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Bayesian Estimation of Parameters under the Constant Shape Bi-Weibull Distribution Using Extension of Jeffreys" Prior Information with Three Loss Functions

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Abstract: ***The Weibull distribution has been observed as one of the most useful distributions, for modeling and analyzing lifetime data in Engineering, Biology, Survival and other fields. Studies have been done vigorously in the literature to determine the best method in estimating its parameters. In this paper, we examine the performance of Maximum Likelihood Estimator and Bayesian Estimator using Extension of Jeffreys' Prior Information with three Loss functions, namely, the Linear Exponential Loss, General Entropy Loss, and Square Error Loss for estimating the Constant Shape Bi-Weibull failure time distribution. These methods are compared using Mean Square Error through Simulation Study with varying sample sizes. The results show that Bayesian Estimator using Extension of Jeffreys' Prior under Linear Exponential (LINEX) Loss function in most cases gives the smallest Mean Square Error and Absolute Bias for both the scale parameter σ and the shape parameter β for the given values of Extension of Jeffreys' Prior. An illustrative example is also provided to explain the concepts.***

Keywords: Constant Shape Bi-Weibull Distribution, MLE, Extension of Jeffreys Prior information, Bayesian method, Lindley"s approximation.

1. Introduction

The Weibull distribution is widely used in Reliability and life data analysis due to its versatility. Depending on the values of the parameters, the Weibull distribution can be used to model a variety of life behaviours. An important aspect of the Weibull distribution is how the values of the shape parameter, $\boldsymbol{\beta}$, and the scale parameter, $\boldsymbol{\sigma}$, affect the characteristics life of the distribution, the shape/slope of the distribution curve, the Reliability Function, and the Failure Rate. It has been found that this distribution is satisfactory in describing the life expectancy of components that involve fatigue and for assessing the Reliability of bulbs, ball bearings, and machine parts according to [13].The primary advantage of Weibull analysis according to [1] is its ability to provide accurate Failure Analysis and Failure Forecasts with extremely small samples. With Weibull, solutions are possible at the earliest indications of a problem without having to pursue further. Small samples also allow cost-effective component testing. Maximum Likelihood Estimation (MLE) has been the most widely used method for estimating the parameters of the Constant Shape Bi-Weibull distribution. In recent, work we developed Functional Relationship between Brier Score and Area Under the Constant Shape Bi-Weibull ROC Curve [8] and Confidence Intervals Estimation for ROC Curve, AUC and Brier Score under the Constant Shape Bi-Weibull Distribution [7].

Now the main objective of this paper is to compare the traditional Maximum Likelihood Estimation of the parameters of the Constant Shape Bi-Weibull distribution with its Bayesian counterpart using Extension of Jeffreys" Prior Information obtained from Lindley"s approximation procedure with three Loss Functions. Recently, Bayesian Estimation approach has received great attention by most researchers among them is [4]. They considered Bayesian Survival Estimator for Weibull distribution with censored data. While [2] studied Bayesian Estimation for the extreme value distribution using progressive censored data and Asymmetric Loss. Bayes Estimator for Exponential distribution with Extension of Jeffreys" Prior Information was considered by [5]. Others including [3, 6, and 10] did some comparative studies on the estimation of Weibull parameters using complete and censored samples and [9] determined Bayes Estimation of the extreme-value Reliability function.

In this paper, the Bayesian Estimation of Parameters under the Constant Shape Bi-Weibull Distribution is studied by Using Extension of Jeffreys" Prior Information with Three Loss Functions. This paper is organized as follows: In Section 2, estimation of parameters under MLE and Extension of Jeffreys" Prior Information with Three Loss functions are discussed. Section 3, provides simulation study for proposed theory. In Section 4, the proposed theory is validated by using real data. Finally conclusions are provided in Section 5.

2. Materials and Methods

Let $t_1, t_2 \ldots t_n$ be a random sample of size **n** with respect to the Constant Shape Bi-Weibull distribution, with $\boldsymbol{\sigma}$ and $\boldsymbol{\beta}$ as the parameters, where $\boldsymbol{\sigma}$ is the scale parameter and $\boldsymbol{\beta}$ is the shape parameter. The Probability density function (pdf), and Cumulative distribution function (cdf), are given, respectively,

$$f(t) = \frac{\beta}{\sigma} t^{\beta-1} e^{-\left[\frac{t^{\beta}}{\sigma}\right]}. \quad (1)$$

The Cumulative distribution function (CDF) is

$$F(t) = 1 - e^{-\left[\frac{t^{\beta}}{\sigma}\right]}. \quad (2)$$

2.1 Maximum Likelihood Estimation of Constant Shape Bi-Weibull Distribution

Since ($t_1, t_2 \ldots t_n$) is the set of $\boldsymbol{n}$ random lifetimes from the Constant Shape Bi-Weibull distribution, with $\boldsymbol{\sigma}$ and $\boldsymbol{\beta}$ as the parameters, where $\boldsymbol{\sigma}$ is the scale parameter and $\boldsymbol{\beta}$ is the shape parameter.

The likelihood function of the pdf is

$$L(t_i, \sigma, \beta) = \prod_{i=1}^{n} \frac{\beta}{\sigma} t_i^{\beta-1} e^{-\left[\frac{t_i^{\beta}}{\sigma}\right]}. \quad (3)$$

The log-likelihood function is

$$lnL = nln\beta + (\beta - 1)\left[\sum_{i=1}^{m} lnt_i\right] - nln\sigma - \frac{1}{\sigma}\sum_{i=1}^{n} t_i^{\beta}. \quad (4)$$

By differentiating the equation (4) with respect to $\boldsymbol{\sigma}$ and $\boldsymbol{\beta}$ and equating to zero, we get

$$\frac{\partial lnL}{\partial \sigma} = -\frac{n}{\sigma} + \frac{\sum_{i=1}^{n} t_i^{\beta}}{\sigma^2} = 0. \quad (5)$$

$$\frac{\partial lnL}{\partial \beta} = \frac{n}{\beta} + \left[\sum_{i=1}^{n} lnt_i\right] - \frac{1}{\sigma}\sum_{i=1}^{n} t_i^{\beta}\, lnt_i = 0. \quad (6)$$

From equation (5), we get

$$\hat{\sigma} = \frac{1}{n}\sum_{i=1}^{n} t_i^{\beta}. \quad (7)$$

First we shall find $\widehat{\boldsymbol{\beta}}$ and so that $\widehat{\boldsymbol{\sigma}}$ can be determined. So that we propose to find $\widehat{\boldsymbol{\beta}}$ by using Newton-Raphson method as given below. Let $f(\boldsymbol{\beta})$ be the same as equation (6) and taking the first differential of $f(\boldsymbol{\beta})$, we have

$$f'(\beta) = -\left(\frac{n}{\beta^2}\right) - \frac{1}{\sigma}\sum_{i=1}^{n} t_i^{\beta} (lnt_i)^2. \quad (8)$$

