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String Dust Bianchi Type III Cosmological Model with Time Dependent Cosmological TermΛ

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Abstract: ***The behaviour of cosmological constant Λ in Bianchi Type III string cosmological model is investigated for dust fluid. To obtain the deterministic solution of field equations we assume that string tension density λ is equal to rest density ρ i.e. λ=ρ. Some physical and geometrical features are also discussed.***

Keywords: Cosmic string, Bianchi type III model, Dust fluid, Cosmological term Λ

1. Introduction

The string theory play an important role in the study of physical situation at the very early stage of formation of universe and the study is more interesting as these models contain isotropic special cases and permit arbitrary small anisotropy levels at some point of time.

Tikekar and Patel [1] have investigated some exact solution of massive string for Bianchi type-III space time in presence and absence of magnetic field. Bali and Dave [2] investigated the Bianchi type III string cosmological model with bulk viscosity. Recently Bali and Pradhan [3] investigated the Bianchi type III string cosmological model with time dependent bulk viscosity.

In modern cosmological theories the cosmological constant Λ remains a focal point of interest. The cosmological models without the cosmological constant are unable to explain satisfactorily problems like structure formation and the age of universe. Several researchers like Zeldovich [4], Bertolami [5], Ozer and Taha [6], Lorenz [7], Friemann and Waga [8], Weinberg [9], Carroll et al. [10] and Pradhan et al. [11] investigated more significant cosmological model with cosmological constant. Ratra and Pebbles [12] discussed in detail the cosmological models and cosmology with time varying constant.

Axially symmetric Bianchi type I string dust cosmological model in presence and absence of magnetic field are discussed by Banerjee et al. [13]. Bali and Upadhyay [14] have investigated LRS Bianchi type I string dust magnetized cosmological models. Pradhan et al. [15] presented the generation of Bianchi type V cosmological model with varying Λ term. Pradhan et al. [16] also discussed some homogeneous cosmological models with electromagnetic field in presence of perfect fluid with variableΛ. Tiwari [17] has discussed Bianchi type III cosmological model filled with perfect fluid in presence of cosmological constant. Singh and Tyagi [18] also investigated magnetized Bianchi type III anti-stiff fluid cosmological models with time dependent Λ and variable magnetic permeability. A cosmological model for barotropic fluid distribution in creation field with varying cosmological constant (Λ) in Bianchi Type III space time is investigated by Tyagi and Singh [19].

In this paper we investigated string dust Bianchi type III cosmological models with time dependent cosmological term Λ. To get determinate solution we assume an equation of state λ=ρ. The physical and geometrical aspects of model are also discussed.

2. The Metric and Field Equations

We consider Bianchi type III metric of the form

$$ds^2 = -dt^2 + A^2 dx^2 + B^2 e^{-2\alpha x} dy^2 + C^2 dz^2 \quad \dots (1)$$

where A, B and C are functions of t alone.
The energy momentum tensor for a cloud of string dust is given by

$$T_i^j = \rho v_i v^j - \lambda x_i x^j \quad \dots (2)$$

$$\text{where } v^i v_i = -x^i x_i = -1 \text{ and } v^i x_i = 0 \quad \dots (3)$$

whereρ is proper energy density, λ is string tension density, v^i is the four velocity of particles and x^i is unit space like vector in direction of string.

The Einstein's field equation (in gravitational write C = 1, 8πG = 1) is

$$R_i^j - \frac{1}{2} R g_i^j + \Lambda g_i^j = -T_i^j \quad \dots (4)$$

where R_i^j is the Ricci tensor, R = $g^{ij}R_{ij}$ is Ricci scalar. In co-moving coordinate system we have
$v^i = (0, 0, 0, 1)$ $x^i = (1/A, 0, 0, 0)$

The field equation (4) with equation (2) lead to following system of equations:

$$\frac{B_{44}}{B} + \frac{C_{44}}{C} + \frac{B_4 C_4}{BC} + \Lambda = \lambda \quad \dots (5)$$

$$\frac{A_{44}}{A} + \frac{C_{44}}{C} + \frac{A_4 C_4}{AC} + \Lambda = 0 \quad \dots (6)$$

$$\frac{A_{44}}{A} + \frac{B_{44}}{B} + \frac{A_4 B_4}{AB} - \frac{\alpha^2}{A^2} + \Lambda = 0 \quad \dots (7)$$

$$\frac{A_4 B_4}{AB} + \frac{A_4 C_4}{AC} + \frac{B_4 C_4}{BC} - \frac{\alpha^2}{A^2} + \Lambda = \rho \quad \dots (8)$$

$$\alpha\left(\frac{A_4}{A} - \frac{B_4}{B}\right) = 0 \quad \dots (9)$$

where a dot stands for first and double dot for second derivative with time.

