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Bianchi Type - I Inhomogeneous String Cosmological Model in General Relativity

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Abstract: ***In this paper we have investigated inhomogeneous Bianchi type I string cosmological model in general relativity. To get the deterministic solution, we assume $B=C^n$ where $C = f(x)\, g(t)$. Various physical and geometrical properties of the model are also discussed.***

Keywords: Inhomogeneous, Cosmology, String, Bianchi Type-I

1. Introduction

In recent years, there has been considerable interest in string cosmology because cosmic strings play an important role in the study of the early universe. These strings arise during the phase transition after the big bang explosion as the temperature goes down below some critical temperature as predicted by grand unified theories. Moreover, the investigation of cosmic strings and their physical processes near such strings has received wide attention because it is believed that cosmic strings give rise to density perturbations which lead to formation of galaxies. These cosmic strings have stress energy and couple to the gravitational field. Therefore, it is interesting to study the gravitational effect which arises from strings by using Einstein's equations.

The general treatment of strings was initiated by Letelier and Stachel [18]. Letelier [11] obtained the general solution of Einstein's field quations for a cloud of strings with spherical, plane and a particular case of cylindrical symmetry. Letelier [12] also obtained massive string cosmological models in Bianchi type-I and Kantowski-Sachs space-times. Benerjee et al. [8] have investigated an axially symmetric Bianchi type I string dust cosmological model in presence and absence of magnetic field using a supplementary condition $\alpha = a\beta$ between metric potential where $\alpha = \alpha(t)$ and $\beta = \beta(t)$ and a is constant. Exact solutions of string cosmology for Bianchi type-Il, VI_0 -VIII and -IX space-times have been studied by Krori et al. [10] and Wang [21]. Bali et al. [1]–[4] have obtained Bianchi type-I, -III, -V and type-IX string cosmological models in general relativity. The string cosmological models with a magnetic field are discussed by Chakraborty [9], Tikekar and Patel [19],[20], Patel and Maharaj [14]. Singh and Singh [15] investigated string cosmological models with magnetic field in the context of space-time with G_3 symmetry. Singh [17] has studied string cosmology with electromagnetic fields in Bianchi type-II, -VIII and -IX space-time. Bali and Upadhaya [7] investigated LRS Bianchi type- I string dust magnetized cosmological models. Bali and Tyagi [5],[6] also obtained cylindrically symmetric inhomogeneous cosmological model and stiff fluid universe with electromagnetic field in general relativity. Sharma et al. [16] have obtained inhomogeneous Bianchi type VI_0 string cosmological model for stiff fluid distribution.

In this paper we have investigated inhomogeneous Bianchi type I string cosmological model in general relativity. To get the deterministic solution, we assume $B= C^n$ where $C = f(x)\, g(t)$. Some physical and geometrical properties of the model are also discussed.

2. Metric and Field Equations

We consider the metric in the form

$$ds^2 = dx^2 - dt^2 + B^2 dy^2 + C^2 dz^2 \quad ...(1)$$

where B and C are both functions of x and t. The energy-momentum tensor is taken as,

$$T_i^j = \rho v_i v^j - \lambda x_i x^j \quad ...(2)$$

With and v_i and x_i satisfy conditions,

$$v_i v^j = x_i x^j = -1 \quad \text{and } v^i x_i = 0 \quad ...(3)$$

Here ρ is the rest energy of the cloud of strings with massive particles attached to them $\rho = \rho_p + \lambda$, ρ_p being the rest energy density of particles attached to the strings and λ the density of tension that characterizes the strings. The unit space like vector x^i represents the string direction in the cloud, i.e. the direction of anisotropy and the unit time like vector v^i describes the four velocity vector of the matter satisfying the following conditions,

$$g_i^j v^i v^j = -1 \quad ... (4)$$

In present scenario, the commoving coordinates are taken as, $v^i = (0,0,0,1)$ and We choose the direction of string parallel to x- axis so that $x^i = (1,0,0,0)$

The Einstein's field equation in the geometrized unit, ($c=1$, $8\pi G=1$)

$$-T_i^j = R_i^j - \frac{1}{2} R g_i^j \quad ..(5)$$

The Einstein's field equations (5) for the line-element (1) are given by,

$$-\lambda = -\frac{B_{44}}{B} - \frac{C_{44}}{C} - \frac{B_4 C_4}{BC} + \frac{B_1 C_1}{BC} \quad ...(6)$$

$$0 = \frac{C_{11}}{C} - \frac{C_{44}}{C} \quad ...(7)$$

$$0 = \frac{B_{11}}{B} - \frac{B_{44}}{B} \quad ...(8)$$

$$\rho = \frac{B_4 C_4}{BC} - \frac{B_{11}}{B} - \frac{C_{11}}{C} - \frac{B_1 C_1}{BC} \quad ...(9)$$

$$\frac{B_{14}}{B} + \frac{C_{14}}{C} = 0 \quad ...(10)$$

where the sub indices 1 and 4 in B,C and elsewhere denote ordinary differentiation with respect to x and t respectively.