By substituting equation (7) into equation (6), we call $f(\boldsymbol{\beta})$ as

$$f(\beta) = \frac{n}{\beta} + \left[\sum_{i=1}^{n} lnt_i\right] - \frac{\sum_{i=1}^{n} t_i^{\beta}\, lnt_i}{\frac{1}{n}\sum_{i=1}^{n} t_i^{\beta}}. \quad (9)$$

Substituting equation (7) into equation (8), we obtain

$$f'(\beta) = -\left\{\frac{n}{\beta^2} + \frac{\sum_{i=1}^{n} t_i^{\beta} (lnt_i)^2}{\frac{1}{n}\sum_{i=1}^{n} t_i^{\beta}}\right\}. \quad (10)$$

Therefore, $\widehat{\boldsymbol{\beta}}$ is obtained from the equation below by carefully choosing an initial value $\boldsymbol{\beta}$ as $\boldsymbol{\beta_i}$ and iterating the process till it converges:

$$\beta_{i+1} = \beta_i - \frac{\frac{n}{\beta} + \left[\sum_{i=1}^{n} lnt_i\right] - \frac{\sum_{i=1}^{n} t_i^{\beta}\, lnt_i}{\frac{1}{n}\sum_{i=1}^{n} t_i^{\beta}}}{-\left\{\frac{n}{\beta^2} + \frac{\sum_{i=1}^{n} t_i^{\beta} (lnt_i)^2}{\frac{1}{n}\sum_{i=1}^{n} t_i^{\beta}}\right\}}. \quad (11)$$

2.2 Bayesian Estimation of Constant Shape Bi-Weibull Distribution

Bayesian Estimation approach has received a lot of attention in recent times for analyzing Failure Time data, which has mostly been proposed as an alternative to that of the traditional methods. Bayesian Estimation approach makes use of once prior knowledge about the parameters as well as the available data. When once prior knowledge about the parameter is not available, it is possible to make use of the noninformative prior in Bayesian analysis. Since we have no knowledge on the parameters, we seek to use the Extension of Jeffreys" Prior Information, where Jeffreys" Prior is the square root of the determinant of the Fisher information. According to [5], the extension of Jeffreys" prior is by taking $u(\theta) \propto [I(\theta)]^c$, $c\epsilon R^+$, so that

$$u(\theta) \propto \left[\frac{1}{\theta}\right]^{2c}. \quad (12)$$

Thus,

$$u(\sigma, \beta) \propto \left[\frac{1}{\sigma\beta}\right]^{2c}. \quad (13)$$

Given a sample t= ($t_1, t_2, \ldots, t_n$) from the likelihood function of the pdf (1) is

$$L(t_i \mid \sigma, \beta) = \prod_{i=1}^{n} \frac{\beta}{\sigma} t_i^{\beta-1} e^{-\left[\frac{t_i^{\beta}}{\sigma}\right]}. \quad (14)$$

With Bayes theorem, the joint posterior distribution of the parameters $\boldsymbol{\sigma}$ and $\boldsymbol{\beta}$ is

$$\pi^*(\sigma, \beta | t) \propto L(t|\, \sigma, \beta) u(\sigma, \beta)$$

$$L(t_i \mid \sigma, \beta) = \frac{k}{(\sigma\beta)^{2c}} \prod_{i=1}^{n} \frac{\beta}{\sigma} t_i^{\beta-1} e^{-\left[\frac{t_i^{\beta}}{\sigma}\right]}, \quad (15)$$

where $\boldsymbol{k}$ is the normalizing constant that makes $\boldsymbol{\pi}^*$ a proper pdf.

2.2.1 Asymmetric Loss Functions

Here we consider two Asymmetric Loss Functions namely Linear Exponential Loss Function (LINEX) and General Entropy Loss Function.

2.2.1(a) Linear Exponential Loss Function (LINEX)

The LINEX Loss Function is under the assumption that the minimal loss occurs at $\hat{\theta} = \theta$ and is expressed as

$$L(\hat{\theta} - \theta) \propto \exp\left(a(\hat{\theta} - \theta)\right) - a(\hat{\theta} - \theta) - 1, \quad (16)$$

where $\widehat{\boldsymbol{\theta}}$ is an estimation of $\boldsymbol{\theta}$ and $a \neq 0$. The sign and magnitude of the shape parameter „**a**" represents the direction and degree of symmetry, respectively. There is overestimation if $\mathbf{a} > 0$ and underestimation if $\mathbf{a} < 0$ but when $\mathbf{a} \cong 0$, the LINEX Loss Function is approximately the Squared Error Loss Function. The posterior expectation of the LINEX Loss Function, according to [10], is

$$E_\theta L(\hat{\theta} - \theta) \propto exp(a\hat{\theta}) E_\theta(\exp(-a\theta))$$

$$-a(\hat{\theta}-E_{\theta}(\theta))-1\,. \qquad (17)$$

The Bayes Estimator of **θ**, represented by $\hat{\boldsymbol{\theta}}_{BL}$ under LINEX Loss Function, is the value of $\hat{\boldsymbol{\theta}}$ which minimizes equation (17) and is given as

$$\hat{\theta}_{BL}=-\frac{1}{a}\ln E_{\theta}(exp(-a\theta)). \qquad (18)$$

Provided $E_{\theta}(exp(-a\theta)$ exists and is finite. The Bayes Estimator $\hat{\boldsymbol{u}}_{BL}$ of a function

$u=u(exp(-a\sigma),exp(-a\beta))$ is given as

$$\hat{u}_{BL}=E(exp(-a\sigma),exp(-a\beta)|t)=\frac{\iint u[exp(-a\sigma),exp(-a\beta)]\pi^{*}(\sigma,\beta)d\sigma d\beta}{\iint \pi^{*}(\sigma,\beta)d\sigma d\beta}. \qquad (19)$$

From (19), it can be observed that ratio of integrals which cannot be solved analytically and for that we employ Lindley‟s approximation procedure to estimate the parameters. Lindley considered an approximation for the ratio of integrals for evaluating the posterior expectation of an arbitrary function $\hat{\boldsymbol{u}}(\boldsymbol{\theta})$ as

$$E[u(\theta)|x]=\frac{\int u(\theta)v(\theta)[L(\theta)]d\theta}{\int v(\theta)[L(\theta)]d\theta}. \qquad (20)$$

According to [11], Lindley‟s expansion can be approximated asymptotically by

$$\hat{\theta}=u+\frac{1}{2}[u_{11}\delta_{11}+u_{22}\delta_{22}]+u_{1}\rho_{1}\delta_{11}+u_{2}\rho_{2}\delta_{22}+\frac{1}{2}[L_{30}u_{1}\delta^{2}{}_{11}+L_{03}u_{2}\delta^{2}{}_{22}], \qquad (21)$$

where L is the log-likelihood function in equation (4),

$u(\sigma)=exp(-a\sigma)$, $u_{1}=\frac{\partial u}{\partial\sigma}=-a\,exp(-a\sigma)$,

$u_{11}=\frac{\partial^{2}u}{\partial^{2}\sigma}=-a^{2}\,exp(-a\sigma)$, $u_{2}=u_{22}=0$,

$u(\beta)=exp(-a\beta)$, $u_{2}=\frac{\partial u}{\partial\beta}=-a\,exp(-a\beta)$,

