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Viscosity Levels and Strength of Chaos

Javier Montenegro Joo

VirtualDynamics: Science & Engineering Virtual Labs, Edo. Astete 574, Las Magnolias, Surco, Lima 33, Perú
Director@VirtualDynamics.Org, www.VirtualDynamics.Org

Abstract: ***It is intuitively expected that the vibrations of an oscillator be hindered when the viscosity of the medium is increased. The higher the viscosity, the higher the hindering effect. This paper reports the detailed way the viscosity of the environment affects the chaotic vibrations of the nonlinear damped and forced oscillator (NLDFO), which transitions towards chaos through a cascade of period bifurcations. It has been found that when the damping of the environment is increased, the mentioned bifurcation cascades become weaker, implying lower chaos intensity, which is corroborated by the corresponding decreasing values of the Lyapunov exponents and by the descent of the Kolmogorov entropy. It is also informed that the value of the extreme displacement as well as that of extreme velocity during a chaotic event decrease when the damping is increased, which is interpreted as a contraction of the orbit in state space, additionally the effect of the damping on the Return Maps of extreme displacements is described. Furthermore this paper gives account of evidence of serial chaos, which is obtained by extending during a very long time the computer simulation after a chaotic event.***

Keywords: Chaos, bifurcation cascades, viscosity, nonlinear, oscillator

1. Introduction

From elementary physics courses and from physical intuition it is expected that when an oscillator vibrates in presence of viscosity (damping), its oscillations become weakened. It is foreseeable that the higher the damping of the environment, the smaller the amplitude of the oscillations, it is also expected that the oscillations tend to die out in a shorter time. From intuition it can also be anticipated that the vibrations of a chaotic oscillator diminish when environment damping is high, but how exactly this weakening of chaotic vibrations takes place remains unknown. This paper reports an investigation on the exact way the damping of the environment affects the chaotic vibrations of the nonlinear damped and forced oscillator (NLDFO). Since the maximum Lyapunov exponent measures the degree of chaos of a chaotic event, it is expected that as the viscosity of the environment is increased, lower values of the Lyapunov exponents will be detected and, since an increment of the environment damping hinders the motion of the vibrator, dropping the degree of chaos, it is expected that the Kolmogorov entropy diminishes as this damping is increased. This paper confirms these two expectations.

2. The Mathematical Model

The investigation here reported is based on computer simulation of the oscillating system depicted in Fig. 1, whose differential equation of motion (Eq. 1) is that of the nonlinear damped and forced (NLDF) oscillator [1], [2]:

$$\frac{d^2\theta}{dt^2} + \left(\frac{b}{mL}\right)\frac{d\theta}{dt} + \omega_o^2 \, Sin\,\theta = \left(\frac{F_o}{mL}\right) Sin\,\Omega t \quad (1)$$

The system is an oscillator immersed in a medium of variable viscosity, such as oil or gelatin (Fig.1). The continuous motion of the oscillator disturbs the temperature of the medium and in this way its density and damping varies while the oscillator vibrates. The oscillator, whose natural frequency is ω_o, is subject to an oscillatory driving motion force F_o whose frequency is Ω .

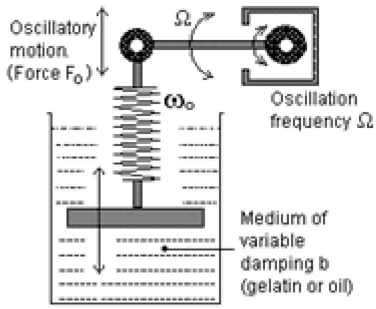

Figure 1: Forced oscillations in a medium of variable viscosity (damping).The oscillating arm imposes forced oscillations on the immersed piece which goes up and down in gelatin or oil. Due to the vibration the temperature of the medium varies and changes the density (viscosity) of the medium, therefore as time elapses the oscillation regime changes. Eventually the conflict between both frequencies, natural (ω_o) and applied(Ω), and the variation of the viscosity unchain chaos.

