_{j=1}^{m}\left(1-e^{-y_j}\right)^{\alpha_2-1} \quad (4)$$

Take Ln to both sides we get:-

$$\mathrm{Ln}(l) = n\,\mathrm{Ln}\alpha 1 + m\,\mathrm{Ln}\alpha 2 - \sum_{i=1}^{n} x_i - \sum_{j=1}^{m} y_j + \sum_{i=1}^{n} Ln\left(1-e^{-x_i}\right)^{\alpha_1-1} + \sum_{j=1}^{m} Ln\left(1-e^{-y_j}\right)^{\alpha_2-1} \quad (5)$$

The derivatives of Ln (l) with respect to α_1 and α_2 and equate to the zero are respectively given as follows:

$$\frac{dLn(l)}{d\alpha_1} = \frac{n}{\alpha_1} + \sum_{i=1}^{n} Ln\left(1-e^{-x_i}\right) = 0 \quad (6)$$

$$\frac{dLn(l)}{d\alpha_2} = \frac{m}{\alpha_2} + \sum_{j=1}^{m} Ln\left(1-e^{-y_j}\right) = 0 \quad (7)$$

The ML's estimator for the unknown shape parameters α_i (i=1, 2) is given by:

$$\hat{\alpha}_{1_{mle}} = \frac{-n}{\sum_{i=1}^{n} Ln\left(1-e^{-x_i}\right)} \quad (8)$$

$$\hat{\alpha}_{2_{mle}} = \frac{-m}{\sum_{j=1}^{m} Ln\left(1-e^{-y_j}\right)} \quad (9)$$

Noted that, $\hat{\alpha}_{i_{mle}}$ is biased estimator, since $E(\hat{\alpha}_{i_{mle}}) = \frac{n\alpha}{n-1} \neq \alpha$.

Thus, $\hat{\alpha}_{i_{ub}} = \frac{n-1}{n}\hat{\alpha}_{i_{mle}}$ will be unbiased estimator of α_i.

That is mean, $E(\hat{\alpha}_{i_{ub}}) = \alpha$, $\mathrm{Var}(\hat{\alpha}_{1_{ub}}) = \frac{(\alpha_1)^2}{(n-2)}$ and $\mathrm{Var}(\hat{\alpha}_{2_{ub}}) = \frac{(\alpha_2)^2}{(m-2)}$.

i.e. ;

$$\hat{\alpha}_{1_{ub}} = \frac{n-1}{-\sum_{i=1}^{n} Ln\left(1-e^{-x_i}\right)} \quad (10)$$

and

$$\hat{\alpha}_{2_{ub}} = \frac{m-1}{-\sum_{j=1}^{m} Ln\left(1-e^{-y_j}\right)} \quad (11)$$

By substituting equations (8) and (9) in equation (3) we get:

$$\hat{R}_{mle} = \frac{\hat{\alpha}_{1_{mle}}}{\hat{\alpha}_{1_{mle}} + \hat{\alpha}_{2_{mle}}} \quad (12)$$

2.2 Moment Method (MOM)

The moment method was one of the exact method used to estimate the parameters. In this subsection we used the (MOM) to estimate the parameter α for GED when the parameter (θ=1) and we need the populations moment for X and Y of GED, which is given below:-[2], [8] and [9]

$$E(x^r) = \begin{cases} \alpha \sum_{i=0}^{\alpha-1} \binom{\alpha-1}{i} (-1)^i (i+1)^{-r-1} \Gamma(r+1) & , if\ \alpha \in N \\ \alpha \sum_{i=0}^{\alpha-1} \frac{\alpha-1^{P_i}}{i!} (-1)^i (i+1)^{-r-1} \Gamma(r+1) & , if\ \alpha \notin N \end{cases}$$

for r = 1,2,3 .... (13)

