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The Impact of Elasticity on the Firm's Revenue

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Abstract: ***The elasticity is an important measure impacting on a form's revenue. Hence, it is important for a firm to know how the proposed change in price of its product can affect its total revenue, when the product is to be sold in the new market condition at the new price. In this context, the measure of elasticity indirectly reflects how the buyers will react to the change in price and the new price to come. This implies that the elasticity of the product becomes a crucial measure to reflect what the percentage of income the firm can gain or lose, when the price change takes place for its respective product.This paper demonstrates in a new mathematically constructive approach as consistent with the existing accepted phenomena of elasticity that elastic product shows negative relationship between price change and change in total revenue; inelastic product can result in positive relationship between price change and change in total revenue; and unit elasticity product has no impact on change in total revenue as the response to a price change. Indicatively, this paper explores three constructive, but similar and alternative,mathematicalmethods for the existing phenomena how the percentage of change in total revenue can be determined with respect to elasticity, and current and new prices and their respective quantities.***

Keywords: elasticity, demand, supply, quantity, market, revenue

JEL code: A1, A2, A22, A23, A29, B21, D4, D41, D42

1. Introduction

Generally, the price, demand and supply analyses determine the equilibrium of a product in a market. The price as an independent variable determines both the demand and supply of a product and leads to market equilibrium. The elasticity is a measure of reflecting the changing rate of a quantity to a changing rate of price. Therefore, from a firm's perspective, the elasticity becomes a crucial measure in determining the price of the product, and any change in price affect the market and revenue of the firm. This implies that elasticity of the product has impacts on revenue of the firm, when there is a change in price of its product in the market. In this context this paper, provides the theoretical explanation and modelling how the elasticity of a product has impacts on the firm's revenue.

In the theoretical term and application, the price elasticity of demand (E_d) and total revenue (TR) have close relationship, since they are determined by the two common variables, P and Q. In general application, if a product iswith elastic demand in the market, the firm can increase its revenue by decreasing the price of its product, where the price decreases at a lower rate and its respective quantity increases at a higher rate, thus resulting in increase in total revenue. On the other hand, for an inelastic product, the firm can increase price and can respectively sell lower amount in quantity to earn more revenue from the product. Hence, knowing the relationship between the revenue of the firm and its product elasticity is vital for its management to make decision on pricing the product.

This paper highlights how the elasticity can be a determinant of assessing the approximate, but mathematically constructive, percentage change in income of a firm in terms of change in price. Further, the paper exhibits and concludes the relationships between the elasticity and percentage change in total revenue, and the role of percentage change in quantity and percentage change in price in determining the percentage change in total revenue.

The rest of this paper is organised as how elasticity becomes a determinant of total revenue of a product, and the concluding remarks.

2. Elasticity as the Determinant of Total Revenue of A Product

As there are different markets and firms in operations, it is important to know about how the elasticity becomes a crucial component in determining the revenue of the firm/market. To explore the relationship of elasticity to firm's revenue, consider the following revenue (TR) function of a firm/market.

$$TR = P \, . \, Q \tag{1}$$

Where
TR = Total Revenue
P= Price of the product
Q= Quantity demanded for the product

Indicatively, variables P and Q are dependent on each other, since quantity becomes the function of price and price becomes the function of price as demand and supply become the two forces in determining the market equilibrium.

Therefore, differentiating TR for Marginal Revenue (MR) with respect to Q can result in

$$\frac{d(TR)}{dQ} = MR = P\left(\frac{dQ}{dQ}\right) + Q\left(\frac{dP}{dQ}\right) \tag{2}$$

So that

$$MR = P + Q\left(\frac{dP}{dQ}\right) \tag{3}$$

But

$$E_d = \left(\frac{dQ}{dP}\right)\left(\frac{P}{Q}\right) \tag{4}$$

$$\rightarrow E_d\left(\frac{Q}{P}\right) = \left(\frac{dQ}{dP}\right)$$

$$\rightarrow \left(\frac{E_d . Q}{P}\right) = \left(\frac{dQ}{dP}\right)$$

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$$\rightarrow \left(\frac{dP}{dQ}\right) = \left(\frac{P}{E_d . Q}\right) \quad (5)$$

Substituting equation (5) in equation (3) can result in

$$MR = P + Q\left(\frac{P}{E_d . Q}\right) \rightarrow MR = P + \left(\frac{P}{E_d}\right)$$

$$MR = P\left(1 + \frac{1}{E_d}\right) \quad (6)$$

Notably, E_d is always negative in mathematical calculation, while interpreting it in positive term.

