

Mathematical Modelling and Control of a Mobile Robot for Path Tracking

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Abstract: An autonomous robot is a machine able to extract information from its environment and use knowledge about its world to move safely in a meaningful and purposive manner. The trajectory tracking task in non holonomic systems can be performed through differentiable control laws. A solution for trajectory tracking control of a differential drive wheeled mobile robot (WMR) based on a Kinematic Controller is implemented here. The most important feature of this work is that the complete modelling and control is done in Simscape software which employs physical network approach which differs from standard Simulink modelling approach and is particularly suited to simulate systems that consist of real physical components. Thus modelling is done with MATLAB- SIMSCAPE in which physical units for parameters and variables and all unit conversions are handled automatically.

Keywords: Wheeled Mobile Robot (WMR), Kinematic Controller, Trajectory Tracking, MATLAB/SIMSCAPE

1. Introduction

Now a day's mobile robot is a rising research topic which is used in applications which range among civilians (automatic cars), industrial (cooperative mobile robots in factories), military (unmanned vehicles to destroy enemy target) and fun (soccer player robots) usages. The control of a differential WMR is done via Kinematic controller which is based on the kinematic model of the mobile robot. There are many methods in path tracking control in the sense of kinematics (velocity control).

In [1] robust feedback controller was proposed but controllers cannot deal with the speed jump. A path tracking stable controller by means of Linearising the static and dynamic state feedback is suggested in [2], but if the error is big this method may be violated. [3] addressed the local tracking problem and the global tracking problem. However backstepping based path tracker cannot resolve the speed jump problems. In Path Tracking with Dynamics, most of researchers consider the kinematics part neglecting the dynamics part.

This project deals with Kinematic control method. The control problem for the path tracking is to design a control rule for the linear and angular velocity/acceleration of the nonholonomic WMR in order to track its desired path. The control scheme aims to minimize the error between the real and the desired path.

2. Modelling of Mobile Robot and Design of Control Algorithm

To design appropriate mobile robot for tasks and to understand how to create control software for an instance of mobile robot hardware, the mechanical behaviour of the robot has to be understood. The different aspects of designing wheeled mobile robot can be depicted as: positioning of the robot model in the environment, maneuverability analysis with respect to kinematic constraints, generalized control of developed Kinematic and Dynamic model, and design of control law after solving the

trajectory tracking problem using integral backstepping algorithm based on a single Lyapunov function for mobile robot navigation.

A. Kinematic Model of Mobile Robot

The model of mobile robot consists of a vehicle chassis with two driving wheels mounted on the same axis and a front point sliding support. Both wheels have the same diameter denoted by '2r' and separated by distance '2R'. The two driving wheels are independently driven by two DC gear motors to achieve the motion and orientation.

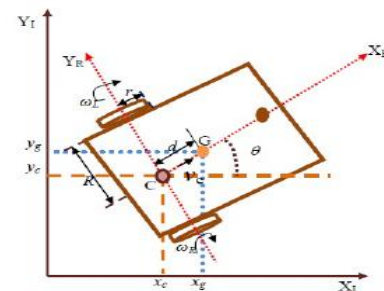


Figure 1: Kinematic Analysis of Mobile Robot

Kinematic equations of the two-wheeled mobile robot are:

$$\begin{bmatrix} \dot{x}_g \\ \dot{y}_g \\ \dot{\theta} \end{bmatrix} = \begin{bmatrix} \cos\theta & -d \sin\theta \\ \sin\theta & d \cos\theta \\ 0 & 1 \end{bmatrix} \begin{bmatrix} v_c \\ \omega \end{bmatrix} \quad (1)$$

And

$$\begin{bmatrix} v_c \\ \omega \end{bmatrix} = \begin{bmatrix} \frac{r}{2} & \frac{r}{2} \\ \frac{r}{2R} & \frac{r}{2R} \end{bmatrix} \begin{bmatrix} \omega_R \\ \omega_L \end{bmatrix} \quad (2)$$

where, x_g and y_g are the coordinates of the center of mass of the platform, θ be the angle between the heading direction and the OX_I axis specifying the orientation of the local platform with respect to the inertial frame. The distance between points G and C is 'd'. v_c and ω are the linear speed

and angular velocity of the mobile robot. ω_R and ω_L are the angular velocities for right and left wheels respectively.