$u_{22}=\frac{\partial^{2}u}{\partial^{2}\beta}=-a^{2}\,exp(-a\beta)$, $u_{1}=u_{11}=0$,

$\rho(\sigma,\beta)=-ln(\sigma^{2c})-ln(\beta^{2c})$,

$\rho_{1}=\frac{\partial\rho}{\partial\sigma}=-\frac{1}{\sigma^{2c}}$, $\rho_{2}=\frac{\partial\rho}{\partial\beta}=-\frac{1}{\beta^{2c}}$,

$\delta_{11}=(-L_{20})^{-1}$, $\delta_{22}=(-L_{02})^{-1}$,

$L_{02}=-\left(\frac{n}{\beta^{2}}\right)-\frac{1}{\sigma}\sum_{i=1}^{n}t_{i}^{\beta}\,(lnt_{i})^{2}$,

$L_{03}=2\left(\frac{n}{\beta^{3}}\right)-\frac{1}{\sigma}\sum_{i=1}^{n}t_{i}^{\beta}\,(lnt_{i})^{3}$,

$L_{20}=\frac{n}{\sigma^{2}}-2\frac{\sum_{i=1}^{n}t_{i}^{\beta}}{\sigma^{3}}$, and $L_{30}=-2\frac{n}{\sigma^{3}}+6\frac{\sum_{i=1}^{n}t_{i}^{\beta}}{\sigma^{4}}$.

2.2.1(b) General Entropy Loss Function

Another useful Asymmetric Loss Function is the General Entropy (GE) Loss which is a generalization of the Entropy Loss and is given as

$$L(\hat{\theta}-\theta)\propto\left(\frac{\hat{\theta}}{\theta}\right)^{k}-k\,ln\left(\frac{\hat{\theta}}{\theta}\right)-1\,. \qquad (22)$$

The Bayes Estimator $\hat{\boldsymbol{\theta}}_{BG}$ of **θ** under the General Entropy Loss is

$$\hat{\theta}_{BG}=[E_{\theta}(\theta^{-k})]^{-\frac{1}{k}}\;, \qquad (23)$$

provided $E_{\theta}(\theta^{-k})$ exists and is finite. The Bayes Estimator for this Loss Function is

$$\hat{u}_{BG}=E\{u[\sigma^{-k},\beta^{-k}]|t\}=\frac{\iint u[\sigma^{-k},\beta^{-k}]\,\pi^{*}(\sigma,\beta)d\sigma d\beta}{\iint \pi^{*}(\sigma,\beta)d\sigma d\beta}. \qquad (24)$$

Applying the same Lindley approach here as in (21) with u_1, u_{11} and u_2, u_{22} are the first and second derivatives for $\boldsymbol{\sigma}$ and $\boldsymbol{\beta}$, respectively, and are given as

$u=[\sigma^{-k}]$, $u_{1}=\frac{\partial u}{\partial\sigma}=-k[\sigma^{-k-1}]$,

$u_{11}=\frac{\partial^{2}u}{\partial^{2}\sigma}=-(-k^{2}-k)\sigma^{-k-2}$,

$u_{2}=u_{22}=0$,

$u=[\beta^{-k}]$, $u_{2}=\frac{\partial u}{\partial\beta}=-k[\beta^{-k-1}]$,

$u_{22}=\frac{\partial^{2}u}{\partial^{2}\beta}=-(-k^{2}-k)\beta^{-k-2}$ and $u_{1}=u_{11}=0$.

2.2.2 Symmetric Loss Function

The Squared Error Loss is given by

$$L(\hat{\theta}-\theta)\propto(\hat{\theta}-\theta)^{2}.$$

This Loss Function is symmetric in nature, that is, it gives equal weightage to both over and under estimation. In real life, we encounter many situations where overestimation may be more serious than underestimation or vice versa. The Bayes Estimator $\hat{\boldsymbol{u}}_{BS}$ of a function $u=u(\sigma,\beta)$ of the unknown parameters under Square Error Loss Function (SELF) is the posterior mean, where

$$\hat{u}_{BS}=E\{u[\sigma,\beta]|t\}=\frac{\iint u[\sigma,\beta]\,\pi^{*}(\sigma,\beta)d\sigma d\beta}{\iint \pi^{*}(\sigma,\beta)d\sigma d\beta}. \qquad (25)$$

Applying the same Lindley approach here as in (21) where u_1, u_{11} and u_2, u_{22} are the first and second derivatives for $\boldsymbol{\sigma}$ and **β**, respectively, and are given as

$u=\sigma$, $u_{1}=\frac{\partial u}{\partial\sigma}=1$, $u_{11}=u_{2}=u_{22}=0$,

$u=\beta$, $u_{2}=\frac{\partial u}{\partial\beta}=1$, $u_{22}=u_{1}=u_{11}=0$.

3. Simulation Study

Since it is difficult to compare the performance of the estimators theoretically and also to validate the data employed in this paper, we have performed extensive simulations to compare the estimators through Mean Squared Errors and Absolute Biases by employing different sample sizes with different parameter values. The Mean Squared Error and Absolute Bias given as

$$MSE=\frac{\sum_{r=1}^{5000}(\hat{\theta}^{r}-\theta)^{2}}{R-1}, and\ Abs=\frac{\sum_{r=1}^{5000}|\hat{\theta}^{r}-\theta|}{R-1}. \qquad (26)$$

In our Simulation study, we chose a sample size of $\boldsymbol{n}$= 25, 50, and 100 to represent small, medium, and large dataset. The scale and shape parameters are estimated for Constant

Shape Bi-Weibull distribution with Maximum Likelihood and Bayesian using Extension of Jeffrey‟s Prior methods.

The values of the parameters chosen are $\sigma = 0.5$ and 1.5, $\beta = 0.8$ and 1.2. The values of Jeffreys‟ Extension are $c = 0.4$ and 1.4. The values for the Loss parameters **(a, k)** are ***a*=*k***=±0.6 and ±1.6. These were iterated (*R)* 5000 times and the scale and shape parameters for each method were calculated. The results are presented below for the estimated parameters and their corresponding Mean Squared Error and Absolute Bias values.

In Table 3.1 we present the estimated values for the scale parameter σ for both the Maximum Likelihood Estimation and Bayesian Estimation using extension of Jeffrey‟s prior information with the three loss functions.