Where αP_i=α (α-1) (α-2)….. (α-i+1) and N is the set of natural number. Thus, the populations mean of x and y will be:

$$E(x) = \begin{cases} \alpha_1 \sum_{i=0}^{\alpha_1-1} \binom{\alpha_1-1}{i} (-1)^i (i+1)^{-2} \Gamma(2) & , if\ \alpha_1 \in N \\ \alpha_1 \sum_{i=0}^{\alpha_1-1} \frac{\alpha_1-1^{P_i}}{i!} (-1)^i (i+1)^2 \Gamma(2) & , if\ \alpha_1 \notin N \end{cases}$$

$$E(Y) = \begin{cases} \alpha_2 \sum_{j=0}^{\alpha_2-1} \binom{\alpha_2-1}{j} (-1)^j (j+1)^{-2} \Gamma(2) & if\ \alpha_2 \in N \\ \alpha_2 \sum_{j=0}^{\alpha_2-1} \frac{\alpha_2-1^{P_j}}{j!} (-1)^j (j+1)^{-2} \Gamma(2) & , if\ \alpha_2 \notin N \end{cases}$$

And when equating the sample mean with the corresponding population mean, we get

$$\bar{X} = \frac{\sum_{i=1}^{n} x_i}{n} = \alpha_1 \sum_{i=0}^{\alpha_1-1} \binom{\alpha_1-1}{i} (-1)^i \Gamma(2)(i+1)^{-2}$$

and,

$$\bar{Y} = \frac{\sum_{j=1}^{m} y_j}{m} = \alpha_2 \sum_{j=0}^{\alpha_2-1} \binom{\alpha_2-1}{j} (-1)^j \Gamma(2)(j+1)^{-2}$$

By simplification, we obtain the estimation of the unknown shape parameters α_1, α_2 using moment method as follows

$$\hat{\alpha}_{1_{mom}} = \frac{\bar{x}}{\sum_{i=0}^{\alpha_1-1} \binom{\alpha_1-1}{i} (-1)^i \Gamma(2)(2)^{-2}} \quad (14)$$

and

$$\hat{\alpha}_{2_{mom}} = \frac{\bar{Y}}{\sum_{j=0}^{\alpha_2-1} \binom{\alpha_2-1}{j} (-1)^j \Gamma(2)(j+1)^{-2}} \quad (15)$$

Substitution the equations (14) and (15) in the equation (3), we get the estimation of (S-S) reliability using moment method as below:

$$\hat{R}_{mom} = \frac{\hat{\alpha}_{1_{mom}}}{\hat{\alpha}_{1_{mom}} + \hat{\alpha}_{2_{mom}}} \quad (16)$$

2.3 Shrinkage Estimation Method (Sh)

Shrinkage estimation method is the Bayesian approach depending on prior information regarding the value of specific parameter α from past experiences or previous studies as initial value α_0 and classical (unbiased) estimator $\hat{\alpha}_{ub}$ through merge them as a linear combination using shrinkage weight factor $\varphi(\hat{\alpha}_i), 0 \le \varphi(\hat{\alpha}_i) \le 1$ as below:

$$\hat{\alpha}_{sh} = \varphi(\hat{\alpha}_i)\hat{\alpha}_{ub} + \left(1-\varphi(\hat{\alpha}_i)\right)\alpha_0 \quad , i=1,2 \quad (17)$$

Where, $\varphi(\hat{\alpha}_i)$ refers to the believe of $\hat{\alpha}_{ub}$, and $(1-\varphi(\hat{\alpha}_i))$ represents to believe of α_0, which may be constant or a function of $\hat{\alpha}_{ub}$, function of sample size or may be found by minimizing the mean square error for $\hat{\alpha}_{sh}$;

As Thompson said, the important reasons to use initial value α_0 refer to:

1) Suppose that the initial value α_0 near to the real value, and then it is necessary to used it.

2) Something bad may be happened if not used α_0, especially when the initial value near to the real value of parameter. See [1], [3], [4], [11] and [12].

There is no doubt, if our assumption to take the moment method as initial value instated of α_0 in this work.