3. Solution of Field Equations

The field equations (5) - (9) are system of five equations with six unknown parameters A, B, C, ρ, λ and Λ.
From equation (9) we have

$$B = \ell A \qquad \text{... (10)}$$

where ℓ is constant of integration.
To obtain the determinate solution of the model we assume that string tension density λ is equal to rest density ρ.

$$\text{i.e. } \lambda = \rho \qquad \text{... (11)}$$

Now using equations (10) and (11) in equation (5) and (8), we get

$$\frac{A_4 C_4}{AC} + \left(\frac{A_4}{A}\right)^2 - \left(\frac{A_{44}}{A}\right) - \left(\frac{C_{44}}{C}\right) - \frac{\alpha^2}{A^2} = 0 \qquad \text{... (12)}$$

For complete solution of model we use an extra condition that the expansion (θ) in the model is proportional to the shear (σ) which is physically plausible condition. This condition leads to

$$A = C^n$$

Now equation (12) leads to

$$CC_{44} - \frac{2n}{(1+n)} C_4^2 = -\frac{\alpha^2}{(1+n)} \frac{1}{C^{2n-2}} \qquad \text{... (13)}$$

Now on putting $C_4 = f(C)$ in equation (13), we get

$$Cff' - \frac{2n}{(1+n)} f^2 = -\frac{\alpha^2}{(1+n)} \frac{1}{C^{2n-2}} \qquad \text{... (14)}$$

Equation (14) can be written as

$$\frac{df^2}{dC} - \frac{4n}{(1+n)} \frac{f^2}{C} = -\frac{2\alpha^2}{(1+n)} \frac{1}{C^{2n-1}} \qquad \text{... (15)}$$

Equation (15) on integration gives

$$f^2 . C^{-\frac{4n}{1+n}} = \frac{\alpha^2}{(n^2+2n-1)} C^{\frac{2(1-2n-n^2)}{1+n}} + L \qquad \text{.. (16)}$$

where L is constant of integration.
Equation (16) can be written as:

$$f^2 = \frac{\alpha^2}{(n^2+2n-1)} C^{2(1-n)} + LC^{\frac{4n}{1+n}} \qquad \text{... (17)}$$

Equation (17) leads to

$$\int \frac{dC}{\sqrt{\frac{\alpha^2}{(n^2+2n-1)} C^{2(1-n)} + LC^{\frac{4n}{1+n}}}} = t + M \qquad \text{... (18)}$$

where M is constant of integration.
Value of C can be determined by equation (18)
After a suitable transformation of coordinate, metric (1) reduces to

$$ds^2 = \frac{-dT^2}{\left[\frac{\alpha^2}{(n^2+2n-1)} T^{2(1-n)} + LT^{\frac{4n}{1+n}}\right]} + T^{2n} dX^2 + \ell^2 e^{-2\alpha x} T^{2n} dY^2 + T^2 dZ^2 \qquad \text{... (19)}$$

where C = T, x = X, y = Y and z = Z

4. Some Physical and Geometrical features

The density for the model (19) is given by

$$\rho = \frac{\alpha^2}{(n^2+2n-1)} . \frac{1}{T^{2n}} + \frac{Ln(n+2)}{T^{2\frac{(1-n)}{1+n}}} + \Lambda \qquad \text{... (20)}$$

Also the string tension density

$$\lambda = \frac{\alpha^2}{(n^2+2n-1)} . \frac{1}{T^{2n}} + Ln(n+2) . \frac{1}{T^{2\frac{(1-n)}{1+n}}} + \Lambda \qquad \text{... (21)}$$

The scalar expansion (θ) is given by

$$\theta = (2n+1)) \sqrt{\frac{\alpha^2}{(n^2+2n-1)} \frac{1}{T^{2n}} + L \frac{1}{T^{2\frac{(1-n)}{1+n}}}} \qquad \text{... (22)}$$

The spatial volume (V) is given by

$$V = lT^{2n+1} \qquad \text{... (23)}$$

The shear scalar is given by

$$\sigma^2 = \frac{1}{3}(n-1)^2 \sqrt{\frac{\alpha^2}{(n^2+2n-1)} \frac{1}{T^{2n}} + L \frac{1}{T^{2\left(\frac{1-n}{1+n}\right)}}} \qquad \text{... (24)}$$

The cosmological parameter Λ is given by

$$\Lambda = + \frac{(2n-1)}{(n^2+2n-1)} \frac{\alpha^2}{T^{2n}} - \frac{(3n^3+5n^2-2n)}{(1+n)} \frac{1}{T^{2\left(\frac{1-n}{1+n}\right)}} \qquad \text{... (25)}$$

5. Conclusion

In this model we observe from equation (25) that cosmological constant is positive and decreasing function of time and approaches a small value in the present epoch. Also from equation (22) it is observed that the model start with big-bang at T=0 and expansion factor (θ) is a decreasing function of T and approaches to 0 as T→∞. We further observe that expansion in the model stops when $n = -\frac{1}{2}$.

Also when T →0 then the spatial volume V →0 and when T →∞ then V→∞. These results show that the universe starts expanding with zero volume and blows up at infinite past and future.

Since $T \to \infty \ \frac{\sigma}{\theta} =$ constant, so the model doesn't approach isotropy for large values of T. However the model isotropizes for n=1. A point type singularity is observed for n>0 as T →0, $g_{11} \to 0$, $g_{22} \to 0$, $g_{33} \to 0$. In general the model represent expanding, shearing and non-rotating universe.

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