Table 3.1: Estimated values for Scale Parameter ***(σ)***

n	σ	c	β	$\hat{\sigma}_{ML}$	$\hat{\sigma}_{BS}$	$\hat{\sigma}_{BL}$	$\hat{\sigma}_{BG}$	$\hat{\sigma}_{BL}$	$\hat{\sigma}_{BG}$	$\hat{\sigma}_{BL}$	$\hat{\sigma}_{BG}$	$\hat{\sigma}_{BL}$	$\hat{\sigma}_{BG}$
						a = k = 0.6		a= k = -0.6		a =k =1.6		a = k = -1.6	
25	0.5	0.4	0.8	0.4924	0.5236	0.7289	1.5027	1.3664	0.6751	0.4264	3.0624	2.2802	0.3613
	0.5	0.4	1.2	0.3891	0.5548	0.7140	1.4681	1.3899	0.6975	0.3995	2.9520	2.3698	0.3992
	0.5	1.4	0.8	0.6006	0.4768	0.7501	1.5786	1.3293	0.6391	0.4618	3.4361	2.1226	0.3099
	0.5	1.4	1.2	0.4824	0.4689	0.7531	1.6035	1.3220	0.6317	0.4654	3.6041	2.0842	0.3038
	1.5	0.4	0.8	1.3972	1.5691	0.3820	0.7798	2.5155	1.3036	0.0685	0.5345	10.771	2.0968
	1.5	0.4	1.2	1.5376	1.5509	0.3878	0.7824	2.4968	1.2955	0.0733	0.5357	10.712	2.0520
	1.5	1.4	0.8	1.3342	1.6278	0.3669	0.7632	2.5971	1.3327	0.0587	0.5072	11.647	2.2212
	1.5	1.4	1.2	1.6934	1.5675	0.3848	0.7748	2.5275	1.3050	0.0726	0.5196	11.204	2.0801
50	0.5	0.4	0.8	0.5082	0.5107	0.7353	1.5101	1.3574	0.6667	0.4387	3.0489	2.2501	0.3443
	0.5	0.4	1.2	0.4254	0.5191	0.7313	1.5016	1.3636	0.6726	0.4314	3.0256	2.2588	0.3483
	0.5	1.4	0.8	0.5650	0.4874	0.7458	1.5501	1.3387	0.6485	0.4560	3.2535	2.1694	0.3191
	0.5	1.4	1.2	0.4071	0.4811	0.7481	1.5732	1.3326	0.6422	0.4583	3.4152	2.1356	0.3149
	1.5	0.4	0.8	1.6224	1.5218	0.3984	0.7836	2.4746	1.2839	0.0830	0.5292	10.861	1.9734
	1.5	0.4	1.2	1.9561	1.5135	0.4012	0.7844	2.4672	1.2804	0.0855	0.5286	10.867	1.9522
	1.5	1.4	0.8	1.4024	1.5549	0.3893	0.7754	2.5177	1.2999	0.0767	0.5170	11.262	2.0463
	1.5	1.4	1.2	1.6692	1.5349	0.3953	0.7792	2.4948	1.2907	0.0813	0.5209	11.120	1.9993
100	0.5	0.4	0.8	0.5551	0.5042	0.7386	1.5139	1.3523	0.6620	0.4445	3.0410	2.2329	0.3347
	0.5	0.4	1.2	0.4654	0.5070	0.7373	1.5113	1.3544	0.6639	0.4431	3.0379	2.2407	0.3384
	0.5	1.4	0.8	0.5928	0.4941	0.7431	1.5317	1.3446	0.6545	0.4525	3.1346	2.1993	0.3248
	0.5	1.4	1.2	0.3900	0.4902	0.7445	1.5463	1.3408	0.6506	0.4537	3.2383	2.1782	0.3224
	1.5	0.4	0.8	1.2553	1.5257	0.3978	0.7815	2.4828	1.2863	0.0874	0.5259	11.006	1.9793
	1.5	0.4	1.2	1.7461	1.5089	0.4031	0.7840	2.4653	1.2788	0.0831	0.5244	10.941	1.9383
	1.5	1.4	0.8	1.4984	1.5228	0.3993	0.7806	2.4833	1.2855	0.0847	0.5208	11.107	1.9689
	1.5	1.4	1.2	1.8155	1.5144	0.4018	0.7821	2.4739	1.2817	0.0867	0.5222	11.065	1.9535

ML: Maximum Likelihood, BS: Squared Error Loss function, BL: LINEX Loss function, BG: General Entropy Loss function.

From Table 3.1 it is observed that Bayes estimator under LINEX and General Entropy Loss functions tend to underestimate the scale parameter with MLE and Bayes estimation with Squared Error loss function slightly underestimating it. In Table 3.2 we present the estimated values for the shape parameter **β** for both the Maximum Likelihood Estimation and Bayesian Estimation using extension of Jeffrey‟s prior information with the three loss functions.

Table 3.2: Estimated values for Shape Parameter (***β***)

n	σ	c	β	$\hat{\beta}_{ML}$	$\hat{\beta}_{BS}$	$\hat{\beta}_{BL}$	$\hat{\beta}_{BG}$	$\hat{\beta}_{BL}$	$\hat{\beta}_{BG}$	$\hat{\beta}_{BL}$	$\hat{\beta}_{BG}$	$\hat{\beta}_{BL}$	$\hat{\beta}_{BG}$
						a=k=0.6		a=k=-0.6		a=k=1.6		a=k=-1.6	
25	0.5	0.4	0.8	0.8838	0.7872	0.6218	1.1671	1.5992	0.8638	0.2783	1.5358	3.4535	0.6898
	0.5	0.4	1.2	1.1513	0.5548	0.7140	1.4681	2.0154	1.1002	0.1461	0.8023	6.3088	1.3124
	0.5	1.4	0.8	0.9366	0.7775	0.6253	1.1763	1.5896	0.8573	0.2822	1.5683	3.3930	0.6768
	0.5	1.4	1.2	1.1012	1.1872	0.4874	0.9122	2.0261	1.1053	0.1432	0.7963	6.3849	1.3311
	1.5	0.4	0.8	0.7788	0.7973	0.6185	1.1550	1.6103	0.8711	0.2752	1.4880	3.5305	0.7019
	1.5	0.4	1.2	1.2821	1.1916	0.4864	0.9096	2.0323	1.1079	0.1426	0.7895	6.4524	1.3378
	1.5	1.4	0.8	0.8861	0.7899	0.6211	1.1626	1.6026	0.8660	0.2780	1.5160	3.4809	0.6922
	1.5	1.4	1.2	1.1041	1.2011	0.4846	0.9018	2.0484	1.1143	0.1425	0.7673	6.6597	1.3496
50	0.5	0.4	0.8	0.8905	0.7941	0.6201	1.1545	1.6082	0.8696	0.2780	1.4799	3.5292	0.6954
	0.5	0.4	1.2	1.2864	1.1887	0.4885	0.9066	2.0341	1.1077	0.1459	0.7769	6.5419	1.3257
	0.5	1.4	0.8	0.9124	0.7887	0.6220	1.1597	1.6028	0.8659	0.2802	1.4979	3.4954	0.6881
	0.5	1.4	1.2	1.2598	1.1930	0.4872	0.9046	2.0395	1.1101	0.1450	0.7724	6.5949	1.3339
	1.5	0.4	0.8	0.8170	0.7988	0.6186	1.1486	1.6134	0.8730	0.2767	1.4561	3.5664	0.7008
	1.5	0.4	1.2	1.1994	1.1987	0.4864	0.8993	2.0498	1.1141	0.1453	0.7570	6.7361	1.3401
	1.5	1.4	0.8	0.7163	0.7945	0.6201	1.1532	1.6089	0.8700	0.2782	1.4731	3.5369	0.6952
	1.5	1.4	1.2	1.1244	1.2011	0.4855	0.8987	2.0522	1.1153	0.1445	0.7563	6.7482	1.3451
100	0.5	0.4	0.8	0.8301	0.7970	0.6194	1.1489	1.6122	0.8722	0.2779	1.4559	3.5646	0.6978
	0.5	0.4	1.2	1.0879	1.1945	0.4876	0.9013	2.0444	1.1117	0.1462	0.7618	6.6852	1.3328
	0.5	1.4	0.8	0.7808	0.7950	0.6202	1.1506	1.6102	0.8708	0.2789	1.4599	3.5514	0.6946

