Some New Families of Prime Labeling of Graphs

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Abstract: *A Graph G with n vertices is said to admit prime labeling if its vertices can be labeled with distinct positive integers not exceeding n such that the labels of each pair of adjacent vertices are relatively prime. A graph G which admits prime labeling is called a prime graph. In this paper we investigate the existence of prime labeling of some graphs related to cycle C_n, wheel W_n,comb P_n^*,crown C_n^*, and helmH_n .We discuss prime labeling in the context ofthe graph operation namely duplication.*

Keywords: Graph Labeling, Prime Labeling, Duplication, Prime Graphs

1. Introduction

In this paper, We consider only finite simple undirected graph. The graph G has vertex set V = V(G) and edge set E= E(G). The set of vertices adjacent to a vertex u of G is denoted byN(u). For notations and terminology we refer to Bondy and Murthy[1].

The notion of prime labeling was introduced by Roger Entringer and was discussed in a paper by Tout[6]. Two integers aandbare said to be relatively prime if their greatest common divisor is 1. Relatively prime numbers play an important role in both analytic and algebraic number theory. Many researchers have studied prime graph.Fu.H [3] has proved that the path $P_n$on n vertices is a prime graph. Deretsky et al [2] have prove that the cycle C_n on n vertices is a prime graph. Around 1980 Roger Etringer conjectured that all trees have prime labeling which is not settled till today.

The Prime labeling for planar grid was investigated by Sundaram et al [5], Lee.S.et.al [4] have proved that the wheel W_n is a prime graph if and only if n is even.

Definition 1.1[7] Duplication of an edge e= uv by a new vertex w in a graph G produces a new graph G' such that N(w) = {u,v}.

Definition 1.2 The graph obtained by duplication all the edges by the vertices of a graph G is called duplication of G.

Definition 1.3 The comb P_n^* is obtained from a path $P_n$by attaching a pendent edge at each vertex of the path P_n

Definition 1.4 The crown graph C_n^* is obtained from a cycle C_n by attaching a pendent edge at each vertex of the n-cycle.

Definition 1.5 The helm H_n is a graph obtained from a wheel by attaching a pendant edge at each vertex of the n-cycle.

In this paper we proved that the graphs obtained by duplication of every edge by a vertex in cycle C_n, the wheel $W_n$and the comb P_n^*, the graph obtained by duplication of every rim edge by a vertex in crown C_n^* and duplicating every edge by a vertex in crown C_n^*, the graph obtained by duplicating every rim edge by a vertex in Helm H_n and duplicating every rim and pendent edge by a vertex in Helm $H_n$are all prime graphs.

2. Main Results

Theorem 2.1
The graph obtained by duplicating every edge by a vertex inC_n , is a prime graph.

Proof
Let$V(C_n) = \{u_i \ / 1 \leq i \leq n\}$
$E(C_n) = \{u_i u_{i+1} / 1 \leq i \leq n-1\} \cup \{u_n u_1\}$.

Let G be the graph obtained by duplicating every edge by a vertex in C_n and let $u_1', u_2', \ldots, u_n'$,be the new vertices by duplicating the edges
$u_1u_2, u_2u_3, \ldots. u_{n-1}u_n, u_nu_1$respectively.
Then$V(G) = \{u_i, u_i' / 1 \leq i \leq n\}$
$E(G) = \{u_i u_{i+1} \ /1 \leq i \leq n-1\} \cup \{u_i u_i' \ / 1 \leq i \leq n\}$
$\cup \{u_i' u_{i+1} /1 \leq i \leq n-1\} \cup \{u_n u_1, u_n' u_1\}$
$|V(G)| = 2n, |E(G)| = 3n.$
Define a labeling$f : V(G) \rightarrow \{1,2,3, \ldots, 2n\}$as follows
Let$f(u_i) = 2i - 1 \ for \ 1 \leq i \leq n,$
$f(u_i') = 2i \ for \ 1 \leq i \leq n,$
$\gcd(f(u_i), f(u_{i+1})) = \gcd(2i-1, 2i+1) = 1 \ for \ 1 \leq i \leq n-1$
$\gcd(f u_n), f(u_1)) = \gcd(2n-1, 1) = 1$
$\gcd(f(u_i), f(u_i')) = \gcd(2i-1, 2i) = 1 \ for \ 1 \leq i \leq n$
$\gcd(f(u_i'), f(u_{i+1})) = \gcd(2i, 2i+1) = 1 \ for \ 1 \leq i \leq n-1$
$\gcd(f(u_n'), f(u_1)) = \gcd(2n, 1) = 1$
Thus$f$ is a prime labeling.
Hence G is a prime graph.