As it can be appreciated, there are two competing frequencies in the system and there exists also an applied force and a variable damping. This constitutes the recipe [3], [4] for a prone-to-chaos system. Note that the same effect may be attained without viscosity and with a variable force.

3. The Investigation

This oscillator has been previously investigated by this researcher and chaotic behavior has been detected. It has been found that this system evolves towards chaos by means of period bifurcation cascades [5], [6]. In the opinion of this researcher this vibrator is the drosophila of chaotic motion in physical oscillators, because its behavior can intuitively be understood, hence it is very easy to imagine what to expect when varying the oscillation parameters in this system.

Table 1: Chaos simulation with chaotic event NLDFO (3).
Time steps: 10^6

	Damping (viscosity)	Maximum Lyapunov Exp.	Average Lyapunov Exp.
N	b	$Max\ \lambda$	$<\lambda>$
1	3.3400	8.7738	2.2389
2	3.3500	8.7566	2.2387
3	3.3600	8.7394	2.2381
4	3.3650	8.7308	2.2376
5	3.3700	8.7223	2.2371
6	3.3750	8.7137	2.2364
7	3.3800	8.7051	2.2356
8	3.3850	8.6966	2.2348
9	3.3900	8.6880	2.2338
10	3.4000	8.6710	2.2314

In this table it can be seen that as the level of damping is increased (from top down), the maximum Lyapunov exponent-indicating degree of chaos- decreases, which is intuitively expected because higher levels of damping hinder the vibrations. Also the average Lyapunov exponent goes down when the damping is increased.

Table 2: Chaos simulation with chaotic event NLDFO (3).
Time steps: 10^6

	Kolmogorov Entropy K	Max Displacement	Max Velocity
N	ln K	Max X	Max V
1	16.27072	8.28180034	9.27400007
2	16.26966	8.23959055	9.26016145
3	16.26845	8.17386207	9.24507490
4	16.26779	8.16052441	9.23821751
5	16.26709	8.13006874	9.23377038
6	16.26635	8.10565211	9.22796250
7	16.26557	8.06503808	9.21986456
8	16.26475	8.04188386	9.21417867
9	16.26389	8.01822079	9.20758956
10	16.26203	7.94588140	9.19332682

In this table it can be seen that since higher levels of damping (from top down) imply higher restrictions to oscillate, the Kolmogorov Entropy lowers from top down. It can also be seen that as intuitively expected, the higher the damping the shorter the maximum displacement and the lower the maximum velocity attained by the oscillator.

Ever since it is known that chaotic happenings in this system are multiple [7]-[9], several chaotic events were studied, it was observed that all of them display similar behavior. This report contains the results obtained from one of these events, which was randomly selected and it is in this paper referred to as NLDFO (3) and, it lasts four-million time steps.

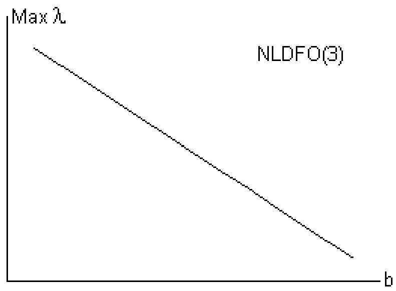

Figure 2: Maximum Lyapunov exponent vs. damping. It is evident that in general, the higher the damping, the lower the degree of chaos.

