

Electric Field Distribution of Wire-Duct Electrostatic Precipitator using FDM and MATLAB

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Abstract: *This paper presents the electric field distribution in three-dimension (3-D) of duct-type electrostatic precipitator (ESP) for different values of geometrical parameters using a new program implemented in MATLAB. Poisson's and Current Continuity equations are solved simultaneously using the Finite Difference Method (FDM) with Dirichlet boundary conditions and programmed in MATLAB. The effects of wire-plate spacing and half wire-wire spacing on the electric field distribution are evaluated. In the present study the software's is promising for getting accuracy results due to less time of executing and increasing the point's number in the mesh of problem domain which is explained electric field distribution of the configuration. Also reducing energy consume for new design(applied voltage of wire-discharge is 60 kV in case of wire-plate distance equal 10 cm),3-D results which have been obtained through their implementation in computer programs are better than the results of FDM or FEM techniques when it use without combined with other techniques. Finally some conclusions are viable in design ESP.*

Keywords: electrostatic precipitator, corona discharge, electric field distribution, numerical simulation, MATLAB program.

1. Introduction

Air pollution is problem of today and its result was climate changes. Particulate emission has been shifted from micrometer- to nanometer-sized particles, the effects of the emission nanoparticle on human health are larger than micro- particle, also the efficiency of removal this particle in an industrial applications is less than micro particle Thonglek and Kiatsiriroat [1].The electrostatic precipitator ESP is an industrial technology removes toxic particles from industrial plants such ascement production, chemical processing, electric power and domestic cleaning air. Corona discharge has been used for charging particles. Abdel-Salam and Eid [2] used Finite Element Method to model the corona characteristics in duct-type ESP, electric potential, electric field and the charge density in the interelectrode space of three-wire ESP are evaluated, the calculated current voltage (I-V) characteristics, electric field and potential distributions agreed well with those published before. The Electric field and charge distribution in the duct ESP were investigated by Back and Cramsky [3] using the commercially available finite element method (FEM) solver Comsol Multiphysics, a good agreement was obtained between numerical and experimental results. A novel ESP compared with conventional performed by Tang et al. [4] ,the comparison show improving operational parameter such as electric field line, current density along the plate, dust speed toward collecting plate, equipment volume may be actually reduced and energy consumption may be reduced, under this conditions high efficiency of collecting particles are promising. Tang et al. [5] using MATLAB PDE to calculate electric-field distribution of the electric-bag composite dust collector of the AHPC electrostatic fabric filter at different operation conditions (diameter of different corona electrode, distance of electric field lines and electric potential) this research can provide theoretical basis for design and modification of corona wire radius ,line-spaces and distance between precipitating plates, because of experimental research is much cost .Leet al. [6] investigated, ESP model with different discharge electrodes, the empirical equations are different since the ion concentration and electric fields

are different, a predictive method is developed to estimate the particle collection efficiency of nano particles and sub-micron particles for multipoint-to-plane ESPs, a good agreement is obtained between the present model predictions and the experiments data obtained from the literature. There are many numerical techniques have been employed to calculate electrical characteristic of ESP, FEM by Popa et al. [7], FEM and Finite Volume Method (FVM) by Farnoosh [8], Finite Difference Method(FDM) by Anagnostopoulos and Bergeles [9],computational model in electro hydrodynamic (EHD) by Zamankhan and Ahmadia[10], FDM and Boundary Element Method (BEM) by Rajanikant and Thirumaran [11], comparison of results lead to advantage to each method. Ionan-Gabriela [12] employed different numerical methods to estimate the electric field distribution of ESP, the principle of each method are briefly presented and the differences between the results obtained using each method separately are investigated, the calculations obtained using both commercial software (e.g. Quick Field or FEMM) and programs created in MATLAB are accuracy and time of executing the program for solving the problem is excellent. Kim et al. [13] used electrospray upstream of ESP of deionized water produced water droplets with a size range from several tens to several hundred μm , efficiency of the ESP with the electrospray was found for the collection of particles larger than about 200 nm, also the combination of the ESP and the electrospray was effective in reducing energy consumption of ESP. This paper aims to study the electric field distributions in(3-D) inside an electrostatic precipitator of Karbala cement plant, the effects of geometrical parameters such as wire-plate distance and half wire-wire distance are predicted .The calculated results, obtained using FDM and our own programs created in MATLAB. The results were accurate and the time required for solving the problem is less, also the energy required for operation is reduced, a good computation is obtained in comparison with the results using other techniques separately.