2.3.1 Shrinkage Weight Function (sh1)

In this subsection we consider the shrinkage weight factor as a function of n and m respectively in equation (17) $K_1=n/(n+10)$ and $K_1=m/(m+10)$.

$$\hat{\alpha}_{i_{sh1}} = K_i \, \hat{\alpha}_{i_{ub}} + (1 - K_i)\hat{\alpha}_{i_{mom}} \text{ for i= 1,2} \quad (18)$$

The corresponding (S-S) reliability using above shrinkage method $\hat{R}_{sh1}$ will be

$$\hat{R}_{sh1} = \frac{\hat{\alpha}_{1_{sh1}}}{\hat{\alpha}_{1_{sh1}} + \hat{\alpha}_{2_{sh1}}} \quad (19)$$

2.3.2 Constant Shrinkage Factor (sh2)

In this subsection the constant shrinkage weight factor will be assumed as $\varphi(\hat{\alpha}_1) = K_3 = 0.3$, and $\varphi(\hat{\alpha}_2) = K_4 = 0.3$, and this implies to the following shrinkage estimators

$$\hat{\alpha}_{1_{sh2}} = K_3\hat{\alpha}_{1_{ub}} + (1 - K_3)\hat{\alpha}_{1_{mom}} \quad (20)$$
$$\hat{\alpha}_{2_{sh2}} = K_4\hat{\alpha}_{2_{ub}} + (1 - K_4)\hat{\alpha}_{2_{mom}} \quad (21)$$

When substitution the equations (20) and (21) in the equation (3), lead to the estimation of (S-S) reliability using shrinkage estimator $\hat{R}_{sh2}$ as below:

$$\hat{R}_{sh2} = \frac{\hat{\alpha}_{1_{sh2}}}{\hat{\alpha}_{1_{sh2}} + \hat{\alpha}_{2_{sh2}}} \quad (22)$$

2.3.3 Modified Thompson type shrinkage weight factor (th):

In this subsection we modified the shrinkage weight factor considered by Thompson in 1968 as follows:

$$\emptyset(\hat{\alpha}_i) = \frac{\left(\hat{\alpha}_{i_{ub}} - \hat{\alpha}_{i_{mom}}\right)^2}{\left(\hat{\alpha}_{i_{ub}} - \hat{\alpha}_{i_{mom}}\right)^2 + var(\hat{\alpha}_{i_{ub}})}(0.01) \quad \text{for i=1,2} \quad (23)$$

Where, $Var(\hat{\alpha}_{i_{ub}})$ defined in section (2-1).

Thus, the modified Thompson type shrinkage estimator will be

$$\hat{\alpha}_{i_{th}} = \emptyset(\hat{\alpha}_i)\hat{\alpha}_{i_{ub}} + \left(1 - \emptyset(\hat{\alpha}_i)\right)\alpha_{i_{mom}}, \text{for i} = 1,2 \quad (24)$$

By substituting equation (24) in the equation (3), we get the modified Thompson type shrinkage estimation of the (S-S) reliability as below

$$\hat{R}_{th} = \frac{\hat{\alpha}_{1_{th}}}{\hat{\alpha}_{1_{th}} + \hat{\alpha}_{2_{th}}} \quad (25)$$

3. Simulation Study

In this section the numerical results were studied to compare the performance of the different estimators of reliability which is obtained in section 2, using different sample size =(10, 30, 50 and 100), based on 1000 replication via MSE criteria. For this purpose Mote Carlo simulation was used the following steps:[7]

Step1: we generate the random sample which is follows the continuance uniform distribution defined on the interval (0,1) as $u_1, u_2, \ldots, u_n$.

Step2: we generate the random sample which is follows the continuance uniform distribution defined on the interval (0, 1) as $w_1, w_2, \ldots, w_m$.