	0.5	1.4	1.2	1.2115	1.1964	0.4870	0.9005	2.0468	1.1128	0.1458	0.7597	6.7075	1.3361
	1.5	0.4	0.8	0.8323	0.7990	0.6187	1.1471	1.6141	0.8735	0.2771	1.4485	3.5746	0.7004
	1.5	0.4	1.2	1.1115	1.1993	0.4865	0.8980	2.0518	1.1147	0.1459	0.7526	6.7730	1.3395
	1.5	1.4	0.8	0.8429	0.7974	0.6194	1.1479	1.6127	0.8725	0.2781	1.4499	3.5684	0.6976
	1.5	1.4	1.2	1.2494	1.2004	0.4861	0.8977	2.0530	1.1153	0.1455	0.7521	6.7849	1.3417

ML: Maximum Likelihood, BS: Squared Error Loss function, BL: LINEX Loss function, BG: General Entropy Loss function.

For the shape parameter **β**, it is clear from Table 3.2. Bayes estimator under LINEX Loss functions provides the smallest values compared to the others in most cases especially **a=k**=1.6.

In Table 3.3 we present the Mean Square Error estimated values for the scale parameter ***σ*** for both the MLE and Bayesian Estimation using extension of Jeffrey‟s prior information with the three loss functions.

Table 3.3: MSE Estimated Parameter (***σ***) of Constant Shape Bi-Weibull Distribution

n	σ	c	β	$\hat{\sigma}_{ML}$	$\hat{\sigma}_{BS}$	$\hat{\sigma}_{BL}$	$\hat{\sigma}_{BG}$	$\hat{\sigma}_{BL}$	$\hat{\sigma}_{BG}$	$\hat{\sigma}_{BL}$	$\hat{\sigma}_{BG}$	$\hat{\sigma}_{BL}$	$\hat{\sigma}_{BG}$
						a = k = 0.6		a= k = -0.6		a =k =1.6		a = k = -1.6	
25	0.5	0.4	0.8	0.01924	3.9e-07	9.4e-08	2.4e-07	2.0e-07	1.81e-07	3.3e-07	2.36e-07	2.5e-06	6.42e-07
	0.5	0.4	1.2	0.01217	4.7e-06	1.0e-06	6.6e-06	2.7e-06	2.47e-06	3.3e-06	8.43e-05	4.0e-05	6.74e-06
	0.5	1.4	0.8	0.01876	2.9e-07	4.3e-08	2.7e-06	2.4e-07	2.49e-07	6.4e-08	0.00012	6.6e-06	1.78e-07
	0.5	1.4	1.2	0.02618	1.9e-07	3.1e-08	1.6e-06	1.5e-07	1.62e-07	5.3e-08	6.84e-05	4.1e-06	1.38e-07
	1.5	0.4	0.8	0.21533	1.1e-06	1.3e-07	5.3e-09	7.7e-07	1.89e-07	1.1e-07	2.51e-08	1.0e-05	7.82e-06
	1.5	0.4	1.2	0.12672	1.1e-05	1.1e-06	3.0e-07	1.3e-05	2.38e-06	7.4e-07	1.64e-07	0.0002	6.91e-05
	1.5	1.4	0.8	0.22339	2.4e-07	2.4e-08	6.0e-08	2.7e-07	4.91e-07	1.5e-08	2.65e-09	4.7e-06	1.44e-06
	1.5	1.4	1.2	0.30899	1.0e-07	1.0e-07	2.0e-08	1.0e-06	2.02e-07	7.4e-08	2.99e-09	8.9e-06	6.38e-06
50	0.5	0.4	0.8	0.15144	1.3e-08	3.5e-09	2.2e-09	6.5e-09	5.57e-09	1.3e-08	8.78e-08	6.4e-08	2.54e-08
	0.5	0.4	1.2	0.12541	2.2e-08	5.7e-09	5.8e-09	1.1e-08	9.53e-09	2.1e-08	6.30e-08	1.1e-07	4.07e-08
	0.5	1.4	0.8	0.11348	4.7e-08	7.5e-09	3.7e-07	3.7e-08	3.82e-08	1.2e-08	1.59e-05	9.7e-07	3.31e-08
	0.5	1.4	1.2	0.18305	2.5e-08	4.0e-09	1.8e-07	1.9e-08	1.97e-08	7.4e-09	7.57e-06	4.9e-07	1.86e-08
	1.5	0.4	0.8	1.39003	7.3e-08	1.0e-08	1.1e-12	3.1e-08	1.08e-08	9.5e-09	8.36e-09	5.2e-06	5.75e-07
	1.5	0.4	1.2	1.42004	6.8e-08	9.9e-09	2.4e-14	2.8e-08	1.01e-08	9.0e-09	8.28e-09	5.2e-06	5.41e-07
	1.5	1.4	0.8	1.03816	4.6e-07	4.5e-08	1.0e-08	5.0e-07	9.11e-08	3.0e-08	3.46e-09	6.9e-06	2.75e-06
	1.5	1.4	1.2	1.84031	1.2e-07	1.3e-08	1.9e-09	1.1e-07	2.30e-08	9.3e-09	1.78e-11	3.5e-07	7.67e-07
100	0.5	0.4	0.8	0.61916	3.4e-09	8.8e-10	5.6e-10	1.6e-09	1.39e-09	3.4e-09	2.19e-08	1.6e-08	6.36e-09
	0.5	0.4	1.2	0.27268	5.5e-08	1.3e-08	4.8e-08	3.0e-08	2.69e-08	4.4e-08	2.30e-07	4.0e-07	8.68e-08
	0.5	1.4	0.8	0.78762	5.8e-09	9.4e-10	4.2e-08	4.5e-09	4.57e-09	1.7e-09	1.74e-06	1.1e-07	4.34e-09
	0.5	1.4	1.2	0.37559	1.5e-08	2.3e-09	1.3e-07	1.2e-08	1.27e-08	3.8e-09	5.70e-06	3.3e-07	1.02e-08
	1.5	0.4	0.8	4.44984	4.9e-09	5.6e-11	3.7e-08	2.0e-08	5.92e-09	4.2e-09	2.07e-09	1.1e-06	2.74e-07
	1.5	0.4	1.2	5.73241	1.7e-08	2.5e-09	5.9e-14	7.4e-09	2.59e-09	2.3e-09	2.07e-09	1.3e-06	1.38e-07
	1.5	1.4	0.8	4.45341	1.0e-07	1.0e-08	2.2e-09	1.0e-07	1.99e-08	6.8e-09	6.34e-10	1.3e-06	6.08e-07
	1.5	1.4	1.2	3.35240	2.3e-07	2.2e-08	6.3e-09	2.7e-07	4.73e-08	1.4e-08	3.90e-09	6.1e-06	1.35e-06

ML: Maximum Likelihood, BS: Squared Error Loss function, BL: LINEX Loss function, BG: General Entropy Loss function.

