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Missing Terms Formula for Tribonacci-Like Sequences

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Abstract: ***Tribonacci sequences are generalization of Fibonacci sequences. Tribonacci sequence is defined by $T_n=T_{n-1}+T_{n-2}+T_{n-3}$, $n\geq3$ with $T_0=0, T_1=1$ and $T_2=1$. In recent years, few research scholars have been introduced Fibonacci-Like sequences which are similar to Fibonacci sequences in recurrence relation, but initial conditions are different. Due to this reason, these are known as Fibonacci-Like sequences. Tribonacci-Like sequences can be established by same pattern. A formula derived by Natividad to find missing terms for Fibonacci-like sequence. In this paper, we present a formula to find consecutive missing terms for Tribonacci-Like Sequences in terms of conventional Tribonacci numbers.***

Keywords: Tribonacci Sequence, Tribonacci-Like Sequence, Missing term, Binet's Formula.

Mathematics Subject Classification 2010: 11B39,11B37

1. Introduction

Sequence is a list of numbers arranged a specific order. It can contain members similar to a set. However sequence can have the same members repeated as much as possible at a divergent locations. Thus patterns are a substantial element of a sequence. Fibonacci sequence is a sequence starting from 0 and 1 where the succeeding terms are taken from two previous terms that are added. Fibonacci sequence [1, 2] is defined by $F_n = F_{n-1} + F_{n-2}$, $n \geq 2$, with $F_0 = 0$, $F_1 = 1$.

Tribonacci sequences are generalization of Fibonacci sequences. Tribonacci sequence [8, 10, 11] is defined by $T_n=T_{n-1}+T_{n-2}+T_{n-3}$, $n\geq3$ with $T_0=0, T_1=1$ and $T_2=1$. The few terms are 0, 1, 1, 2, 4, 7……. This only means that the previous three terms are added to find the next term. It is also referred as series of Tribonacci numbers.

In recent years, few research scholars have been introduced Fibonacci-Like sequences which are similar to Fibonacci sequences in recurrence relation, but initial conditions are different. Due to this reason, these are known as Fibonacci-Like sequences [3, 4, 5, 6, 8]. Tribonacci-Like sequences can be established by same pattern. Noe, Piezas and Weisstein [4] listed two explicit formulas for Tribonacci sequence. The implicit formula is

$$T_n = \frac{\alpha^{n+1}}{(\alpha-\beta)(\alpha-\gamma)} + \frac{\beta^{n+1}}{(\beta-\alpha)(\beta-\gamma)} + \frac{\gamma^{n+1}}{(\gamma-\alpha)(\gamma-\beta)}, \quad (1.1)$$

where α, β and γ are the three roots of the polynomial $x^3 - x^2 - x - 1$. They gave also another explicit formula in the form of

$$T_n = \left[3\frac{\left\{\frac{1}{3}(19+3\sqrt{33})^{1/3} + \frac{1}{3}(19-3\sqrt{33})^{1/3} + \frac{1}{3}\right\}^{1/n}\left(586+102\sqrt{33}\right)^{1/3}}{\left(586+102\sqrt{33}\right)^{2/3} + 4 - 2\left(586+102\sqrt{33}\right)^{1/3}} \right] \quad (1.2)$$

P. Howell [9] presented a proof for finding n^{th} term of Fibonacci sequence using vectors and eigen values but did not account Fibonacci-like sequence. M. Agnes. et al. [7] provided a formula for inclusion of missing terms in Fibonacci-like sequence but only for three consecutive missing terms. A formula derived by Natividad [6] in solving Fibonacci-like sequence from Binet's formula regardless of the number of consecutive missing terms.

In this paper, we present a formula to find consecutive missing terms for Tribonacci-Like Sequences in terms of conventional Tribonacci numbers.

2. Main Results

Before moving to the general formula, it is imperative to observe the specific formula from the basic problem. Basic formula will be studied and from this a general will be deduced. In this derivation, x_1 stands for the first term, x_2 stands for second term given and l stands for last term given in any Tribonacci-like sequence.

2.1. One Missing Term

Consider the Tribonacci-like sequence x_1, x_2, x_3,l where in there is one term missing denoted by x_3, we could solve the missing term because the next number in Tribonacci sequence is found by adding up the three number before it. In mathematical statement $x_1 + x_2 + x_3 = l$, rearranging the terms

$$x_3 = l - x_2 - x_1 \quad (2.1)$$

This basic formula will be used in finding the other formulas.

2.2. Two Consecutive Mission Terms.

Now consider Tribonacci-like sequence as x_1, x_2, x_3, x_4, l, where x_3 and x_4 are two consecutive missing terms. By formula (2.1), we have