B. Modelling of DC Motor

For modelling and simulation of a DC Motor, simple circuit of its electrical diagram as shown in Figure 1 is to be considered.

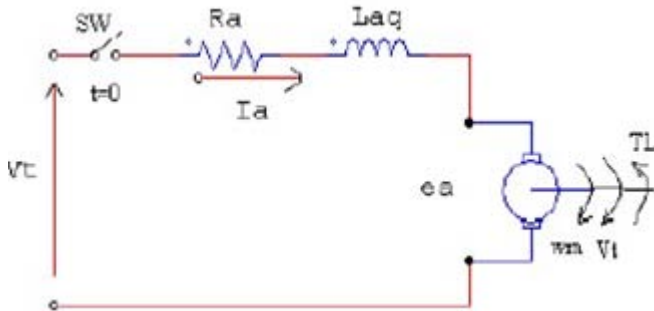


Figure 2: Schematic Diagram of a DC Motor

The motor torque T_m is related to the armature current, i_a by a torque constant K_t as,

$$T_m = K_t * i_a \tag{3}$$

The generated voltage, e_a , is related to angular velocity ω_m by;

$$e_a = K_b * \omega_m = K_b * \frac{d\theta_m}{dt} \tag{4}$$

Using these basic equations, dc motor can be modelled using simscape.

C. Kinematic Control Design

The design of the kinematic controller is based on the kinematic model of the robot, assuming that the disturbance term is a zero vector. Kinematic model of the robot is given by (1) whose outputs are the coordinates of the point of interest, thus meaning

$$\dot{h} = \begin{bmatrix} x_c \\ y_c \end{bmatrix} \begin{bmatrix} \cos\theta & -d \sin\theta \\ \sin\theta & d \cos\theta \end{bmatrix} \begin{bmatrix} v_c \\ \omega \end{bmatrix} = A \begin{bmatrix} v_c \\ \omega \end{bmatrix} \tag{5}$$

with
$$A^{-1} = \begin{bmatrix} \cos\theta & \sin\theta \\ -1/d \sin\theta & 1/d \cos\theta \end{bmatrix} \tag{6}$$

There the kinematic control law applied to the robot is given by:

$$\begin{bmatrix} v_{cref} \\ \omega_{cref} \end{bmatrix} = A^{-1} \begin{bmatrix} \dot{x}_g + I_x \tanh\left(\frac{K_x}{I_x} \bar{x}\right) \\ \dot{y}_g + I_y \tanh\left(\frac{K_y}{I_y} \bar{y}\right) \end{bmatrix} \tag{7}$$

Here, $\bar{x} = x_g - x$ and $\bar{y} = y_g - y$ are the current position errors in the axes X and Y, respectively,

$K_x > 0$ and $K_y > 0$ are the gains of the controller, $I_x \in \mathfrak{R}$ and $I_y \in \mathfrak{R}$ are saturation constants, and (x, y) and (x_g, y_g) are the current and the desired coordinates of the point of interest, respectively.

3. Modelling of Mobile Robot in Simscape

The overall modelling is reduced into a single circuit and is shown in figure 3.

When the right motor is subjected to step input with a value of 12 V, and Left motor of step input with a value of 7V, the robot moves to the side with higher velocity whose response curve is shown in Figure 5(a). Alternatively, when right motor is subjected to a step input with a value of 7V and Left motor when subjected to a step input of 12V, the response curve is as shown in figure 5(b).

Applying the step input to right wheel and ramp input on left wheel, it result in infinite radius and thus the robot moves in a circular motion as shown in figure 5(c).

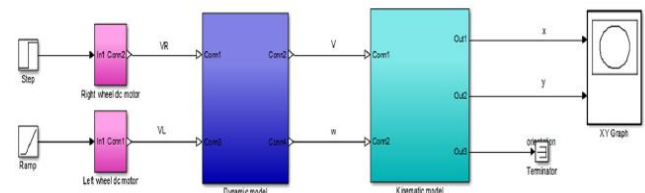


Figure 3: Kinematic model of a mobile robot

4. Trajectory Tracking of Mobile Robot in Simscape

The complete modelling of trajectory tracking is shown in figure 4 which is implemented using Kinematic Controller. In this control, the actual output is compared with the desired output created by modelling normal equations of a circle and a straight line and the output signal is fed to the control section (Kinematic Controller) whose output is then fed to both right and left motors.