$$\text{Therefore, } MR = P\left(1 - \frac{1}{E_d}\right) \quad (7)$$

In equation 7, we can term that the MR is the result of multiplication of two components, namely: price (P) and elasticity$E_C = \left(1 - \frac{1}{E_d}\right)$.

$$\text{Since} \qquad E_C = \left(1 - \frac{1}{E_d}\right), \quad (8)$$

$$\text{Then} \qquad MR = P \,.\, E_C \quad (9)$$

As the TR is calculated as the quantity of a good sold (Q) multiplied by its price (P), it gives a measure of money, without considering the cost incurred, a company can make by selling its product. Since a firm intends to maximize profits, and it has to aim for increasing the TR. In this context, the firm can sell more items or raise the price of the product to increase its TR.

Now, it is possible to explore the relationship between the revenue and elasticity in the context of perfect (pure) competition market. In a perfect completion market, there might be various products, which can be primarily categorised into three types, based on the elasticity: (a) Inelastic, (b) Elastic, and (c) Unit-elastic products. As the MR in equation 7 consists of multiplying components of price and elasticity, the elasticity component $E_C = (\mathbf{1} - \mathbf{1}\boldsymbol{Ed}$of equation 7 can be used to explore the impacting nature of elasticity on revenue.

Suppose consider an exhibit that the demand function of a product is given by
Q_d = -0.6P + 90 (in '000) and the initial quantity is 30 (000).

Accordingly, it is known that the initial price of the product is:
30 = -0.6P + 90 $\rightarrow$ P = (90 – 30)/0.6 = 100 and

The respective elasticity is:
$E_d = \left(\frac{dQ}{dP}\right).\left(\frac{P}{Q}\right) = (0.6).\left(\frac{100}{30}\right) \rightarrow E_d = 2.$

Substituting E_d = 2in equation (8) can result $E_C$in a positive term, i.e.,
E_C=1 – (1/2)=(1/2).

As the elasticity of the product is Ed = 2 (Ed > 1 and elastic in the market), the change in price for an elastic product implies that its elastic nature will definitely have the negative impacts on the firm's revenue.

Suppose, the initial market price increases by 5% from P = 100 to 105, the quantity can fall to 27 (=-0.6x105 + 90) from 30 as 10% decrease. This implies that:

The revenue of the initial price (P = 100) and quantity (Q =30) is TR_0 = 3000 and the revenue for the price changed (P = 105) and respective quantity changed (Q = 27) is TR_1 = 2835. Hence, the change in revenue as a percentage is:

$$\%\Delta TR = \frac{TR_1 - TR_0}{TR_0} = \frac{2835 - 3000}{3000} = \frac{-165}{3000} = -0.055$$

However, the reaction of suppliers to the price increase in the market has not been considered. It is obvious that the market equilibrium for a price increase reaches with the reaction of buyers and suppliers of the product to the price change. In this context, the approximate, but theoretically constructive, change in revenue of a firm can be determined with equation (9) above, i.e.

$$MR = P \,.\, E_C \quad (9)$$

$$\text{As } MR = \frac{\Delta TR}{\Delta Q} = P \,.\, E_C \rightarrow \frac{\Delta TR}{\Delta Q} = P \,.\, E_C$$

$$\Delta TR = P \,.\, E_C \,.\Delta Q \quad (10)$$

So that

$$\%\Delta TR = \frac{\Delta TR}{TR} = \frac{P \,.\, E_C \,.\Delta Q}{TR} \quad (11)$$

According to the exhibit, P = 100, E_C = (1/2) = 0.5, and ΔQ = (27 – 30) = -3.

Therefore, substituting the above values, since the price change from P = 100 leads to a change in income as:

$$\Delta TR = 100\,.(0.5).(-3) = -150$$

Hence, the approximate percentage change in TR

$$\%\Delta TR = \frac{\Delta TR}{TR} = \frac{-150}{(100)(30)} = \frac{-150}{3000} = -0.05 = -5\%$$

To find the percentage change in TR = %ΔTR, the equation (11) can be constructively used as a mathematical instrument as shown below.

$$\%\Delta TR = \frac{\Delta TR}{TR} = \frac{P\,.E_C\,.\Delta Q}{TR}$$

Equation (11) can be further simplified with TR = P.Q as shown below.

$$\%\Delta TR = \frac{\Delta TR}{TR} = \frac{P\,.E_C\,.\Delta Q}{TR} = \frac{P\,.E_C\,.\Delta Q}{P.Q}$$

$$\%\Delta TR = \frac{E_C\,.\Delta Q}{Q} = E_C\,.(\%\Delta Q) \quad (12)$$

In the above exhibit, E_C = 0.5 and %ΔQ = (ΔQ/Q) = (-3/30) = -0.1.