2. Corona Discharge

Ionization phenomenon near electrodes occurs if high voltage applied on it which is called a corona effect. Corona discharge is a source of negative ion that is used in charging the toxic particle in duct ESP. The number of electrons acquired depends on the intensity of the corona generated around the electrodes, and this is proportional to the electrical Field E. Thus, charge is proportional to electric field.

The process of corona generation in the air atmospheric conditions requires a non-uniform electrical field, which can be obtained by the use of a small diameter of discharge wire located between collecting plates in ESP. Once charged, particles are subject to a transverse electrostatic force that pulls them toward the collecting plates (see Figure 1).

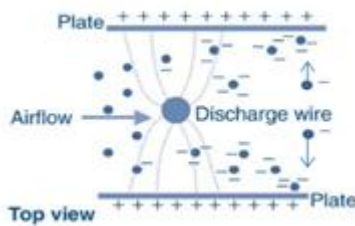


Figure 1: Corona generation

Plates are periodically “rapped” (vibrated) to make the collected particles fall down into a receiver basket. High electric field can easily remove the charging toxic particle from the stream of gas in ESP, the electric field reduces significantly.

With the increasing distant away from the surface of the wire, the reduced electric field near the collecting electrode thus helps to prevent an initiation of the electric arc or sparking due to the electron bridging across the inter electrode spaces. Many techniques have been applied successfully and several designs of corona diffusion chargers are described in the literature, corona wire diffusion chargers used by Biskoset al. [14]. Porous discharge electrode with water supply has been studied by Zhang et al. [15] to get high efficiency of ESP. Kimet al. [16] suggested a novel computation method which is used to calculate the plasma region thickness, instead of the prior studying, the estimated plasma region thickness was approximately 1.5–2.5 times greater than the wire radius in the range of 0.15 mm to 1.6 mm. Brociloet al.[17] investigated a mathematical model to predict the effect of different shapes of discharge wires (round, threaded, rectangular, and rigid) on corona onset voltage. Ziedanet al. [18] developed numerical modeling to investigate the corona current-voltage characteristic in a wire-duct electrostatic precipitator, the influenced of the number of discharge wires, the wire radius and the spacing between the collecting plates also determined.

3. Numerical Solution

In order to calculate the electric field and charge density distributions in ESP, governing equations (Poisson's and current continuity equations) must be solved under suitable

boundary conditions such as Dirichlet conditions ,the ground collecting plates potential $u = 0$ and the potential at discharge wire is taken as the ESP operating voltage with negative polarity . The electric potential distribution in problem domain is estimated at all points in the grid using Cooperman's equation[19]which is depending as first boundary conditions

$$u(x, y) = \frac{\sum_{m=-w}^w \ln \left\{ \frac{\cosh \left[\frac{\pi(y-2mS_y)}{2S_x} \right] - \cos \left(\frac{\pi x}{2S_x} \right)}{\cosh \left[\frac{\pi(y-2mS_y)}{2S_x} \right] + \cos \left(\frac{\pi x}{2S_x} \right)} \right\}}{\sum_{m=-w}^w \ln \left\{ \frac{\cosh \left(\frac{\pi m S_x}{S_x} \right) - \cos \left(\frac{\pi r}{2S_x} \right)}{\cosh \left(\frac{\pi m S_x}{S_x} \right) + \cos \left(\frac{\pi r}{2S_x} \right)} \right\}} \quad (1)$$

Where: u_0 = the applied voltage on the wire

S_x = wire - plate spacing

S_y = half wire-wire spacing

m = the number of discharge wires.

x, y = the coordinate position in meters measured from the discharge wire as origin

Evaluating the charge density ρ_0 on the discharge wire depends on boundary conditions which has been used by McDonald et al.[19], the plasma region surrounding the electrical discharge as cylindrical can be evaluated in terms of Estimation average current density J_p

$$\rho_0(i, j) = \frac{2S_y J_p}{\pi b E r_i} \quad (2)$$

J_p is estimation current density along the plate (A/m^2) (input data), b is effective mobility of charge carriers ($m^2/V \cdot s$) and r_i is radius of the plasma region around the discharge electrode (0.001 m) and approximately equal to the wire radius of discharge electrode , E is calculated from the corona onset gradient given by Peek's formula[21].

$$E = 32.3 * 10^5 \delta + 0.846 * 10^5 \sqrt{\delta/r} \quad (3)$$

The electric field related to the voltage as follow

$$E = -\nabla u \quad (4)$$

From equations (1) and (4) the electric field density is calculated at any point in the grid