From Table 3.3 it is observed that Bayes estimation with LINEX loss function provides the smallest MSE values in most cases especially compared when the loss parameter values are (0.6, 1.6). Also sample size increases MLE has increased and Bayes estimation under all loss functions have decreases in MSE values.

In Table 3.4 we present the Mean Square Error estimated values for the shape parameter **β** for both the MLE and Bayesian Estimation using extension of Jeffrey‟s prior information with the three loss functions.

Table 3.4: MSE Estimated Parameter (***β***) of Constant Shape Bi-Weibull Distribution

n	σ	c	β	$\hat{\beta}_{ML}$	$\hat{\beta}_{BS}$	$\hat{\beta}_{BL}$	$\hat{\beta}_{BG}$	$\hat{\beta}_{BL}$	$\hat{\beta}_{BG}$	$\hat{\beta}_{BL}$	$\hat{\beta}_{BG}$	$\hat{\beta}_{BL}$	$\hat{\beta}_{BG}$
						a=k=0.6		*a=k=-0.6*		*a=k=1.6*		*a=k=-1.6*	
25	0.5	0.4	0.8	0.00025	3.8e-08	2.23e-09	1.2e-07	6.4e-08	2.7e-08	5.4e-11	2.5e-06	4.6e-06	2.4e-08
	0.5	0.4	1.2	0.00017	1.1e-07	3.34e-09	8.2e-08	3.3e-07	5.2e-08	5.1e-11	6.8e-07	5.8e-05	1.5e-07
	0.5	1.4	0.8	0.00010	9.9e-08	9.01e-09	1.9e-07	1.3e-07	5.7e-08	4.7e-09	3.3e-06	7.5e-06	1.1e-07
	0.5	1.4	1.2	0.00033	2.7e-08	3.45e-11	4.7e-08	1.4e-07	1.8e-08	2.7e-09	4.7e-07	3.6e-05	6.2e-09
	1.5	0.4	0.8	9.67e-05	2.5e-09	5.57e-12	4.3e-08	1.1e-08	4.1e-09	2.0e-09	1.0e-06	1.3e-06	8.3e-10
	1.5	0.4	1.2	0.00023	1.3e-08	1.57e-11	3.3e-08	9.3e-08	1.1e-08	2.9e-09	3.4e-07	2.5e-05	2.1e-10
	1.5	1.4	0.8	5.98e-05	2.0e-08	1.25e-09	6.5e-08	3.3e-08	1.4e-08	5.6e-11	1.2e-06	2.3e-06	1.3e-08
	1.5	1.4	1.2	9.26e-05	4.7e-10	9.80e-10	5.6e-09	6.1e-09	2.0e-10	3.4e-09	8.1e-08	5.0e-06	2.7e-08
50	0.5	0.4	0.8	0.00034	6.9e-09	3.50e-10	2.7e-08	1.2e-08	5.3e-09	4.5e-12	5.6e-07	9.7e-07	3.4e-09
	0.5	0.4	1.2	0.00056	2.3e-08	5.44e-10	1.9e-08	7.7e-08	1.1e-08	9.5e-11	1.7e-07	1.4e-05	2.7e-08
	0.5	1.4	0.8	0.00023	2.5e-08	2.23e-09	5.1e-08	3.4e-08	1.4e-08	1.1e-09	8.8e-07	1.9e-06	2.7e-08
	0.5	1.4	1.2	0.00050	2.7e-09	1.88e-11	8.1e-09	2.1e-08	2.4e-09	8.8e-10	8.5e-08	6.3e-06	4.3e-11
	1.5	0.4	0.8	5.64e-05	1.2e-10	1.04e-11	4.4e-09	9.2e-10	3.3e-10	3.5e-10	1.1e-07	1.3e-07	3.6e-10
	1.5	0.4	1.2	8.84e-05	3.9e-10	3.61e-11	2.4e-09	5.5e-09	5.5e-10	4.2e-10	2.7e-08	1.9e-06	6.1e-10

	1.5	1.4	0.8	7.70e-05	5.1e-09	2.96e-10	1.7e-08	5.0e-07	9.1e-08	3.9e-12	3.5e-07	6.3e-07	3.1e-09
	1.5	1.4	1.2	8.29e-05	1.7e-10	2.12e-10	9.3e-10	8.3e-10	1.5e-11	6.7e-10	1.4e-08	8.4e-07	6.0e-09
100	0.5	0.4	0.8	0.00031	1.7e-09	9.02e-11	6.5e-09	3.0e-09	1.2e-09	1.5e-13	1.3e-07	2.3e-07	9.1e-10
	0.5	0.4	1.2	0.00031	6.7e-09	1.78e-10	5.2e-09	2.1e-08	3.2e-09	1.1e-11	4.4e-08	3.7e-06	8.5e-09
	0.5	1.4	0.8	0.00033	5.5e-09	4.67e-10	1.1e-08	7.6e-09	3.2e-09	1.9e-10	2.1e-07	4.5e-07	5.7e-09
	0.5	1.4	1.2	0.00032	1.7e-09	5.76e-12	2.8e-09	8.8e-09	1.1e-09	1.3e-10	2.7e-08	2.1e-06	6.1e-10
	1.5	0.4	0.8	6.45e-05	4.7e-11	2.71e-12	1.4e-09	3.1e-10	1.1e-10	1.0e-10	3.8e-08	4.5e-08	1.0e-10
	1.5	0.4	1.2	0.00010	1.2e-10	8.85e-12	7.0e-10	1.6e-09	1.6e-10	1.1e-10	7.9e-09	5.6e-07	1.3e-10
	1.5	1.4	0.8	6.38e-05	1.0e-09	6.06e-11	3.4e-09	1.7e-09	7.2e-10	1.3e-12	6.8e-08	1.2e-07	6.5e-10
	1.5	1.4	1.2	0.00019	1.2e-11	8.71e-11	7.1e-10	9.5e-10	4.7e-11	3.4e-10	9.8e-09	6.2e-07	2.3e-09

ML: Maximum Likelihood, BS: Squared Error Loss function, BL: LINEX Loss function, BG: General Entropy Loss function.

From Tables 3.4, Bayesian estimation under LINEX loss gives smaller Mean Squared Error as compared to the others. It is observed again from Table 3.4 that as the sample size increases, the Mean Squared Error values of the MLE has increased and Bayes estimation under all loss functions have decreases.

In Table 3.5 we present the Absolute Bias estimated values for the scale parameter σ for both the Maximum Likelihood Estimation and Bayesian Estimation using extension of Jeffrey‟s prior information with the three loss functions.