Therefore, alternatively using equation (12) can result in
%TR = E_C (%ΔQ) = (0.5)(-0.1) = -0.05 = -5%.

Further, it is possible to substitute TR = P.Q and $E_C = \left(1 - \frac{1}{E_d}\right)$.

$$\text{So that,} \%\Delta TR = \frac{P\,.\left(1 - \frac{1}{E_d}\right).\Delta Q}{P\,.Q} = \frac{\Delta Q}{Q}\left(1 - \frac{1}{E_d}\right) = \left(\frac{\Delta Q}{Q} - \frac{\Delta Q}{Q.E_d}\right)$$

$$\text{Since mathematically} E_d = -\left(\frac{dQ}{dP}\right).\left(\frac{P}{Q}\right) = -\left(\frac{\Delta Q}{\Delta P}\right).\left(\frac{P}{Q}\right),$$

$$\%\Delta TR = \frac{\Delta Q}{Q} - \frac{\Delta Q}{Q}\left(\left[\frac{1}{\left(-\frac{\Delta Q}{\Delta P}.\frac{P}{Q}\right)}\right]\right) = \frac{\Delta Q}{Q} + \frac{\Delta Q}{Q}\left(\frac{\Delta P.Q}{\Delta Q.P}\right)$$

$$\%\Delta TR = \frac{\Delta Q}{Q} + \frac{\Delta P}{P} \quad (13)$$

$$\%\Delta TR = \%\Delta Q + \%\Delta P \quad (14)$$

In the exhibit, when price change from P = 100 to P = 105, then the change in quantity is moving from Q = 30 to Q = 27.

Hence, $\%\Delta Q = \frac{27-30}{30} = -0.1 = -10\%$ and $\%\Delta P = \frac{105-100}{100} = 0.05$

Using equation (14), now we can determine the percentage change in revenue (%ΔTR):

$$\%\Delta TR = \%\Delta Q + \%\Delta P = (-0.10 + 0.05) = -0.05 = -5\%.$$

Notably, for a change in price of a product, how the revenue of a firm can get affected and determined in relation to respective change in quantity and relative elasticity is shown through two formulas above:

$$\%\Delta TR = \frac{P\ .\ E_C\ .\Delta Q}{TR} \quad (11)$$

$$\%\Delta TR = \left(1 - \frac{1}{E_d}\right).(\%\Delta Q) = E_C\ .(\%\Delta Q) \quad (12)$$

$$\%\Delta TR = \%\Delta Q + \%\Delta P \quad (14)$$

It is also possible to explore how a price decrease can be useful to determine the percentage change in revenue (%ΔTR). Now consider the same exhibit of demand function above: Q_d = -0.6P + 90 (in '000) with the initial quantity 30 (000). If the price falls by 10% and moves from P = 100 to P = 90, the respective quantity can increase from Q = 30 to Q = 36.

For using equation (11), the values of P = 100, Q = 30, TR = (100).(30)=3000$E_C = \left(\mathbf{1} - \frac{\mathbf{1}}{\boldsymbol{E_d}}\right) = \left(\mathbf{1} - \frac{\mathbf{1}}{\mathbf{2}}\right) = \mathbf{0.5}$, and$\Delta Q = (36 - 30) = 6$.

Accordingly,

$$\%\Delta TR = \frac{P\ .\ E_C\ .\Delta Q}{TR} = \frac{(100)\ .(0.5).(6)}{3000} = 0.1 = 10\%\ .$$

In the above exhibit, E_C = 0.5 and %ΔQ = (ΔQ/Q) = (6/30) = 0.2.

Therefore, alternatively, using equation (12) can result in

%TR = E_C (%ΔQ) = (0.5)(0.2) = 0.10 = 10%.

Similarly, for using equation (14), the values of %ΔQ =(ΔQ/Q)=(36 – 30)/30=0.2=20% and %ΔP =(ΔP/P)=(90 – 100)/100= -0.1 = -10% needs to be substituted.