The current continuity equation is given by Oglesby and Nichols [22]

$$\nabla \cdot \vec{J} = 0 \quad (5)$$

The relation between current density and charge density is given by

$$\vec{J} = \rho b \vec{E} \quad (6)$$

Eq.(5) became

$$\nabla \cdot (\rho E b) = 0$$

Eq. (5) expanded using vector algebra, is given by

$$(\rho b) \nabla \cdot E + b(E \cdot \nabla \rho) + \rho(E \cdot \nabla b) = 0$$

Since the mobility of charge carriers is assumed to be constant, the last term will be zero, rewriting the equation

$$(\rho b) \nabla \cdot E + b(E \cdot \nabla \rho) = 0$$

The above equation can be further expand [23] as rewriting in the form

$$(\rho b) \rho / \epsilon + b \left(E_x \frac{\partial \rho}{\partial x} + E_y \frac{\partial \rho}{\partial y} \right) = 0$$

$$\left(\frac{\rho^2}{\epsilon} + E_x \frac{\partial \rho}{\partial x} + E_y \frac{\partial \rho}{\partial y} \right)_{i,j} = 0 \quad (7)$$

Using finite difference method

$$\frac{\partial \rho_{(i,j)}}{\partial x} = \frac{\rho_{(i,j)} - \rho_{(i-1,j)}}{\Delta x} \quad \text{and} \quad \frac{\partial \rho_{(i,j)}}{\partial y} = \frac{\rho_{(i,j)} - \rho_{(i,j-1)}}{\Delta y} \quad (8)$$

Substituting equation (8) in equation (7)

$$\frac{\rho_{(i,j)}^2}{\epsilon} + E_x \frac{\rho_{(i,j)} - \rho_{(i-1,j)}}{h} - E_y \frac{\rho_{(i,j)} - \rho_{(i,j-1)}}{h} = 0$$

$$\frac{\rho_{(i,j)}^2}{\epsilon} + \frac{E_x \rho_{(i,j)} - E_x \rho_{(i-1,j)} + E_y \rho_{(i,j)} - E_y \rho_{(i,j-1)}}{h} = 0$$

$$\frac{h}{\epsilon} \rho_{(i,j)}^2 + (E_x + E_y) \rho_{(i,j)} - (E_x \rho_{(i-1,j)} + E_y \rho_{(i,j-1)}) = 0 \quad (9)$$

From equations(2) and (9), $\rho(i, j)$ at any point in the grid can be obtained.

Poisson's equation relates the electric field intensity to the space charge density ρ as

$$\nabla^2 u = -\frac{\rho}{\epsilon} \quad (10)$$

Where, ρ is the charge density and ϵ is permeability of free space

In two dimensions equation (10) becomes:

$$\frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2} = -\frac{\rho}{\epsilon} \quad (11)$$

In applying the methods of finite difference (central difference method)

$$\frac{\Delta^2 u}{\Delta x^2} = \frac{u_4 + u_2 - 2u_0}{(\Delta x)^2} \text{ and } \frac{\Delta^2 u}{\Delta y^2} = \frac{u_1 + u_3 - 2u_0}{(\Delta y)^2} \quad (12)$$

Substituting equation (12) in Eq. (11) and letting $(\Delta x = \Delta y = h)$ obtained

$$\frac{u_4 + u_2 - 2u_0}{h^2} + \frac{u_1 + u_3 - 2u_0}{h^2} = -\frac{\rho}{\epsilon} \quad (13)$$

$$u = \frac{1}{4} \left[(u_1 + u_2 + u_3 + u_4) + \frac{h^2 \rho}{\epsilon} \right] \quad (14)$$

By discretizing equation (14) using Finite Differences around an interior node, (i, j) , for a uniform mesh both on the x -axis direction (of step h) and on the y -axis direction (of step h)

$$u(i, j) = \frac{1}{4} \left[u_{i,j+1} + u_{i,j-1} + u_{i+1,j} + u_{i-1,j} + \frac{h^2 \rho}{\epsilon} \right] \quad (15)$$

From equation (15) and see figure 2 it is clear that voltage at the central node $u(i, j)$ is mean of the voltages at other four nodes, the values of potential at all point in the grid can be calculated by repeating the central node at another point by iteration method. The difference between the values of electric potential calculated from Eq.(1) and Eq.(15) must equal one volt or less, this condition is satisfied the numerical solution, sufficient to get high efficiency of ESP.