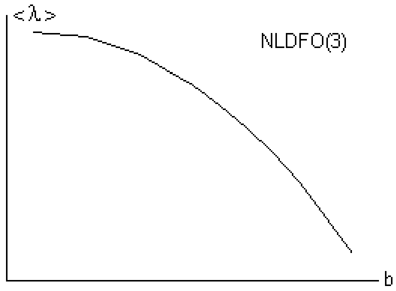

Figure 3: Average Lyapunov exponent vs. damping. Since higher levels of environment damping imply a restriction to manifest the chaotic behavior of the oscillator, in general the average Lyapunov exponent falls down as the damping is increased

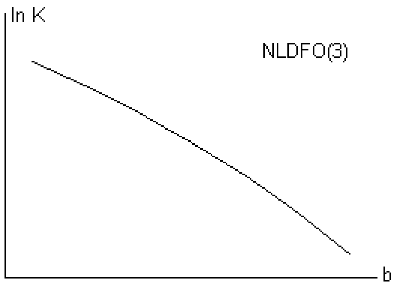

Figure 4: Logarithm of Kolmogorov entropy vs damping. Since higher levels of damping are associated to lower degrees of chaos then the entropy must also decrease when the damping is high. Values of Ln K instead of those of K are used because the values of K are too large.

The investigation consisted of many computer simulations with the selected chaotic event and, for each one of them the viscosity (damping) of the environment was fixed at a different value and kept constant during simulation while the applied force was gradually varied. In order to quantify the level of chaos, the maximum Lyapunov exponent was obtained from each experiment. Seeing that chaotic events are finite [7]-[9], in order to compute the maximum Lyapunov exponent [10]-[13] of each happening the complete chaotic phenomenon was scanned. The shortest among the chaotic events investigated by this researcher lasts one million time steps, as a consequence, obtaining the whole spectrum of the Lyapunov exponents becomes extremely difficult and, it has not been incorporated into this research.

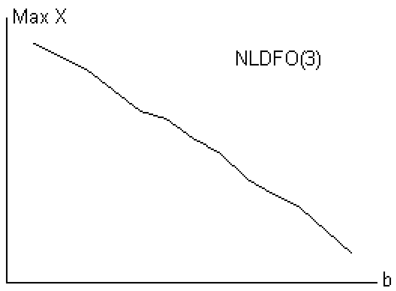

Figure 5: Maximum displacement versus damping. When simulating chaotic events with increasing values of the environment viscosity (damping), it is observed that the maximum amplitude attained by the oscillator reduces. This means that the state space flow shrinks

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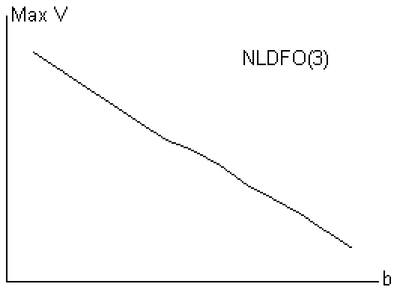


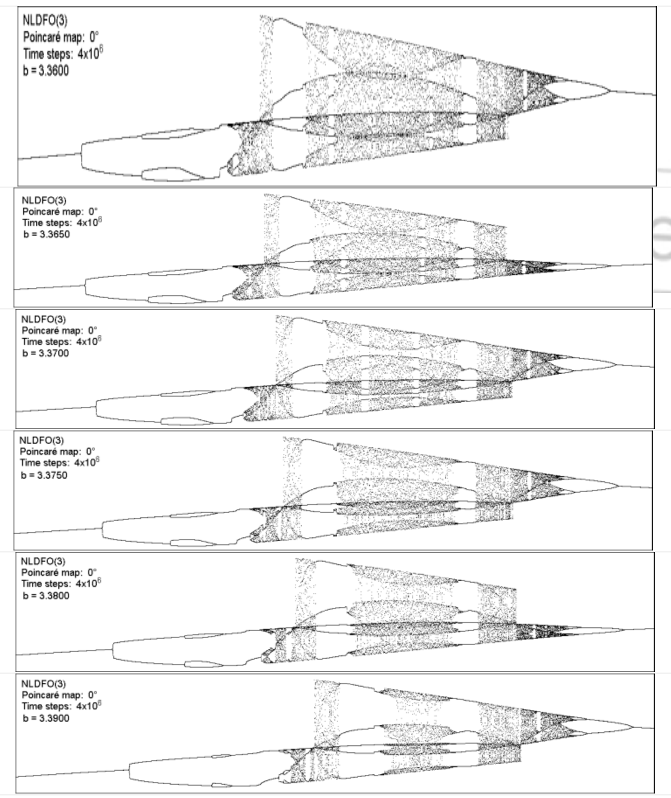
Figure 6: Maximum velocity attained by the chaotic oscillator as a function of increasing environment viscosity (damping).