Accordingly,

$\Delta TR = \%\Delta Q + \%\Delta P = 20\% + (-10\%) = 10\%.$

From the above, the following can be concluded:

For an Elastic Product (E_d> 1), there is negative relationship between the price change and total revenue change for a product of a firm, i.e.,

Increase in price for the product leads to decrease in revenue of a firm; and

Decrease in price for the product leads to increase in revenue of a firm.

The same approach can be shown for the products with E_d< 1 and E_d = 1.

Suppose consider the demand function of a product Q_d = -0.6P + 90 (in '000) and the initial quantity is 60 (000).

Accordingly, it is known that the initial price of the product is:

60 = -0.6P + 90 ➔ P = (90 – 60)/0.6 = 50 and

The respective elasticity is:

$E_d = \left(\frac{dQ}{dP}\right).\left(\frac{P}{Q}\right) = (0.6).\left(\frac{50}{60}\right)$➔$E_d$ = 0.5 (the product is ***inelastic***).

Now assume that the price increases from P = 50 to P = 55, and this leads to decrease in quantity from 60 ('000) to 57 ('000).

For using equation (11), the values of P = 50, Q = 60, TR = (50) . (60) = 3000 $E_C = \left(\mathbf{1} - \frac{\mathbf{1}}{\boldsymbol{E_d}}\right) = \left(\mathbf{1} - \frac{\mathbf{1}}{\mathbf{0.5}}\right) = (\mathbf{1} - \mathbf{2}) = -\mathbf{1}$, and$\Delta Q = (57 - 60) = -3$.

Accordingly,

$$\%\Delta TR = \frac{P\ .\ E_C\ .\Delta Q}{TR} = \frac{(50)\ .(-1).(-3)}{3000} = 0.05 = 5\%\ .$$

In the above exhibit, E_C = -1 and %ΔQ = (ΔQ/Q) = (-3/60) = -0.05.

Therefore, alternatively, using equation (12) can result in

%TR = E_C (%ΔQ) = (-1)(-0.05) = 0.05 = 5%.

Similarly, in using equation (14), the values of %ΔQ =(ΔQ/Q)=(57 – 60)/60= -0.05= -5% and %ΔP =(ΔP/P)=(55 – 50)/50= 0.1 = 10% needs to be substituted.

Accordingly,

$\%\Delta TR = \%\Delta Q + \%\Delta P = (-5\%) + (10\%) = 5\%.$

Now assume that the price decreases from P = 50 to P = 45, and this leads to increase in quantity from 60 ('000) to 63 ('000).

For using equation (11), the values of P = 50, Q = 60, TR = (50) . (60) = 3000 $E_C = \left(\mathbf{1} - \frac{\mathbf{1}}{\boldsymbol{E_d}}\right) = \left(\mathbf{1} - \frac{\mathbf{1}}{\mathbf{0.5}}\right) = (\mathbf{1} - \mathbf{2}) = -\mathbf{1}$, and$\Delta Q = (63 - 60) = 3$.

Accordingly,

$$\%\Delta TR = \frac{P\ .\ E_C\ .\Delta Q}{TR} = \frac{(50)\ .(-1).(3)}{3000} = -0.05 = -5\%\ .$$

In the above exhibit, E_C = -1 and %ΔQ = (ΔQ/Q) = (3/60) = 0.05.

Therefore, alternatively, using equation (12) can result in

%TR = E_C (%ΔQ) = (-1)(0.05) = -0.05 = -5%.

Similarly, in using equation (14), the values of %ΔQ =(ΔQ/Q)=(63 – 60)/60= 0.05= 5% and %ΔP =(ΔP/P)=(45 – 50)/50= -0.1 = -10% needs to be substituted.

Accordingly,

$$\%\Delta TR = \%\Delta Q + \%\Delta P = (5\%) + (-10\%) = -5\%.$$

From the above, the following can be concluded:

For an Inelastic Product (E_d< 1), there is positive relationship between the price change and total revenue change for a product of a firm, i.e.,

Increase in price for the product leads to increase in revenue of a firm; and

Decrease in price for the product leads to decrease in revenue of a firm.

Now consider the same demand function of a product Q_d = -0.6P + 90 (in '000) and the initial quantity is 45 (000).

Accordingly, it is known that the initial price of the product is:

45 = -0.6P + 90 ➔ P = (90 – 45)/0.6 = 75 and

The respective elasticity is:
$E_d = \left(\frac{dQ}{dP}\right).\left(\frac{P}{Q}\right) = (0.6).\left(\frac{75}{45}\right) \rightarrow E_d = 1$ (***unit elasticity*** product).