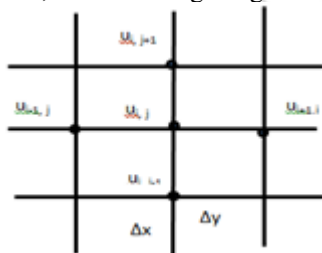


Figure 2: Finite difference solution region

Due to the symmetry, see Figure 3 Only a quarter section of the Electrostatic precipitator channel needs to be modeled. The distributions of electric field are evaluated at area $(10 \times 15) \text{ cm}^2$ for uniform meshes which contains (60×60) points. The program which is used in this work gives additional free for choosing the number of point in the grid of problem domain, and then the procedure is valid for getting high efficiency of ESP.

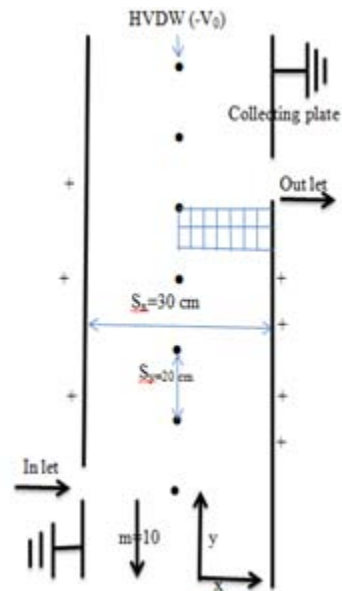


Figure 3: Show computational domain of new design electrostatic precipitator for quarter section

4. Results and Discussion

After the corona discharge occurs between two electrodes, discharge wires and collecting plates in ESP, the electric field distribution play an important role in charging the toxic particles which emitted from industry application. To obtain maximum electrical field for charging the dust particles it is necessary to apply maximum voltage between ESP electrodes for operation time. A new MATLAB program employed for determined the electric field distributions at any point in the grid for ESPs domain (see Figure 3). The problem domain was divided into 60×60 cells and solving equations. (1, 2, 4, 9, 15) using the program. Assumed initial voltage at the wire, wire radius, half wire-to-wire spacing, wire to plate spacing, measured (or assumed) current density at the plate, effective mobility of charge carriers, air density factor at NTP and roughness factor of the wire are the required input data for dust free simulation. The electric field distributions in (3-D) which has been estimated in this paper according to various input data are available in comparison with analytical methods given by G. Cooperman [24], Sekar and Stomberg [25] and McLean [26].

The input data which are used in this work from Kerbala cement plant such as distance between wire-plate $S_x = 0.15 \text{ m}$, half distance between wire-wire $S_y = 0.15 \text{ m}$, the radius of discharge wire $a = 0.001 \text{ m}$, mobility of charge carrier $b = 2 \times 10^{-4} \text{ m}^2/\text{V.s}$, current density $J = 4 \times 10^{-4} \text{ A/m}^2$, in case of dust free conditions and the applied voltage on the discharge wire is 60 kV and the area of problem domain is $(0.15 \times 0.15 \text{ m}^2)$ with 60×60 point in the grid, see Figure 4. Figures 5, 6, 7, 8 show electric field distribution at different values for wire-plate spacing in the electrostatic precipitator ($S_x = 0.20, 0.17, 0.13, 0.10$) m respectively. Figure 5. display electric field distribution between discharge wire and collecting plate of the ESP which is the problem domain has area $(20 \times 15 \text{ cm}^2)$, The high value of electric field intensity decrease slowly toward the collecting plate according to the fact, electric field depends on the voltage drop. the electric field intensity along the wire-plate distance has different behavior due it's values of plate-wire spacing. The same results were obtained

in figures 6-8. Figures 9-12 show the effect of half wire to wire spacing on the electric field distribution, the distance were ($S_y = 0.20, 0.17, 0.13, 0.10$) m, the figures show also increase in peak value of electric field (near the discharge wire) with decreasing the spacing of half wire to wire but the values of electric field along the collecting plate S_y decreases as the wire to wire spacing decreasing therefore figure 12 Illustrated the optimization design of ESP which is showing the promising new geometrical configuration of ESP. Because of no or less back corona occurring along the collecting plate due to low value of voltage drop also reducing energy consumption, this is clear from using the same value of applied voltage on the discharge electrode 60 kV at all geometrical configuration in this work. Table 1 show the summary of this paper which is predicted the new design of ESP; the value of the half wire-wire distance (10 cm) investigated no back corona and less energy consumes (60keV). The model, FDM and MATLAB program can be used as tool for determine the electric field distributions in any geometrical domain if the design parameters was known.