It is observed that higher values of the damping reduce the maximum velocity reached by the chaotic oscillator.

4. Results of the Investigation

The data in tables (1) and (2) has been obtained from computer simulations with the chaotic event identified as NLDFO(3), which has a damping of b = 3.3700. This is the reference chaotic event, it occupies position number 5 in the tables and it runs along four-million time steps. The other measurements have been obtained to construct a panorama of the general behavior of the chaotic oscillator when damping is changed.

The graphs in figures 2, 3 and 4 were constructed with data from both tables and they display the general behavior of the chaotic oscillator when the environment damping is increased. It can be seen that as expected, the level of chaos diminishes when the damping increases because higher levels of damping imply more restriction to motion, consequently the entropy of the system must also decrease.



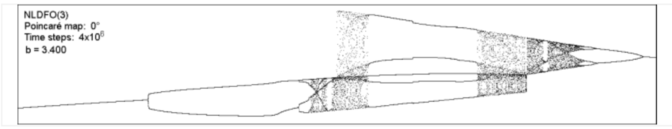


Figure 7: Sequence of seven chaotic bifurcation cascades of the NLDFO (3) for each time higher damping. It can be seen, that as the damping increases the chaos intensity weakens and some bifurcations vanish. It can be foreseen that chaos will eventually disappear when viscosity becomes high enough

Figures 5 and 6, show the influence of the damping in the maximum displacement and in the maximum velocity detected in the chaotic NLDF Oscillator, it can be seen that when the damping of the environment is higher, a decrease in both, the maximum amplitude of oscillation and the maximum velocity attained by the chaotic oscillator, takes place. This behavior was intuitively expected. In particular, the shrinking of the maximum displacements is in agreement with the lower values of the maximum Lyapunov exponents previously detected for increased values of the damping, because the Lyapunov exponent is a measure of the degree of divergence of the flow (state space orbit) in a chaotic system. In figure 6 it can be seen that the motion of the chaotic oscillator is slowed down in environments of high viscosity, the higher the viscosity, the slower the vibration, which also happens in non-chaotic oscillators.

Figure 7 displays the effect of the environment damping on the period bifurcation cascades (Poincaré maps at0^o) of the chaotic NLDFO. The images show from top to bottom, higher levels of the damping, it can be seen that as the damping increases some bifurcations disappear and the chaos intensity reduces.

The effect of the viscosity on the chaos intensity is also shown in figure 8, where Return Maps of extreme displacements for different levels of the damping are shown, and where it is observed that the return maps reduce its population of points (become thinner) as the viscosity increases.

