

Identification of Proper Distribution for Flood Frequency Analysis of Sarda River using L-Moments

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Abstract: Flood frequency analysis of River Sarda (Banbasa Barrage) site in Champawat district in Uttarakhand was carried out using annual flood peak series for 86 years (1930-2015). The objective of the study was to determine the proper probability distribution using Generalized Extreme Value distribution (GEV), generalized logistic distribution (GLO) and generalized Pareto distribution (GPA), Pearson type 3rd distribution (PE3) and wakeby five parameter distribution whose parameters were estimated using the method of L-moments. The proper fit distribution is selected on the basis Z^{dist} - statistic test. The present study shows that wakeby five parameter distribution identified as the proper distribution for analyzing the flood frequency analysis of Sarda river using L-moment

Keywords: flood frequency analysis; L-moments; probability distribution; proper distribution

1. Introduction

In Hydraulic engineering since more than thirty years design of flood frequency analysis using statistical approaches has become one of the robust and active field of application in research work in water resource engineering. Concept of L-moments first time given by Hosking(1990).Hosking defined L-moments as the linear combination of probability weighted moments. Hosking estimated the parameters of different types of distribution using the linear combination of order of statistics on the basis of L-moments.The role of the both probability weighted moments and L-moments are find out the theoretical distribution for the sample data.

Recent years a number of regional flood frequency analysis studies have been carried out based on the L-moments approach. The L-moment methods are demonstrated superior to those that have been used previously, and are now being adopted by many organization worldwide (Hosking and Wallis, 1997). The L-moments give significant benefit over traditional moments such as:

- Regardless of the probability distribution from which the observations arise, Location, scale and shape parameters are calculated by L-moment are nearly unbiased, (Hosking, 1990)
- L-coefficient of variation, L-skewness, and L-kurtosis can exhibit lower bias as compared conventional product moment ratios, especially for highly skewed samples.
- L-coefficient of variation and L-skewness do not have any liability on sample size as in case of ordinary product moment ratio estimators of co-efficient of variation and skewness.
- L-moment estimators are the linear combinations of the observations and hence they are less sensitive to the largest observed sample
- L-moment ratio diagrams helps for summarizing the distributional properties of highly skewed data, whereas ordinary product moment diagrams are not muchused for the task (Vogel and Fennessey, 1993).

2. Study Area and Data Availability

The Sarda River originates from the Greater Himalayas at Kalapaani, at an altitude of 3600 m, in the Champawat District of Uttarakhand. The river borders the Nepalese Mahakali Zone and the Indian state of Uttarakhand.Mahakali flows for a length of 223 km length in Nepal and 323.5 km in India up to its confluence with Ghaghra River.

Sarda, amajor tributary of Ghaghara River pierces through the Siwalik Hills and emerges into the Ganga Plain in the vicinity of Tanakpur in the Champawat district of Uttarakhand (as shown in figure).

The maximum annual rainfall data of 86 years provided from Banbasa barrage at Sarda River from (1930-2014). The existing Upper Sarda Barrage at Banbasa at latitude **29°4'26.14"** andlongitude **80°7'10.61"**



Figure1: Location Map of Banbasa Barrage

3. Methods and Material

Probability distributions which are selected for the present study as generalized extreme value distribution(GEV), generalized logistic distribution(GLO), generalized Pareto distribution(GPA), generalized normal (GNO), Pearson type 3rd (PE3) and wakeby five parameter distribution. The generalized extreme value distribution method used over the world for flood frequency analysis (Cunnane, 1989). Generalized logistic distribution proposed as a distribution for flood frequency analysis in UK first given by Hosking and Wallis (1997).

Basic theory of L-moments

The following aspects of methodology used for development of L-moment based flood frequency relationship for Sarda river maximum flow data are discussed as follows.

Probability weighted moments (PWMs) and L-moments

L-moments of a random variable were first introduced by Hosking (1990). According the Hosking definition of L-moments is the linear combination of order of statics it is an alternative method for defining the shapes of probability distributions.