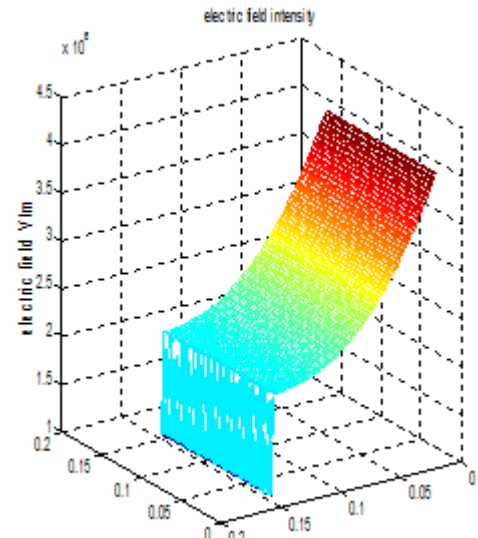


Figure 6: Electrical field distribution for quarter section of ESP $S_x=0.17$ m

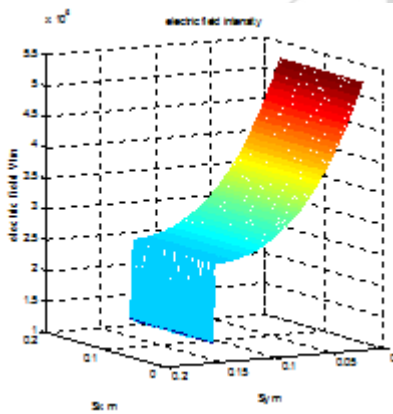


Figure 4: Electrical field distribution for quarter section of ESP $S_x=0.15$ m

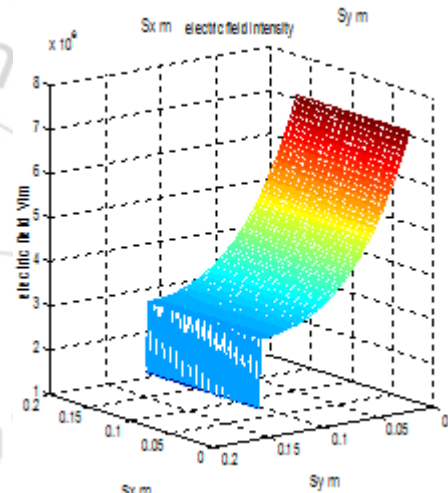


Figure 7: Electrical field distribution for quarter section of ESP $S_x=0.13$ m

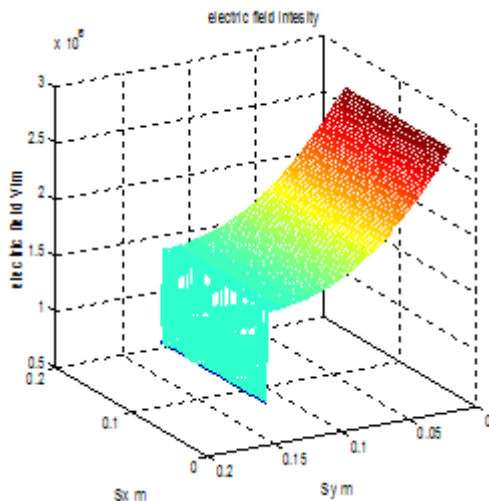


Figure 5: Electrical field distribution for quarter section of ESP $S_x=0.20$ m

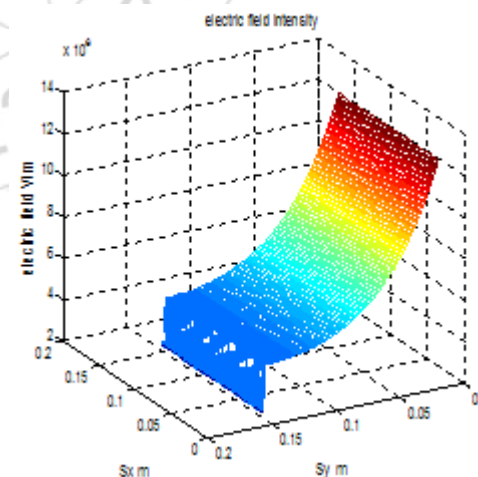


Figure 8: Electrical field distributions for quarter section of ESP $S_x=0.10$ m,