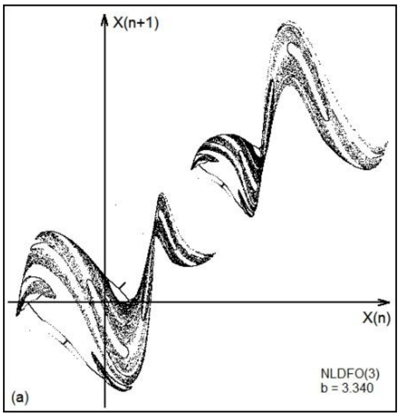
4.1 Displacement Return Maps

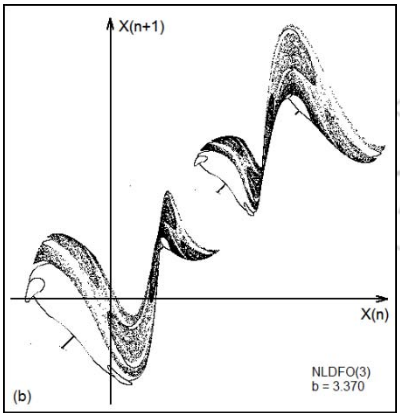
A return map [14] as its name indicates is a map that allows tracking down the path followed to arrive to a certain position. Usually return maps are intuitively understandable as long as the data are one-dimensional, for higher dimensions the return maps are so intricate that they are impossible to interpret and hardly any useful information may be extracted from them. As an example, in the case of the Logistic equation the return map is a parabola.

In the case of oscillations, the extremes (peaks or valleys)X(n) of the displacement may be used to construct return maps by plotting X(n+1) vs. X(n). In this way, for the case of free oscillations where the values of the maximum displacements are constant the return map is a single dot and, when the oscillations are damped, the return map is a series of aligned dots at $45^o$and directed towards the origin of the plotting because in this case the amplitudes tend to shrink.

When the oscillations are chaotic the Return Map does not allow straightforward interpretation, making it impossible to determine future states from previous states. Notwithstanding in this research return maps of oscillation extremes were constructed with the hope of detecting some information. In a previous paper [8], [9] this researcher reported about the asymmetry of the chaotic oscillations of the NLDFO, for this reason in this investigation return maps of both displacement extremes (peaks and valleys) have been constructed.

With the aim on creating return maps, Poincaré maps at 0^o *and* 180^o were previously extracted and the data was used to construct the three return maps appearing in figure 8, for three different values of the damping. In the three instances shown in figure 8 appear two objects in each one, the upper object is that of the points from the Poincaré map at 0^o (oscillation peaks) while the lower object is that of the points from the Poincaré map at 180^o (oscillation valleys), in all cases the resulting objects seem to have a fractal structure.





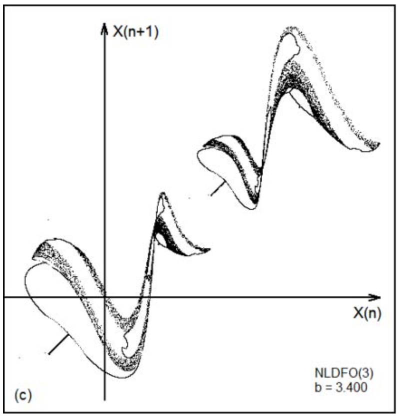


Figure 8: Return Maps of the chaotic oscillation (peaks and valleys) of the NLDF Oscillator. Images (a), (b) and (c), show the return maps for each time higher values of the damping. In the three images the object at the top is the return map of the peaks while the object at the bottom corresponds to the valleys. It can be seen that in image (a) which shows the lowest damping, the return map is the thickest (the most populated) of the three, while in image (c), where the damping is the highest, the return map is rather tenuous (the least populated). Image (b) shows an intermediate situation.

In the three images shown in figure 8, the upper objects are different to their associated lower objects, which confirms that oscillation extremes are different at both sides of the equilibrium position of the oscillator and consequently chaotic oscillations in the NLDFO are far from being symmetrical [8], [9].

It is intuitively expected that an oscillator in a low-damped environment has the opportunity of visiting very many points, for this reason in figure 8, the return map in the least-damped environment (b = 3.340) is the most populated (the thickest) of the three, while the return map of the highest-damped environment (3.400) is the least populated (the most tenuous) of the three. In a few words, the population of points in the return map is inversely proportional to the viscosity (damping) of the environment.

It has been mentioned above that the objects appearing in the return map seem to have fractal structure, if this were the case, and, this researcher is not yet fully convinced, their fractal dimension would be greater than 1.00 and smaller than 2.00. A question that would arise if the fractal dimension were to be calculated is which fractal dimension should be obtained from the return map, whether that of the upper object or that of the lower object or that of both.