3. Solution of Field Equations

From equations (7) and (8), to find the deterministic solution of line element (1) we assume $B = C^n$ where $C = f(x)\, g(t)$

$$\frac{C_{44}}{C} - \frac{B_{44}}{B} = \frac{C_{11}}{C} - \frac{B_{11}}{C} = l(\text{constant}) \quad ...(11)$$

From equation (11), we have

$$\frac{C_{44}}{C} - \frac{B_{44}}{B} = 1 \quad ...(12)$$

Equation (12) leads to

$$gg_{44} + ng_4^2 = \frac{1}{1-n} g^2 \quad ...(13)$$

where is ℓ is a constant. Integrating equation (13), we obtain

$$g = \beta^{\frac{1}{n+1}} \sinh^{\frac{1}{n+1}} (bt + t_0) \quad ...(14)$$

Where t_0, b and β are integrating constants.

Again From equation (11), we have

$$ff_{11} + nf_1^2 = \frac{1}{1-n} f^2 \quad ...(15)$$

where is ℓ is a constant. Integrating equation (15), we obtain

$$f = \beta^{\frac{1}{n+1}} \sinh^{\frac{1}{n+1}} (bx + x_0) \quad ...(16)$$

Where x_0, b and β are integrating constants.

Hence, we obtain

$$C = \beta^{\frac{2}{n+1}} \sinh^{\frac{1}{n+1}} (bt + t_0) \sinh^{\frac{1}{n+1}} (bx + x_0) \quad ...(17)$$

$$B = \beta_0^{\frac{2n}{n+1}} \sinh^{\frac{n}{n+1}} (bt + t_0) \sinh^{\frac{n}{n+1}} (bx + x_0) \quad ...(18)$$

Therefore after suitable transformation of coordinates, the metric (1) reduces to

$$ds^2 = (dX^2 - dT^2) + \sinh^{\frac{2n}{n+1}} (bT) \sinh^{\frac{2n}{n+1}} (bX) dY^2 \quad ...(19)$$
$$+ \sinh^{\frac{2}{n+1}} (bT) \sinh^{\frac{2}{n+1}} (bX) dZ$$

Which may be considered as an Inhomogeneous Bianchi Type - I String Cosmological Model In General Relativity.

4. Some Physical and Geometrical Features

The physical and geometrical properties of the model (19) are given as follows:

The magnitude of rotation ω is zero i.e.

$$\omega = 0 \quad ...(20)$$

String Tension λ of the model is given by,

$$\lambda = b^2 - \frac{b^2 n}{(n+1)^2} \coth^2 bT - \frac{b^2 n}{(n+1)^2} \coth^2 bX \quad ...(21)$$

The Energy density ρ of the model is given by,

$$\rho = \frac{b^2 n}{(n+1)^2} \coth^2 bT + \frac{b^2 n}{(n+1)^2} \coth^2 bX - b^2 \quad ...(22)$$

The Scalar expansion θ of the model is given by,

$$\theta = b \coth bT \quad ...(23)$$

The Shear Scalar σ of the model is given by,

$$\sigma^2 = b^2 \coth^2 bT \left[\frac{1}{3} - \frac{n}{(n+1)^2} \right] \quad ...(24)$$

The proper volume V the model is given by,

$$V^3 = Q \sinh bT \sinh bX \quad ...(25)$$

The Deceleration parameter q of the model is given by,

$$q = -3 \tanh^2 bT + 2 \quad ...(26)$$

The particle density ρ_p of the model is given by

$$\rho_p = \rho - \lambda = 2\frac{b^2 n}{(n+1)^2} \coth^2 bT + 2\frac{b^2 n}{(n+1)^2} \coth^2 bX - 2b^2 \quad ...(27)$$

From equation (23) and (24) we obtain,

$$\frac{\sigma^2}{\theta^2} = \left[\frac{1}{3} - \frac{n}{(n+1)^2} \right] = \text{constant} \quad ...(28)$$

5. Conclusion

The model (19) starts with big bang at T = 0 and goes on expanding till T = ∞ when θ becomes zero. It is clear that as T increases, the ratio of the shear scalar σ and expansion θ tends to finite value i.e. $\frac{\sigma}{\theta} \to$ constant. Hence the model does not approach isotropy for large value of T. since the deceleration parameter q<0, hence the model (19) represents an accelerating universe. In general the model represents expanding, shearing and non rotating universe.

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