Illustration 2.1

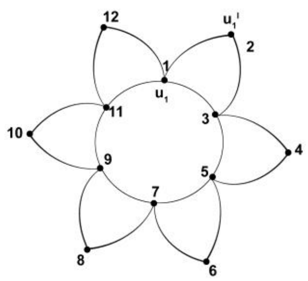

Figure 1: Prime labeling of duplication of every edge by a vertex in C_6

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Theorem 2.2
The graph obtained by duplicating every edge by a vertex in Wheel W_n is a prime graph.

Proof
Let$V(W_n) = \{ c,u_i \ / 1 \le i \le n\}$
$E(W_n) = \{cu_i \ / 1 \le i \le n \} \cup \{u_i u_{i+1} / \ 1 \le i \le n-1\} \cup \{u_n u_1 \}$
Let G be the graph obtained by duplicating every edge by a vertex in Wheel W_n and let $v_1, v_2, \dots, v_n$ and $w_1, w_2, \dots, w_n$ be the new verticesby duplicating the edges $u_1u_2, u_2u_3, \dots, u_{n-1}u_n, u_nu_1$and $cu_1, cu_2, \dots, cu_n$ respectively

$$V(G) = \{ c, u_i, v_i, w_i \ / 1 \le i \le n \}$$
$$E(G) = \{ cu_i, cw_i / 1 \le i \le n\} \cup \{ u_iu_{i+1}, v_iu_{i+1} / 1 \le i \le n-1\} \cup \{u_i v_i , u_i w_i \ /1 \le i \le n\} \cup \{ u_nu_1, v_nu_1\}$$
$|V(G)| = 3n + 1, |E(G)| = 6n.$
Define a labeling $f : V(G) \to \{1,2,3, \dots, 3n + 1\}$as follows.
Let $f(c) = 1, f(u_1) = 3, f(v_1) = 4$ and $f(w_1) = 2$.
$$f(u_i) = \begin{cases} 3i-2 \ if \ i \ is \ odd, \ 3 \le i \le n \\ 3i-1 \ if \ i \ is \ even, 2 \le i \le n \end{cases} \quad f(v_i) = \begin{cases} 3i+1 \ if \ i \ is \ odd, 1 \le i \le n \\ 3i+2 \ if \ i \ is \ even \ , 2 \le i \le n \end{cases}$$
$f(w_i) = 3i, for \ 2 \le i \le n$
since$f(c) = 1$.
$\gcd(f(c), f(u_i)) = 1, for \ 1 \le i \le n$
$\gcd(f(c), f(w_i)) = 1, for \ 1 \le i \le n$
$\gcd(f(u_n), f(u_1))$= $\gcd(3n-2,3)$= 1if n is odd
$\gcd(f(u_n), f(u_1))$= $\gcd(3n-1,3)$= 1 if n is even
$\gcd(f(v_n), f(u_1))$= $\gcd(3n+1,3)$= 1 if n is odd
$\gcd(f(v_n), f(u_1))$= $\gcd(3n+2,3)$= 1 if n is even
clearly,
$\gcd(f(u_i), f(u_{i+1}))$= $\gcd(3i-2, 3(i+1)-1)$
= $\gcd(3i-2, 3i+2)$ = 1 for3$\le i \le n$, i is odd
as these two numbers are odd and their differences is 4 .
$\gcd(f(u_i), f(u_{i+1}))$=$\gcd(3i-1, 3(i+1)-2)$
= $\gcd(3i-1, 3i+1)$=1 f or $2 \le i \le n$, i is even
as they are consecutive odd integers
$\gcd(f(u_i), f(v_i))$= $\gcd(3i-2, 3i+1)$= 1 for $3 \le i \le n$, i is odd
$\gcd(f(u_i), f(v_i))$= $\gcd(3i-1, 3i+2)$= 1 for $2 \le i \le n$, i is even
as one of these numbers is even and the other number is odd, their difference is 3 and they are not multiples of 3.
$\gcd(f(v_i), f(u_{i+1}))$= $\gcd(3i+1, 3i+2)$ = 1
for 3$\le i \le n-1$, i is odd.
$\gcd(f(v_i), f(u_{i+1}))$= $\gcd(3i+2, 3i+1)$= 1
for 2$\le i \le n-1$, i is even
$\gcd(f(u_i), f(w_i))$= $\gcd(3i-2, 3i)$= 1
for 3$\le i \le n$, i is odd.
$\gcd(f(u_i), f(w_i))$= $\gcd(3i-1, 3i)$= 1
for 2$\le i \le n$, i is even.
as they are consecutive integers
Thus f is a prime labeling.
Hence G is a prime graph.