L-moments

According to the definition of Hosking (1990), linear combinations of weighted probability moments called L-moments.

$$\lambda_{r+1} = (-1)^r \sum_{k=0}^r p_{r,k}^* \alpha_k = \sum_{k=0}^r p_{r,k}^* \beta_k \quad (1)$$

Where, $p_{r,k}^r$ is an orthogonal polynomial (shifted Legendre polynomial) expressed as:

$$p_{r,k}^* = (-1)^{r-k} {}^r C_k {}^{r+k} C_k = \frac{(-1)^{r-k} (r+k)!}{(k!)^2 (r-k)!} \quad (2)$$

L-moments are easily computed in terms of probability weighted moments (PWMs) as given below Table No. 1.

Table 1: First four probabilities weighted moments (PWMs)

$\lambda_1 = \alpha_0$	$= \beta_0$
$\lambda_2 = \alpha_0 - 2\alpha_1$	$= 2\beta_1 - \beta_0$
$\lambda_3 = \alpha_0 - 6\alpha_1 + 6\alpha_2$	$= 6\beta_2 - 6\beta_1 + \beta_0$
$\lambda_4 = \alpha_0 - 12\alpha_1 + 30\alpha_2 - 20\alpha_3$	$= 20\beta_3 - 30\beta_2 + 12\beta_1 + \beta_0$

The procedure based on PWMs and L-moments are equivalent, as the L-moments are more convenient as these are measure directly the value of shape and scale of probability distributions.

It is often convenient to standardise the higher moments so that they are independent of units of measurement.

$$\tau_r = \frac{\lambda_r}{\lambda_2} \quad \text{for } r = 3, 4 \quad (3)$$

Analogous to conventional moment ratios, such as coefficient of skewness τ_3 is the L-skewness and reflects the degree of symmetry of a sample. Similarly τ_4 is a measure of peakedness and are referred to as L-kurtosis. The values calculations are given in Table No 2 below:

Table 2: The formula for L-moment ratio calculations

L-coefficient of variation L-CV(τ_2)	λ_2 / λ_1
L-coefficient of skewness(τ_3)	λ_3 / λ_2
L-coefficient of kurtosis, L-kurtosis (τ_4)	λ_4 / λ_2

Symmetric distributions have $\tau_3 = 0$ and its values lie between -1 and +1.

Z_i^{dist} Statistic as a goodness-of-fit measure

In this method also the objective is to identify the most suitable distribution for the available data. The goodness-of-fit measure for a distribution is given by statistic Z_i^{dist} .

$$Z_i^{\text{dist}} = \frac{(\bar{\tau}_i^R - \tau_i^{\text{dist}})}{\sigma_i^{\text{dist}}} \quad (4)$$

Where $\bar{\tau}_i^R$ - weighted regional average of L-moment statistic i, τ_i^{dist} and σ_i^{dist} are the simulated regional average and standard deviation of L-moment statistics i for a given distribution. The distribution giving the minimum $|Z_i^{\text{dist}}|$ value is considered as the best fit distribution. When all the three L-moment ratios are considered in the goodness-of-fit test, the distribution that gives the best overall fit when all the three statistics are considered together is selected as the underlying regional frequency distribution. According to Hosking (1993), distribution is considered to give good fit if $|Z_i^{\text{dist}}|$ is sufficiently close to zero, a reasonable criteria being $|Z_i^{\text{dist}}| \leq 1.64$.