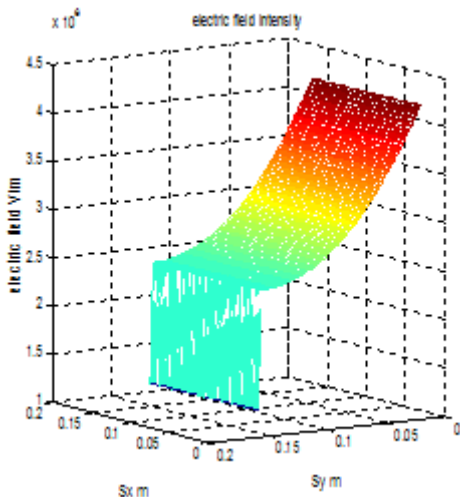


Figure 9: Electrical field distribution for quarter section of ESP $S_y = 0.20\text{m}$

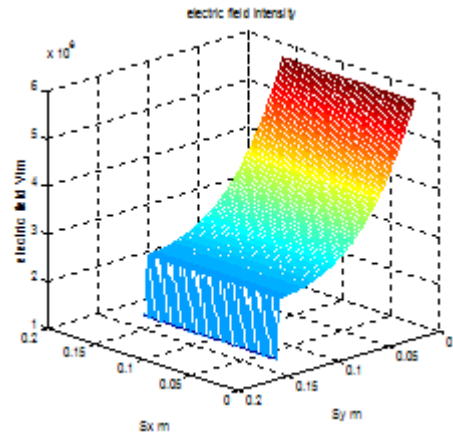


Figure 11: Electric field distribution for quarter section of ESP $S_y = 0.13\text{m}$.

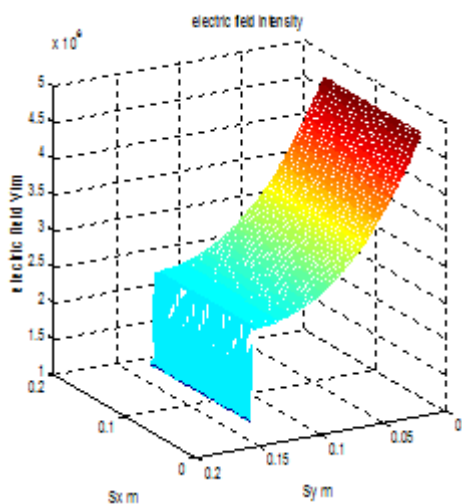


Figure 10: Electrical field distribution for quarter section of ESP $S_y = 0.17\text{m}$

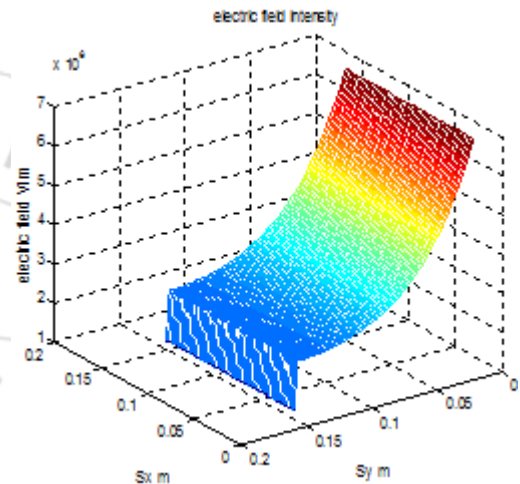


Figure 12: Electrical field distribution for quarter section of ESP $S_y = 0.10\text{m}$

Table 1: Operation and geometrical parameters of ESP and number of cells in the grid of ESP domain

Wire radius mm	Half wire-wire distance cm	Wire-plate spacing cm	Mobility of charge carrier $\text{m}^2/\text{V}\cdot\text{s}$	Applied voltage kV	Current density A/m^2	Number of cells in Math lab	description
1	15	15	2×10^{-4}	90	4×10^{-4}	3600	G. Cooperman[24], Sekar and Stomberg [25] and McLean[26]
1	10	15	$10^{-4} \times 2$	60	4×10^{-4}	3600	New Design

5. Conclusion

A numerical model can be used as a tool for estimation of the electric fields in various geometrical configurations which differ in geometrical and operation parameters such as radius of electrical discharge, number of discharge wires and migration velocity. The model investigates a new design of ESP that satisfied good results for charging particles due to electric field distribution and reduces the effects of back corona, which is more difficult to determine for high collection efficiency of ESP.

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