Illustration 2.2

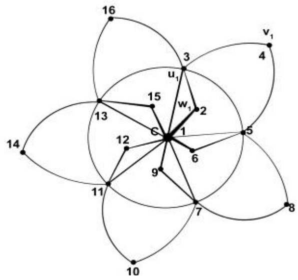

Figure 2: Prime labeling of duplication of every edge by a vertex in W_5

Theorem 2.3
The comb P_n^* is a prime graph.

Proof
Let $V(P_n^*) = \{u_i/1 \le i \le n\}$
$E(P_n^*) = \{ u_iu_{i+1}/1 \le i \le n-1\} \cup \{ u_iv_i/1 \le i \le n\}$
Then
$|V(G)| = 2n, |E(G)| = 2n-1.$
Define a labeling$f : V(G) \to \{1,2,3, \dots ,2n\}$ as follows
Let $f(u_i) = 2i-1, for \ 1 \le i \le n$
$f(u_i) = 4i-3, for \ 2 \le i \le n,$
$\gcd(f(u_i), f(u_{i+1}))$=$\gcd(2i-1, 2i+1)$=1
for 1$\le$ i $\le$ n
as these two number are consecutive odd integers
$\gcd(f(u_i), f(v_i))$= $\gcd(2i-1, 2i)$= 1
for 1$\le i \le n$
Thus f is prime labeling.
Hence P_n^* is a prime graph.

Theorem 2.4
The graph obtained by duplicating every edge by a vertex in Comb P_n^* is a prime graph.

Proof
Let$V(P_n^*) = \{u_i, w_i \ / 1 \le i \le n\}$
$E(P_n^*) = \{u_i u_{i+1} / \ 1 \le i \le n-1\} \cup \{u_iw_i \ /1 \le i \le n\}$
Let G be the graph obtained by duplicating every edge by a vertex in $P_n^*$and let $v_1, v_2, \dots, v_{n-1}$ and $x_1, x_2, \dots, x_n$ be the new vertices by duplicating the edges $u_1u_2, u_2u_3, \dots, u_{n-1}u_n$ and $u_1w_1, u_2w_2, \dots, u_nw_n$ respectively.
Then,
$V(G) = \{ u_i, w_i, x_i \ / \ 1 \le i \le n\} \cup \{v_i \ /1 \le i \le n-1 \}$
$$E(G) = \{ u_iu_{i+1}, \ u_iv_i/1 \le i \le n-1\} \cup \{u_i w_i , w_i x_i , u_i x_i \ /1 \le i \le n\}$$
$$|V(G)| = 4n-1, |E(G)| = 6n-3,$$
Define a labeling $f : V(G) \to \{1,2,3, \dots, 4n-1\}$ as follows
Let $f(u_1) = 1, f(w_1) = 2, f(x_1) = 3$, and $f(v_1) = 4$.
$f(u_i) = 4i-3, for \ 2 \le i \le n, i \not\equiv 0 (mod \ 3)$
$f(x_i) = 4i-2, for \ 2 \le i \le n,$
$f(w_i) = 4i-1, for \ 2 \le i \le n, i \not\equiv 0 (mod \ 3)$
$f(v_i) = 4i, for \ 1 \le i \le n-1,$
$f(u_i) = 4i-1, for \ 2 \le i \le n, i \equiv 0 (mod \ 3)$
$f(w_i) = 4i-3 \ for \ 2 \le i \le n, i \equiv 0 (mod \ 3)$
Since $f(u_1) = 1$
$\gcd(f(u_1), f(v_1))$= 1
$\gcd(f(u_1), f(u_2))$= 1
$\gcd(f(u_1), f(x_1))$= 1
$\gcd(f(u_1), f(w_1))$=1