4. Results and Discussion

Probability weighted moments and statistics for maximum discharge for Banbasa barrage at Sarda River are given in below Table No 3:

Table 3: Showing the PWMs Parameters and Statistics

Parameters		Value
PWMs	β_0	7903.669
	β_1	4763.152
	β_2	3466.395
	β_3	2741.712
L-Moments	λ_1	7903.669
	λ_2	1622.635
	λ_3	123.127
	λ_4	96.545
L-Moment ratios	L-CV(τ)	0.206
	L-skew(τ_3)	0.075
	L-kurtosis(τ_4)	0.052

Identification of proper distribution

Table No 4 shows that GPA distribution Z^{dist} –statistic value is 1.81 lower than all the distribution as generalized extreme value(GEV), Pearson type III(PE3), generalized normal(GNO), generalized logistic (GLO). GPA distribution will be the robust distribution for site data. But according to the Z^{dist} –statistic test the acceptable value of the most suitable distribution should not be more than 1.64. Hence for further data analysis considered the five parameters distribution seen as Wakeby distribution is the proper distribution.

Table 4: Z^{dist} –statistic for different types of distributions

S.No	Different type of Distribution	Z^{dist} –statistic value
1	Generalized pareto (GPA)	1.81
2	Generalized extreme value (GEV)	1.82
3	Pearson type III (PE3)	1.92
4	Generalized normal (GNO)	2.05
5	Generalized logistic (GLO)	3.68

Estimated of Parameters for wakeby distribution

Wakeby distribution is a five parameters distribution used more than the other distributions reason for which is that it can attain wider range of distributional shapes as other ones. The estimated parameters of wake by distribution are given below in Table No 5.

Table 5: Estimated of Parameters for Wakeby distribution

Distribution	Parameters of the distribution				
WAK	$\zeta=0.41$	$\alpha=1.110$	$\beta=1.307$	$\gamma=0.081$	$\delta=0.243$

The values obtained in the above Table such as location parameter 0.41 and $\alpha=1.110$, Scale parameter $\beta=1.307$, $\gamma=0.081$ and $\delta=0.243$ parameters of Wakeby distributions, further can be used for estimating the design discharge value for Sarda River.

5. Conclusion

It is concluded that 86 year maximum annual discharge data for single site (Sarda river annual maximum discharge at Banbasa Barrage) analyzed using L-moment method. Five parameters distributions methods such as Wakeby distribution and other three parameters distribution like GLO(generalized logistic), GPA(generalized pareto), GEV(generalized extreme value) and GNO(generalized normal) used for the study. Further for finding proper distribution Goodness of Fit test are applied and found that wakeby distribution is the most suitable distribution for Sarda river.

References

[1] Abhijit Bhuyan (2015), “Rainfall frequency analysis of north east India”, International Conference on Frontiers in Mathematics 2015 March 26-28, 2015, Guwahati, Assam, India.
 [2] Amit Dubey (2014), “Regional Flood Frequency Analysis Utilizing L-Moments: A Case Study of Narmada Basin”, S.G.S.I.T.S, Indore.
 [3] Cunnane C (1989), “Statistical distributions for flood frequency analysis”, W.M.O. No.718, Operational Hydrology Report No. 33, Geneva.

[4] H. Malekinezhad (2014), Yazd University, Yazd, Islamic republic of Iran, journal of agriculture and science technology, 2011, Volume13:1183-1196
 [5] Hosking J.R.M. and Wallis (1997), “Regional Flood Frequency Analysis an Approach Based On L-Moment”, Cambridge University Press, N.Y.
 [6] Hosking J.R.M. and Wallis (1993), “Some statistics useful in regional frequency analysis”. Water Resour. Res., 29(2), 271-281.
 [7] Hosking J.R.M. (1990), “L – moments: analysis and estimation of distribution using linear combination of order statistic”, J. Royal Stat. Soc. B, 52, 105-124.
 [8] Rakesh Kumar, C Chatterjee (2005), “Regional Flood Frequency Analysis Using L-Moments for North Brahmaputra Region of India”. Journal of hydrologic engineering February 2005.
 [9] Ramin Rostami (2013), “Iran Regional Flood Frequency Analysis Based on L-Moment Approach (Case Study West Azarbaijan Basins)”, Journal of Civil Engineering and Urbanism, Miandoab, India. Volume 3.
 [10] Vogel R.M. and Fennessey (1993), “L-Moments should replace Product Moments Diagrams”, Water Resour. Res., 29(6), 1745-1752.