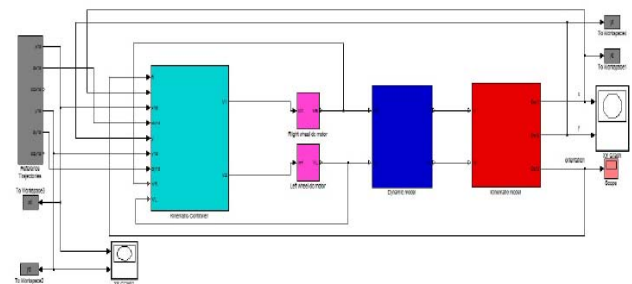


Figure 4: Closed loop control of a mobile robot

5. Simulation Results

The open loop system trajectories are shown in figure 5.

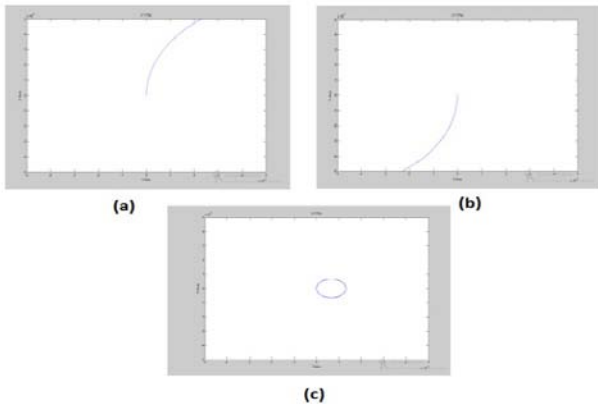


Figure 5: Different trajectories of mobile robot

It shows different path trajectories of mobile robot for different inputs just in order to check the working of robot.

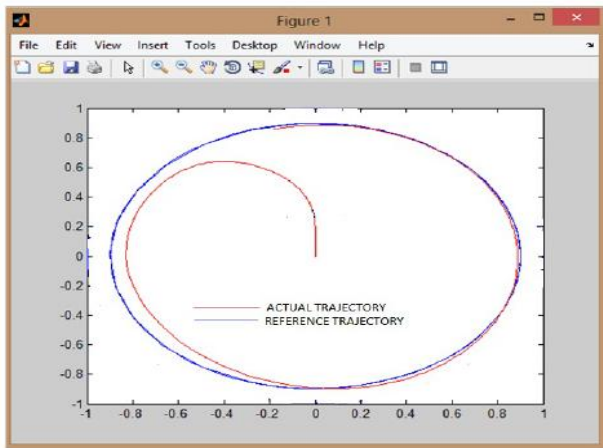


Figure 6: Closed loop control waveform of a mobile robot

Here, the actual output of robot is shown in red colour which tracks the reference given in blue colour. The tracking can be checked with different trajectories like circle (figure 6) and straight line (figure 7).

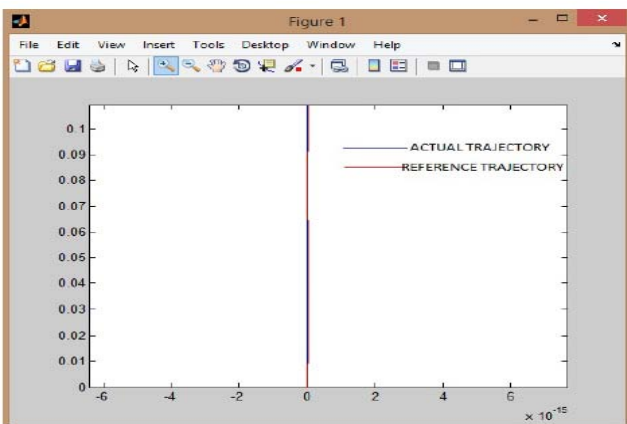


Figure 7: Closed loop control waveform of a mobile robot

6. Conclusion and Future Work

Simulation results shows that the control rule makes good performance in trajectory tracking of mobile robot. Thus with the help of kinematic controller, the system is made closed which helps the robot track the defined trajectory. Most Artificial intelligence researchers that study robotics

are working on mobile robots. Mobile robots pose a unique challenge to the Artificial intelligence community, since they are inherently autonomous and force the researcher to deal with issues such as uncertainty in sensing and action, planning, learning, reliability, and real-time response. By improving and expanding the knowledge of how to successfully integrate these issues into one single system, fundamental contributions can be made to Artificial intelligence research.

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