Step3: transform the above uniform random samples to random samples follows GED using the cumulative distribution function (c.d.f.) as follow:

$$F(x) = (1 - e^{-x_i})^{\alpha_1}$$
$$Ui = (1 - e^{-x_i})^{\alpha_1}$$
$$x_i = [-\ln(1 - U_i^{\frac{1}{\alpha_1}})]$$

And, by the same method, we get

$$y_j = [-\ln(1 - W_j^{\frac{1}{\alpha_2}})]$$

Step4: we compute the maximum likelihood estimator of R using equation (12).

Step5: we compute the moment method of R using equation (16).

Step6: we compute the three shrinkage estimators of R using equations (19), (22) and (25).

Step7: based on (L=1000) Replication, we calculate the MSE as follows:

$$MSE = \frac{1}{L}\sum_{i=1}^{L}(\hat{R}_i - R)^2$$

Where, $\hat{R}$ refer the proposed estimators of real value of Reliability R. The results are put it in tables (1-8) below:

Table 1: Shown estimation when R = 0.400000, alpha1= 2, alpha2= 3

n	m	$\hat{R}_{sh1}$	$\hat{R}_{sh2}$	$\hat{R}_{tho}$
10	10	0.389963	0.399997	0.399968
	30	0.396632	0.400001	0.400028
	50	0.398241	0.400002	0.400069
	100	0.399930	0.400003	0.400073
30	10	0.387929	0.399994	0.399915
	30	0.396825	0.399997	0.399992
	50	0.397234	0.399998	0.400003
	100	0.399229	0.399999	0.400034
50	10	0.393669	0.399993	0.399923
	30	0.397233	0.399996	0.400020
	50	0.397359	0.399996	0.400040
	100	0.399273	0.399997	0.400054
100	10	0.389700	0.399991	0.399939
	30	0.397959	0.399995	0.399981
	50	0.398398	0.399995	0.399963
	100	0.399370	0.399996	0.400008

Table 2: Shown MSE values when R = 0.400000 alpha1= 2, alpha2= 3

n	m	$\hat{R}_{sh1}$	$\hat{R}_{sh2}$	$\hat{R}_{tho}$	Best
10	10	0.005272	19E-10	61E-7	$\hat{R}_{sh2}$
	30	0.001776	12E-10	70E-7	$\hat{R}_{sh2}$
	50	0.001047	11E-10	65E-7	$\hat{R}_{sh2}$
	100	0.000580	11E-10	71E-7	$\hat{R}_{sh2}$
30	10	0.005506	15E-10	72E-7	$\hat{R}_{sh2}$
	30	0.001786	67E-11	76E-7	$\hat{R}_{sh2}$
	50	0.001125	52E-11	73E-7	$\hat{R}_{sh2}$
	100	0.000581	47E-11	77E-7	$\hat{R}_{sh2}$
50	10	0.005571	16E-10	75E-7	$\hat{R}_{sh2}$
	30	0.001978	69E11	78E-7	$\hat{R}_{sh2}$
	50	0.001183	55E-11	81E7	$\hat{R}_{sh2}$
	100	0.000603	44E-11	78E-7	$\hat{R}_{sh2}$
100	10	0.006354	21E-10	74E-7	$\hat{R}_{sh2}$
	30	0.001854	63E-11	78E-7	$\hat{R}_{sh2}$
	50	0.001144	52E-11	79E-7	$\hat{R}_{sh2}$
	100	0.000614	42E-11	85E-7	$\hat{R}_{sh2}$

Table 3: Shown estimation when R = 0.192307, alpha1= 2, alpha2= 8.4

n	m	$\hat{R}_{sh1}$	$\hat{R}_{sh2}$	$\hat{R}_{tho}$
10	10	0.189551	0.192304	0.192313
	30	0.189319	0.192306	0.192387
	50	0.191242	0.192307	0.192376
	100	0.192463	0.192307	0.192379
30	10	0.189670	0.192302	0.192273
	30	0.191860	0.192304	0.192299
	50	0.192873	0.192305	0.192312
	100	0.192036	0.192304	0.192337
50	10	0.188697	0.192301	0.192255
	30	0.189804	0.192303	0.192331
	50	0.191714	0.192304	0.192308
	100	0.192581	0.192304	0.192308
100	10	0.189195	0.192300	0.192233
	30	0.190203	0.192302	0.192298
	50	0.191642	0.192303	0.192313
	100	0.191952	0.192303	0.192309