Now assume that the price increases from P = 75 to P = 80, and this leads to decrease in quantity from 45 ('000) to 42 ('000).
For using equation (11), the values of P = 75, Q = 45, TR = (75) . (45) = 3375, $E_C = \left(\mathbf{1}-\frac{\mathbf{1}}{E_d}\right) = \left(\mathbf{1}-\frac{\mathbf{1}}{\mathbf{1}}\right) = (\mathbf{1}-\mathbf{1}) = 0$, and$\Delta Q = (42-45) = -3$.
Accordingly,
$\%\Delta TR = \frac{P\ .\ E_C\ .\Delta Q}{TR} = \frac{(75)\ .(0).(-3)}{3375} = 0\%$ (***no change*** in total revenue).

In the above exhibit, $E_C = 0$ and %ΔQ = (ΔQ/Q) = (-3/45) = 0.067.

Therefore, alternatively, using equation (12) can result in
%TR = E_C (%ΔQ) = (0)(0.067) = 0 = 0%.

Similarly, in using equation (14), the values of %ΔQ=(ΔQ/Q)=(42 – 45)/45=-0.0667 = -06.67% and %ΔP=(ΔP/P)=(80 – 75)/75=0.0667=6.67% needs to be substituted in it.
Accordingly,
$\%\Delta TR = \%\Delta Q + \%\Delta P = (-6.667\%) + (6.667\%) = 0\%.$

From the above, the following can be concluded:
For a Unit Elastic Product (E_d= 1), there is no change in total revenue for the change in price of the product of a firm, i.e.,

Increase in price for the product leads to no change in revenue of a firm; and

Decrease in price for the product leads to no change in revenue of a firm.

Indicatively, the products with different elasticity measures are available in perfect/ pure competition market, where huge numbers of buyers and sellers are available. Also, the market has no product differentiation, since all products have almost the same features and the buyers have no choice at all to buy a product selectively. In these markets, no buyer or seller can change the price of a product.

Similarly, in monopolistic and monopolistic competition market, the products have unique features of product differentiation with free entry to new entrants. Also there would be an artificial boundary to restrict new firms entering into the markets;and hence, there might be a considerably low level of influence in the supply on similar types of products.In these markets, the elasticity of the products always be greater than one (E_d= 1). [Equation (7) above, i.e., $\boldsymbol{MR = P\left(1-\frac{1}{E_d}\right)}$, always replicate the theoretically emphasising condition that should exist in a monopolistic or monopolistic completion markets, where a firm cannot exist in the market, if it experiences MR < 0. The equation (7) implies that if E_d< 0 (like E_d = 0.5 for example) for a product of a firm, the MR of the firm would be negative, since elasticity makes$\frac{\mathbf{1}}{E_d} > 1$, and $\left(1-\frac{\mathbf{1}}{E_d}\right)$result in negative$\left(1-\frac{\mathbf{1}}{E_d}\right) < 0$. Hence, for monopolistic and monopolistic competition markets the change in total revenue should be dealt with elasticity greater than one (E_d> 1) as the nature of the markets]. Therefore, it is important to know, such as the types of products (whether the products are subject to product differentiation), which market (whether the market is perfect competition), etc., to make conclusion on change in revenue of a firm with respect to change in price and its relative elasticity.

3. Concluding Remarks

As the elasticity becomes a one of the important factors in determining a form's revenue with respect to a change in price of its product, it is important for the firm to know, before taking a decision on changing the price of its product, how the proposed change in price can affect its total revenue from the product to be sold in the new market at the new price. In this context, the measure of elasticity is very important that reflect how the buyers will react to the change in price and the new price to come. In this context, the elasticity of the product becomes a crucial measure to reflect what the percentage of income the firm can gain or lose, when the price change takes place for its respective product.

This paper demonstrates in a new mathematically constructive approach, but existing accepted fact, how the elasticity of a product can affect a firm's total revenue. This paper illustrates with an exhibit of considering three different types of products with elastic (Ed > 1), inelastic (Ed < 1), and unit elastic (Ed = 1) nature.

The above said mathematical constructive method is also consistent with the existing accepted phenomena of elasticity that elastic product shows negative relationship between price change and change in total revenue, inelastic product can result in positive relationship between price change and change in total revenue, and unit elasticity product has no impact on change in total revenue as the response to a price change. In this context, this paper explores three mathematically constructive, but similar and alternative, methods for the existing phenomena how the percentage of change in total revenue can be determined with respect to elasticity, and current and new prices and their respective quantities.

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