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Clearly,
$\gcd(f(u_i), f(u_{i+1}))= \gcd(4i-3), 4(i+1)-3)$
$= \gcd(4i-3, 4i+1)= 1$
for $2\le i \le n-1, i \not\equiv 0 (mod\ 3)$
$\gcd(f(u_i), f(u_{i+1}))= \gcd(4i-1, 4i+1)= 1$
for $2\le i \le n-1, i \equiv 0 (mod\ 3)$
as these two numbers are odd and their differences are 4,2 respectively.
$\gcd(f(u_{i-1}), f(u_i))= \gcd(4(i-1)-3, 4i-1)$
$= \gcd(4i-7, 4i-1)= 1$
for $2\le i \le n-1, i \equiv 0 (mod\ 3)$
as these two numbers are odd and they are not multiples of 3 and their difference is 6
$\gcd(f(u_i), f(w_i))= \gcd(4i-3, 4i-1) = 1$
for $2\le i \le n$, $i \not\equiv 0 (mod\ 3)$
$\gcd(f(u_i), f(w_i))= \gcd(4i-1, 4i-3) = 1$
for $2\le i \le n, i \equiv 0 (mod\ 3)$
as these two numbers are odd and differences is 2
$\gcd(f(u_i), f(x_i))= \gcd(4i-3, 4i-2)= 1$
for $2\le i \le n$, $i \not\equiv 0 (mod\ 3)$
$\gcd(f(u_i), f(x_i))= \gcd(4i-1, 4i-2)= 1$
for $2\le i \le n, i \equiv 0 (mod\ 3)$
$\gcd(f(w_i), f(x_i))= \gcd(4i-1, 4i-2)= 1$
for $2\le i \le n$, $i \not\equiv 0 (mod\ 3)$
$\gcd(f(w_i), f(x_i))= \gcd(4i-3, 4i-2)= 1$
for $2\le i \le n, i \equiv 0 (mod\ 3)$
$\gcd(f(u_i), f(v_i))= \gcd(4i-1, 4i)= 1$
for $2\le i \le n, i \equiv 0 (mod\ 3)$
$\gcd(f(u_i), f(v_{i-1}))= \gcd(4i-3, 4i-4)= 1$
for $2\le i \le n$, $i \not\equiv 0 (mod\ 3)$
as they are consecutive integers
$\gcd(f(u_i), f(v_i))= \gcd(4i-3, 4i)= 1$
for $2\le i \le n$, $i \not\equiv 0 (mod\ 3)$
$\gcd(f(u_i), f(v_{i-1}))= \gcd(4i-1, 4i-4) = 1$
for $2\le i \le n, i \equiv 0 (mod\ 3)$
as in all these two numbers one is odd and other is even and also their differences are 3, and they are not multiple of 3.
Thus f is a prime labeling.
Hence G is a prime graph.

Illustration 2.4

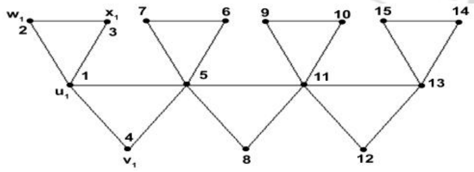

Figure 3: Prime labeling of duplication of every edge by a vertex in P_4^*

Theorem 2.5
The graph obtained by duplicating every rim edge by a vertex in Crown C_n^* is a prime graph.

Proof
Let $V(C_n^*) = \{u_i, w_i\ / 1 \le i \le n\}$
$E(C_n^*) = \{u_i w_i / 1 \le i \le n\} \cup \{u_i u_{i+1} /\ 1 \le i \le n-1\} \cup \{u_n u_1\}$
Let G be the graph obtained by duplicating every rim edge by a vertex in $C_n^*$and let the new vertices be $v_1, v_2, \dots, v_n$ by duplicating the edges $u_1u_2, u_2u_3, \dots . u_{n-1}u_n, u_nu_1$.

Then $V(G) = \{u_i, v_i, w_i\ / 1 \le i \le n\}$
$E(G) = \{u_iv_i, u_iw_i / 1 \le i \le n\} \cup \{u_i u_{i+1}, v_i u_{i+1} / 1 \le i \le n-1\} \cup \{u_n u_1, v_n u_1\}$
$|V(G)| = 3n, |E(G)| = 4n.$
Define a labeling $f : V(G) \to \{1,2,3, \dots, 3n\}$as follows
Let $f(u_1) = 1, f(v_1) = 2, f(w_1) = 3n$

$$f(u_i) = \begin{cases} 3i-3, for\ 2 \le i \le n, i\ is\ even \\ 3i-4, for\ 3 \le i \le n, i\ is\ odd \end{cases}$$

$$f(v_i) = \begin{cases} 3i+1, for\ 2 \le i \le n, i\ is\ even \\ 3i-1, for\ 1 \le i \le n, i\ is\ odd \end{cases}$$

$$f(w_i) = \begin{cases} 3i-2, for\ 2 \le i \le n, i\ is\ even \\ 3i-3, for\ 3 \le i \le n, i\ is\ odd \end{cases}$$