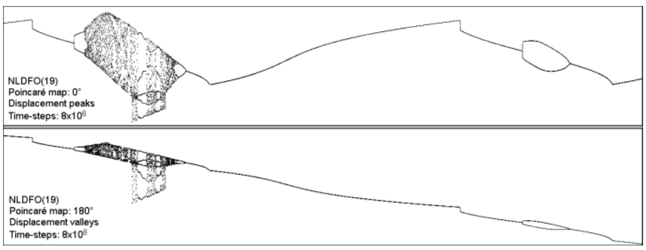


Figure 9: Completely developed chaotic event (left side) followed –very long after- by what seems to be a bifurcation corresponding to another incipient chaotic event. This might be evidence of serial chaos.

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The use of return maps may be generalized to investigate also the velocity and acceleration of a chaotic oscillator. This researcher has previously reported about the behavior of velocity during chaotic events in the NLDFO [15], [16]. In addition to the results then published, return maps of velocity may be constructed to get additional insight. The Runge-Kutta algorithm to solve the second-order differential equation generates values of displacement, velocity and acceleration at every instant of time, which may be used to construct return maps of these magnitudes.

4.2 Chaos beyond chaos – Serial chaos

In order to find out what happens once a chaotic event concludes, the simulation time after a chaotic event finished was considerably extended (extra time), but nothing unexpected was initially encountered[16]. However, after following the track of the detected chaotic event during a very long extra time, a bifurcation indicating the birth of another chaotic event was observed. The situation for a chaotic event identified as NLDFO (19) is shown in figure 9which shows at the top, the Poincaré map at $0^o$and at the bottom, the Poincaré map at 180^o .

In figure 9 the simulation is controlled by the conditions that led to the chaotic event at the left side. From the experience acquired when investigating the dependence of chaos on the environment viscosity, it can be concluded that if the damping when reaching the bifurcation at the right side were smaller, or if the applied external force were higher at that moment, this bifurcation would appear as an openly declared chaotic event. Investigation on this theme is currently under way.

5. Conclusion

By studying the comportment of the nonlinear damped and forced oscillator this paper reports how the viscosity of the environment (damping) impacts on the chaotic behavior of a system experiencing chaos through a period bifurcation cascade. It has been encountered that –as intuitively expected- higher levels of damping tend to reduce the degree of chaos, which has been verified by calculating the maximum Lyapunov exponents. It has been found as well, that the higher the environment damping, the lower the entropy of the chaotic event, which has been confirmed by the fall of the Kolmogorov entropy. It has also been found that when the viscosity of the environment increases the chaotic motion of the oscillator slows down and that the oscillations amplitude is narrowed. Return maps of extreme displacements have been extracted and it has been encountered that their population is inversely proportional to the viscosity of the medium. Additionally, glimpses of serial chaos have been reported.

In general the above described behavior sheds light on the expected comportment in other systems susceptible to undergo chaos by a period bifurcation cascade. It is opportune to mention here that not all systems prone to chaos experiment it by a bifurcation cascade, this is the case of the Duffing equation, where simulations carried out by this researcher have never generated such cascade.

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Author Profile

Javier Montenegro Joo (JMJ), studied Physics and Computation (Computer Science) at San Marcos University in Lima, Perú. His graduate studies were in USA (Physics) and in Brazil (Pattern recognition). Currently JMJ is founding Director of Virtual Dynamics: Science & Engineering Virtual Labs. JMJ has developed virtual labs in the areas of Physics, Mathematics, Digital image processing, Chaos, Cellular Automata, Generic Algorithms, etc. JMJ's most successful brainchild is the Physics Virtual Lab (PVL), a collection of at least 191 intuitively-easy-to-use physics simulation modules, used by several educative institutions worldwide. In his spare time JMJ creates images of Algorithmic Art.