$$x_3 = x_4 - x_1 - x_2 \quad (2.2)$$
$$x_4 = l - x_3 - x_2 \quad (2.3)$$

Substituting (2.2) in (2.3) to find x_4

$x_4 = l - (x_4 - x_1 - x_2) - x_2$ or

$2x_4 = l + x_1$ or

$$x_4 = \frac{l + x_1}{2} \quad (2.4)$$

Substituting above in (2.2), we obtain

$$x_3 = \frac{l + x_1}{2} - x_1 - x_2 \text{ or}$$

$$x_3 = \frac{l - x_1 - 2x_2}{2} \quad (2.5)$$

2.3. Three Consecutive Missing Terms

Consider Tribonacci-like Sequence as $x_1, x_2, x_3, x_4, x_5, l$

$$x_3 = x_4 - x_1 - x_2 \quad (2.6)$$

$$x_4 = x_5 - x_2 - x_3 = x_5 - x_2 - (x_4 - x_1 - x_2)$$

$$x_4 = \frac{x_5 + x_1}{2} \quad (2.7)$$

$$x_5 = l - x_4 - x_3 \quad (2.8)$$

Using (2.6) and (2.7) in (2.8), we get

$$x_5 = \frac{2l + 2x_2}{4} = \frac{l + x_2}{2} \quad (2.9)$$

Substituting (2.9) in (2.7), we get

$$x_4 = \frac{\frac{l + x_2}{2} + x_1}{2}$$

$$x_4 = \frac{l + x_2 + 2x_1}{4} \quad (2.10)$$

Substitution above in (2.6) to find x_3

$$x_3 = \frac{l + x_2 + 2x_1}{4} - x_1 - x_2 = \frac{l - 2x_1 - 3x_2}{4} \quad (2.11)$$

2.4 Four Consecutive Missing Terms

Consider Tribonacci-like sequence. $x_1, x_2, x_3, x_4, x_5, x_6, l$ and missing terms can be determine by

$$x_3 = \frac{l - 4x_1 - 6x_2}{7} \quad (2.12)$$

$$x_4 = \frac{l + 3x_1 + x_2}{7} \quad (2.13)$$

$$x_5 = \frac{2l - x_1 + 2x_2}{7} \quad (2.14)$$

$$x_6 = \frac{4l - 2x_1 - 3x_2}{7} \quad (2.15)$$

3. The General Formula for Consecutive Missing Terms

Consecutive missing terms of Tribonacci-Like sequence are tabulated below:

Formula of consecutive missing terms for Tribonacci-Like sequence

Number of missing terms	Formula			
	x_3	x_4	x_5	x_6
1.	$l - x_1 - x_2$	-	-	-
2.	$\frac{l - x_1 - 2x_2}{2}$	$\frac{l + x_1}{2}$	-	-
3.	$\frac{l - 2x_1 - 3x_2}{4}$	$\frac{l + x_1 + 2x_2}{4}$	$\frac{2l + 2x_2}{4}$	-
4.	$\frac{l - 4x_1 - 6x_2}{7}$	$\frac{l + 3x_1 + x_2}{7}$	$\frac{2l - x_1 + 2x_2}{7}$	$\frac{4l - 2x_1 - 3x_2}{7}$

Depend upon above tabulated expressions, observe pattern regarding coefficients of x_1 and x_2 in numerator and the denominator value of the formulas for x_3. This pattern tabulated below:

General terms of coefficients of x_1 and x_2 and the denominator value of the formulas for x_3

Number of missing terms	Coefficient of x_1 in Numerator	Coefficient of x_2 in Numerator	Coefficient of Denominator
1	1	1	1
2	1	2	2
3	2	3	4
4	4	6	7
:	:	:	:
N	T_n	$T_{n-1} + T_n$	T_{n+1}

Using the observation, the general formula for x_3 is

$$x_3 = \frac{l - T_n x_1 - (T_{n-1} + T_n)x_2}{T_{n+1}}, \ n \geq 1 \text{ or}$$

$$x_3 = \frac{l - (1.837)^n (0.336) x_1 - \left[(1.837)^{n-1} + (1.837)^n\right](.336) x_2}{(1.837)^{n+1}(.336)} \quad (3.1)$$

where x_3 is the first missing term in Tribonacci-like sequence, x_1 and x_2 are the first and second terms given, l is the last term given and n is the number of missing terms.

Note that the formula for finding the n[th] term of Tribonacci sequence (also known as Binet's formula) is

$$T_n = \left[3\frac{\left\{\frac{1}{3}(19+3\sqrt{33})^{1/3} + \frac{1}{3}(19-3\sqrt{33})^{1/3} + \frac{1}{3}\right\}^n \left(586+102\sqrt{33}\right)^{1/3}}{\left(586+102\sqrt{33}\right)^{2/3} + 4 - 2\left(586+102\sqrt{33}\right)^{1/3}} \right]$$

Or

$$T_n = (1.837)^n (.336) \ \ldots \ (3.2)$$

To fully understand the formula, we answer the presented problem by inserting five terms between 2, 3 and 138 to make it a Tribonacci-like sequence. In mathematical statement the sequence is 2, 3, x_3, x_4, x_5, x_6, x_7, 138, …
This problem will be solved by finding x_3 using (3.1)
Given $x_1 = 2$, $x_2 = 3$, $n = 5$, $l = 138$

$$x_3 = \frac{l - (1.837)^n (.336) x_1 - \left[(1.837)^{n-1} + (1.837)^n\right](.336) x_2}{(1.837)^{n+1}(.336)}$$

$$x_3 = \frac{138 - (1.837)^5 (.336)2 - \left[(1.837)^{5-1} + (1.837)^5\right](.336)3}{(1.837)^{5+1}(.336)} = 7$$

Now that we have solved from x_3, we could easily find x_4 which is $x_4 = x_3 + 3 + 2 = 12$.

Similarly, we can find x_5, x_6, x_7…
2, 3, 7, 12, 22, 41, 75, 138,...

4. Conclusion

In this paper we have presented formula for finding consecutive missing terms of Tribonacci-Like Sequences in terms of conventional Tribonacci numbers.

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