Table 3.5: Absolute Bias Estimated Parameter (σ) of Constant Shape Bi-Weibull Distribution

n	σ	c	β	$\hat{\sigma}_{ML}$	$\hat{\sigma}_{BS}$	$\hat{\sigma}_{BL}$	$\hat{\sigma}_{BG}$	$\hat{\sigma}_{BL}$	$\hat{\sigma}_{BG}$	$\hat{\sigma}_{BL}$	$\hat{\sigma}_{BG}$	$\hat{\sigma}_{BL}$	$\hat{\sigma}_{BG}$
						a = k = 0.6		a= k = -0.6		a =k =1.6		a = k = -1.6	
25	0.5	0.4	0.8	0.00196	8.8e-06	4.3e-06	6.9e-06	6.4e-06	6.02e-06	8.1e-06	6.87e-06	2.2e-05	1.13e-05
	0.5	0.4	1.2	0.00156	3.0e-05	1.4e-05	3.6e-05	2.3e-05	2.22e-05	2.5e-05	0.00012	9.0e-05	3.67e-05
	0.5	1.4	0.8	0.00193	7.6e-06	2.9e-06	2.3e-05	6.9e-06	7.06e-06	3.5e-06	0.00015	3.6e-05	5.97e-06
	0.5	1.4	1.2	0.00228	6.3e-06	2.5e-06	1.7e-05	5.6e-06	5.76e-06	3.2e-06	0.00011	2.8e-05	5.27e-06
	1.5	0.4	0.8	0.00656	1.5e-05	5.2e-06	1.0e-06	1.2e-05	6.16e-06	4.7e-06	2.24e-06	4.5e-05	3.95e-05
	1.5	0.4	1.2	0.00503	4.8e-05	1.5e-05	7.8e-06	5.2e-05	2.18e-05	1.2e-05	5.73e-06	0.0002	0.00011
	1.5	1.4	0.8	0.00668	2.2e-05	6.9e-06	3.4e-06	2.3e-05	9.91e-06	5.6e-06	2.30e-06	9.7e-05	5.38e-05
	1.5	1.4	1.2	0.00786	1.4e-05	4.6e-06	2.0e-06	1.4e-05	6.36e-06	3.8e-06	7.74e-07	4.2e-05	3.57e-05
50	0.5	0.4	0.8	0.00550	1.6e-06	8.4e-07	6.7e-07	1.1e-06	1.05e-06	1.6e-06	4.19e-06	3.6e-06	2.25e-06
	0.5	0.4	1.2	0.00500	2.1e-06	1.0e-06	1.0e-06	1.4e-06	1.38e-06	2.0e-06	3.55e-06	4.8e-06	2.85e-06
	0.5	1.4	0.8	0.00476	3.0e-06	1.2e-06	8.6e-06	2.7e-06	2.76e-06	1.6e-06	5.64e-05	1.3e-05	2.57e-06
	0.5	1.4	1.2	0.00605	2.2e-06	9.0e-07	6.0e-06	1.9e-06	1.98e-06	1.2e-06	3.89e-05	9.9e-06	1.93e-06
	1.5	0.4	0.8	0.01667	3.8e-06	1.4e-06	1.4e-08	2.5e-06	1.47e-06	1.3e-06	1.29e-06	3.2e-05	1.07e-05
	1.5	0.4	1.2	0.01685	3.7e-06	1.4e-06	2.2e-09	2.4e-06	1.42e-06	1.3e-06	1.28e-06	3.2e-05	1.04e-05
	1.5	1.4	0.8	0.01441	9.6e-06	3.0e-06	1.4e-06	1.0e-05	4.27e-06	2.4e-06	8.32e-07	3.7e-05	2.34e-05
	1.5	1.4	1.2	0.01918	4.9e-06	1.6e-06	6.1e-07	4.8e-06	2.14e-06	1.3e-06	5.96e-08	8.4e-06	1.23e-05
100	0.5	0.4	0.8	0.01112	8.2e-07	4.2e-07	3.3e-07	5.7e-07	5.28e-07	8.2e-07	2.09e-06	1.8e-06	1.12e-06
	0.5	0.4	1.2	0.00738	3.3e-06	1.6e-06	3.1e-06	2.4e-06	2.32e-06	2.9e-06	6.78e-06	8.9e-06	4.16e-06
	0.5	1.4	0.8	0.01255	1.0e-06	4.3e-07	2.9e-06	9.5e-07	9.56e-07	5.8e-07	1.86e-05	4.7e-06	9.31e-07
	0.5	1.4	1.2	0.00866	1.7e-06	6.9e-07	5.1e-06	1.5e-06	1.59e-06	8.7e-07	3.37e-05	8.1e-06	1.42e-06
	1.5	0.4	0.8	0.02983	2.7e-06	9.9e-07	1.0e-07	2.0e-06	1.08e-06	9.1e-07	6.43e-07	1.5e-05	7.40e-06
	1.5	0.4	1.2	0.03386	1.8e-06	7.1e-07	3.4e-09	1.2e-06	7.20e-07	6.7e-07	6.44e-07	1.6e-05	5.26e-06
	1.5	1.4	0.8	0.02984	4.5e-06	1.4e-06	6.6e-07	4.6e-06	1.99e-06	1.1e-06	3.56e-07	1.6e-05	1.10e-05
	1.5	1.4	1.2	0.02589	6.8e-06	2.1e-06	1.1e-06	7.4e-06	3.07e-06	1.6e-06	8.83e-07	3.5e-05	1.64e-05

ML: Maximum Likelihood, BS: Squared Error Loss function, BL: LINEX Loss function, BG: General Entropy Loss function.

From Table 3.5 it is observed that Bayes estimation with LINEX loss function provides the smallest Absolute Bias values in most cases and as the sample size increases Absolute values of the MLE increased and Bayes estimation under all loss functions decreases.

In Table 3.6 we present the Absolute Bias estimated values for the shape parameter β for both the Maximum Likelihood Estimation and Bayesian Estimation using extension of Jeffrey‟s prior information with the three loss functions.

Table 3.6: Absolute Bias Estimated Parameter (β) of Constant Shape Bi-Weibull Distribution

n	σ	c	β	$\hat{\beta}_{ML}$	$\hat{\beta}_{BS}$	$\hat{\beta}_{BL}$	$\hat{\beta}_{BG}$	$\hat{\beta}_{BL}$	$\hat{\beta}_{BG}$	$\hat{\beta}_{BL}$	$\hat{\beta}_{BG}$	$\hat{\beta}_{BL}$	$\hat{\beta}_{BG}$
						a=k=0.6		a=k=-0.6		a=k=1.6		a=k=-1.6	
25	0.5	0.4	0.8	0.00022	2.7e-06	6.7e-07	5.0e-06	3.5e-06	2.3e-06	1.0e-07	2.2e-05	3.0e-05	2.2e-06
	0.5	0.4	1.2	0.00018	4.7e-06	8.1e-07	4.0e-06	8.2e-06	3.2e-06	1.0e-07	1.1e-05	0.0001	5.5e-06
	0.5	1.4	0.8	0.00014	4.4e-06	1.3e-06	6.2e-06	5.1e-06	3.3e-06	9.7e-07	2.5e-05	3.8e-05	4.7e-06
	0.5	1.4	1.2	0.00025	2.3e-06	8.3e-08	3.0e-06	5.4e-06	1.9e-06	7.3e-07	9.7e-06	8.5e-05	1.1e-06
	1.5	0.4	0.8	0.00013	7.2e-07	3.3e-08	2.9e-06	1.4e-06	9.1e-07	6.4e-07	1.4e-05	1.6e-05	4.0e-07
	1.5	0.4	1.2	0.00021	1.6e-06	5.6e-08	2.5e-06	4.3e-06	1.5e-06	7.6e-07	8.3e-06	7.2e-05	2.0e-07
	1.5	1.4	0.8	0.00010	2.0e-06	5.0e-07	3.6e-06	2.5e-06	1.6e-06	1.0e-07	1.6e-05	2.1e-05	1.6e-06
	1.5	1.4	1.2	0.00013	3.0e-07	4.4e-07	1.0e-06	1.1e-06	2.0e-07	8.2e-07	4.0e-06	3.1e-05	2.3e-06
50	0.5	0.4	0.8	0.00026	1.1e-06	2.6e-07	2.3e-06	1.6e-06	1.0e-06	3.0e-08	1.0e-05	1.3e-05	8.3e-07
	0.5	0.4	1.2	0.00033	2.1e-06	3.3e-07	1.9e-06	3.9e-06	1.5e-06	1.3e-07	5.8e-06	5.3e-05	2.3e-06