Table 4: Shown MSE values when R = 0.192307, alpha1= 2, alpha2= 8.4

n	m	$\hat{R}_{sh1}$	$\hat{R}_{sh2}$	$\hat{R}_{tho}$	Best
10	10	0.002170	83E-11	24E-7	$\hat{R}_{sh2}$
	30	0.000817	54E-11	29E-7	$\hat{R}_{sh2}$
	50	0.000541	49E-11	31E-7	$\hat{R}_{sh2}$
	100	0.000246	40E-11	29E-7	$\hat{R}_{sh2}$
30	10	0.002355	81E-11	32E-7	$\hat{R}_{sh2}$
	30	0.000771	32E-11	30E-7	$\hat{R}_{sh2}$
	50	0.000493	27E-11	33E-7	$\hat{R}_{sh2}$
	100	0.000245	25E-11	34E-7	$\hat{R}_{sh2}$
50	10	0.002273	88E-11	31E-7	$\hat{R}_{sh2}$
	30	0.000831	39E-11	32E-7	$\hat{R}_{sh2}$
	50	0.000471	29E-11	32E-7	$\hat{R}_{sh2}$
	100	0.000247	25E-11	33E-7	$\hat{R}_{sh2}$
100	10	0.002229	90E-11	30E-7	$\hat{R}_{sh2}$
	30	0.000785	40E-11	30E-7	$\hat{R}_{sh2}$
	50	0.000518	34E-11	35E-7	$\hat{R}_{sh2}$
	100	0.000243	28E-11	32E-7	$\hat{R}_{sh2}$

Table 5: Shown estimation when R = 0.464285, alpha1= 2.6, alpha2= 3

n	m	$\hat{R}_{sh1}$	$\hat{R}_{sh2}$	$\square_{tho}$
10	10	0.460779	0.464286	0.464263
	30	0.459692	0.464287	0.464335
	50	0.463116	0.464289	0.464376
	100	0.462579	0.464289	0.464348
30	10	0.457404	0.464282	0.464213
	30	0.462231	0.464285	0.464279
	50	0.462789	0.464286	0.464288
	100	0.462397	0.464287	0.464329
50	10	0.450298	0.464281	0.464223
	30	0.457909	0.464284	0.464324
	50	0.462014	0.464285	0.464286
	100	0.464033	0.464286	0.464286
100	10	0.452935	0.464281	0.464165
	30	0.458697	0.464284	0.464270
	50	0.461882	0.464285	0.464313
	100	0.463015	0.464286	0.464258

Table 6: Shown MSE values when R = 0.464285, alpha1= 2.6, alpha2= 3

n	m	$\hat{R}_{sh1}$	$\hat{R}_{sh2}$	$\hat{R}_{tho}$	Best
10	10	0.005812	20E-10	73E-7	$\hat{R}_{sh2}$
	30	0.001967	12E-10	79E-7	$\hat{R}_{sh2}$
	50	0.001275	12E-10	83E-7	$\hat{R}_{sh2}$
	100	0.000594	12E-10	79E-7	$\hat{R}_{sh2}$
30	10	0.006051	13E-10	80E-7	$\hat{R}_{sh2}$
	30	0.002064	54E-11	85E-7	$\hat{R}_{sh2}$
	50	0.001171	42E-11	85E-7	$\hat{R}_{sh2}$
	100	0.000660	40E-11	85E-7	$\hat{R}_{sh2}$
50	10	0.006376	13E-10	80E-7	$\hat{R}_{sh2}$
	30	0.002216	39E-11	84E-7	$\hat{R}_{sh2}$
	50	0.001213	31E-11	82E-7	$\hat{R}_{sh2}$
	100	0.000631	27E-11	85E-7	$\hat{R}_{sh2}$
100	10	0.006038	11E-10	78E-7	$\hat{R}_{sh2}$
	30	0.002061	30E-11	79E-7	$\hat{R}_{sh2}$
	50	0.001267	20E-11	83E-7	$\hat{R}_{sh2}$
	100	0.000618	14E-11	87E-7	$\hat{R}_{sh2}$

Table 7: Shown estimation when R = 0.236363, alpha1= 2.60, alpha2= 8.40

n	m	$\hat{R}_{sh1}$	$\hat{R}_{sh2}$	$\hat{R}_{tho}$
10	10	0.232445	0.236360	0.236370
	30	0.235672	0.236363	0.236390
	50	0.233498	0.236363	0.236432
	100	0.235738	0.236364	0.236427
30	10	0.230048	0.236358	0.236300
	30	0.234928	0.236361	0.236352
	50	0.234998	0.236361	0.236407
	100	0.235842	0.236362	0.236343
50	10	0.234062	0.236358	0.236348
	30	0.237488	0.236361	0.236354
	50	0.234086	0.236361	0.236318
	100	0.235859	0.236361	0.236387
100	10	0.232680	0.236358	0.236287
	30	0.235834	0.236361	0.236285
	50	0.234282	0.236361	0.236345
	100	0.236211	0.236361	0.236369

Table 8: Shown MSE values when R = 0.236363, alpha1= 2.60, alpha2= 8.40

n	m	$\hat{R}_{sh1}$	$\hat{R}_{sh2}$	$\hat{R}_{tho}$	Best
10	10	0.002938	1.0E-10	3.0E-7	$\hat{R}_{sh2}$
	30	0.001061	7.0E-11	3.0E-7	$\hat{R}_{sh2}$
	50	0.000675	6.0E-11	4.0E-7	$\hat{R}_{sh2}$
	100	0.000308	5.0E-11	4.0E-7	$\hat{R}_{sh2}$
30	10	0.003001	8.0E-11	4.0E-7	$\hat{R}_{sh2}$
	30	0.001039	3.0E-11	4.0E-7	$\hat{R}_{sh2}$
	50	0.000661	2.0E-11	4.0E-7	$\hat{R}_{sh2}$
	100	0.000300	1.0E-11	4.0E-7	$\hat{R}_{sh2}$
50	10	0.003171	7.0E-11	4.0E-7	$\hat{R}_{sh2}$
	30	0.001088	2.0E-11	4.0E-7	$\hat{R}_{sh2}$
	50	0.000601	1.0E-11	4.0E-7	$\hat{R}_{sh2}$
	100	0.000331	1.0E-11	4.0E-7	$\hat{R}_{sh2}$
100	10	0.002924	8.0E-11	4.0E-7	$\hat{R}_{sh2}$
	30	0.000948	1.0E-11	4.0E-7	$\hat{R}_{sh2}$
	50	0.000607	1.0E-11	4.0E-7	$\hat{R}_{sh2}$
	100	0.000331	8.0E-12	4.0E-7	$\hat{R}_{sh2}$

4. Numerical Results

For all n=(10,30,50,100) and for all m=(10,30,50,100), in this work, the minimum mean square error (MSE) for the (S-S) reliability estimator of the Generalized Exponential distribution holds using the shrinkage estimator based on constant shrinkage weight function ($\hat{R}_{sh2}$). This implies that that shrinkage estimator of (S-S) reliability ($\hat{R}_{sh2}$) is the best and follows by using Thompson type shrinkage estimator$\hat{R}_{th}$.

5. Conclusion

It's clear from the tables of numerical results that the proposal shrinkage estimation method using constant shrinkage weight function ($\hat{R}_{sh2}$) which is depend on unbiased estimator and prior estimate (moment method) is the best estimator than the others in the sense of MSE.

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