	0.5	1.4	0.8	0.00021	2.2e-06	6.6e-07	3.2e-06	2.6e-06	1.7e-06	4.6e-07	1.3e-05	1.9e-05	2.3e-06
	0.5	1.4	1.2	0.00031	7.3e-07	6.1e-08	1.2e-06	2.0e-06	7.0e-07	4.1e-07	4.1e-06	3.5e-05	9.3e-08
	1.5	0.4	0.8	0.00010	1.6e-07	4.5e-08	9.4e-07	4.2e-07	2.5e-07	2.6e-07	4.8e-06	5.2e-06	2.7e-07
	1.5	0.4	1.2	0.00013	2.8e-07	8.5e-08	6.9e-07	1.0e-06	3.3e-07	2.9e-07	2.3e-06	1.9e-05	3.4e-07
	1.5	1.4	0.8	0.00012	1.0e-06	2.4e-07	1.8e-06	1.3e-06	8.5e-07	2.8e-08	8.4e-06	1.1e-05	7.9e-07
	1.5	1.4	1.2	0.00013	1.8e-07	2.0e-07	4.3e-07	4.0e-07	5.6e-08	3.6e-07	1.6e-06	1.3e-05	1.1e-06
100	0.5	0.4	0.8	0.00025	5.8e-07	1.3e-07	1.1e-06	7.8e-07	5.0e-07	5.4e-09	5.1e-06	6.8e-06	4.2e-07
	0.5	0.4	1.2	0.00025	1.1e-06	1.8e-07	1.0e-06	2.0e-06	8.0e-07	4.7e-08	2.9e-06	2.7e-05	1.3e-06
	0.5	1.4	0.8	0.00026	1.0e-06	3.0e-07	1.5e-06	1.2e-06	8.1e-07	1.9e-07	6.4e-06	9.4e-06	1.0e-06
	0.5	1.4	1.2	0.00026	5.9e-07	3.3e-08	7.4e-07	1.3e-06	4.8e-07	1.6e-07	2.3e-06	2.0e-05	3.5e-07
	1.5	0.4	0.8	0.00011	9.7e-08	2.3e-08	5.4e-07	2.5e-07	1.5e-07	1.4e-07	2.7e-06	3.0e-06	1.4e-07
	1.5	0.4	1.2	0.00014	1.6e-07	4.2e-08	3.7e-07	5.7e-07	1.8e-07	1.5e-07	1.2e-06	1.0e-05	1.6e-07
	1.5	1.4	0.8	0.00011	4.5e-07	1.1e-07	8.3e-07	5.8e-07	3.8e-07	1.6e-08	3.7e-06	4.9e-06	3.6e-07
	1.5	1.4	1.2	0.00019	4.9e-08	1.3e-07	3.7e-07	4.3e-07	9.7e-08	2.6e-07	1.4e-06	4.9e-08	6.8e-07

ML: Maximum Likelihood, BS: Squared Error Loss function, BL: LINEX Loss function, BG: General Entropy Loss function.

Similarly, it has also been observed from Table 3.6 that the estimator that gives the minimum Absolute Bias over all the other estimators in majority of the cases is Bayes estimator under LINEX loss function.

4. Illustration

The real data set is about Worcester Heart Attack (WHA) Study extracted from [14]. The data represent study is to describe factors associated with trends over time in the incidence and survival rates following hospital admission for acute myocardial infarction (MI). Data have been collected during thirteen 1-year periods beginning in 1975 and extending through 2001 on all MI patients admitted to hospitals in the Worcester, Massachusetts Standard Metropolitan Statistical Area. This data consists of a total of 100 respondents of which 49 are alive and 51 are dead. Here we considered Follow up Survival Time is the most factors. Since we do not have any prior information on the hyper parameters, we assume as 0.01. Table 4.1 depicts the Estimated values for Scale Parameter (σ) and scale parameter (β) using WHA Study Data.

Table 4.1: Estimated values for Scale Parameter (σ) and scale parameter (β) using WHA Study Data

Parameters	MLE	BS	BL (a=k=0.6)	BG (a=k=0.6)	BL (a=k=1.6)	BG (a=k=1.6)
$\hat{\sigma}$	9853.3	0.4999	0.4408	1.5157	0.4493	3.0314
$\hat{\beta}$	1.2136	1.2000	0.4867	0.8963	0.1466	0.7469

From Table 4.1, we observe that, Bayesian estimator under LINEX loss function has the smallest values for both the scale parameter σ and the shape parameter β. So that the Bayes estimators of parameters under LINEX loss function is best estimation method for Constant Shape Bi-Weibull Distribution using WHA Study Data.

5. Conclusion

In this paper, we have addressed the problem of Bayesian estimation for the Constant Shape Bi-Weibull distribution, under Asymmetric and Symmetric loss functions and that of Maximum Likelihood Estimation. Bayes estimators were obtained using Lindley approximation while MLE were obtained using Newton-Raphson method. A Simulation study was conducted to examine and compare the performance of the estimates for different sample sizes with different values for the extension of Jeffreys‟ prior and the loss functions. From the results, we observe that in most cases, Bayesian estimator under LINEX loss function has the smallest Mean Squared Error values and minimum Bias for both the scale parameter σ and the shape parameter β in most cases especially compared when the loss parameter values are (0.6, 1.6), for both values of the extension of Jeffreys‟ prior information. As the sample size increases the Mean Squared Error and the Absolute Bias for Maximum Likelihood Estimator has increased and Bayes estimator under all the loss functions decreases correspondingly.

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Author Profile

Dr. T. Leo Alexander currently serving as an Associate Professor in the Department of Statistics, Loyola College, Chennai, India. For the past three decades. Teaching Under Graduate, Post Graduate courses and guiding research students for their M.Phil and Ph.D. Served as Controller of Examinations as well as Registrar i/c at University of Madras. He has a number of published research papers in reputed journals. He has also participated in many conferences in India.

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