Since $f(u_1) = 1$
$\gcd(f(u_1), f(u_2))= 1$
$\gcd(f(u_n), f(u_1))= 1$
$\gcd(f(u_1), f(v_1))= 1$
$\gcd(f(u_1), f(w_1))=1$
$\gcd(f(v_n), f(u_1))=1$
$\gcd(f(u_i), f(u_{i+1}))= \gcd(3i-3, 3(i-1)+4)$
$=\gcd(3i-3, 3i-1) =1$
for $2\le i \le n$, i is even
$\gcd(f(u_i), f(u_{i+1}))= \gcd(3i-4, 3(i+1)-3)$
$=\gcd(3i-4, 3i) = 1$
for $3\le i \le n$, i is odd
as these two numbers are odd and their differences are 2 and 4 respectively.
$\gcd(f(u_i), f(v_i))= \gcd(3i-3, 3i+1) = 1$
for $2\le i \le n$,i is even.
as these two numbers are odd and their differences is 4
$\gcd(f(u_i), f(v_i))= \gcd(3i-4, 3i-1)= 1$
for $3\le i \le n$,i is odd
as in these two numbers one is odd and other is even and their differences is 3
$\gcd(f(v_i), f(u_{i+1}))= \gcd(3i+1, 3i-1)= 1$
for $2\le i \le n-1$,i is even
as these two numbers are odd and their differences is 2
$\gcd(f(v_i), f(u_{i+1}))= \gcd(3i-1, 3i)= 1$
for$2\le i \le n-1$,i is odd
$\gcd(f(u_i), f(w_i))= \gcd(3i-3, 3i-2)= 1$
for $2\le i \le n$,i is even.
$\gcd(f(u_i), f(w_i))= \gcd(3i-4, 3i-3)= 1$
for $2\le i \le n$,i is odd.
as they are two consecutive numbers.
Thus $f$is a prime labeling
Hence G is a prime graph.

Illustration 2.5

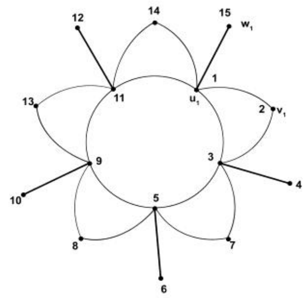

Figure 4: Prime labeling of duplication of every rim edge by a vertex in C_5^*

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Theorem 2.6

The graph obtained by duplicating every edge by a vertex in C_n^* is a prime graph.

Proof

Let $V(C_n^*) = \{u_i, w_i \ / 1 \le i \le n\}$

$E(C_n^*) = \{u_i w_i / 1 \le i \le n\} \cup \{u_i u_{i+1} / \ 1 \le i \le n-1\} \cup \{u_n u_1\}$

Let G be the graph obtained by duplicating every edge by a vertex in C_n^* and let the new vertices be $v_1, v_2, \dots, v_n$ and $x_1, x_2, \dots, x_n$ by duplicating the edges $u_1u_2, u_2u_3, \dots . u_{n-1}u_n, u_nu_1$ and $u_1w_1, u_2w_2, \dots . u_nw_n$ respectively. Then $V(G) = \{u_i, v_i, w_i, x_i \ / 1 \le i \le n\}$

$$E(G) = \{u_iv_i, u_iw_i, u_ix_i, x_iw_i \ / 1 \le i \le n\} \cup \{u_i u_{i+1}, v_i u_{i+1} / 1 \le i \le n-1\} \cup \{u_nu_1, v_nu_1\}$$

$$|V(G)| = 4n, |E(G)| = 6n$$

Define a labeling $f : V(G) \to \{1,2,3,\dots,4n\}$ as follows

Let $f(u_1) = 1, f(v_1) = 4. f(w_1) = 2$, and $f(x_1) = 3$.

$f(u_i) = 4i - 3, for\ 2 \le i \le n, i \not\equiv 0(mod\ 3)$

$f(w_i) = 4i - 2, for\ 1 \le i \le n,$

$f(x_i) = 4i - 1, for\ 1 \le i \le n, i \not\equiv 0(mod\ 3)$

$f(v_i) = 4i, for\ 1 \le i \le n,$

$f(u_i) = 4i - 1, for\ 1 \le i \le n, i \equiv 0(mod\ 3)$

$f(x_i) = 4i - 3\ for\ 1 \le i \le n, i \equiv 0(mod\ 3)$

Since $f(u_1) = 1$

$\gcd(f(u_1), f(u_2)) = 1$

$\gcd(f(u_n), f(u_1)) = 1$

$\gcd(f(v_n), f(u_1)) = 1$

Clearly,

$\gcd(f(u_{i-1}), f(u_i)) = \gcd(4(i-1) - 3, 4i - 1)$

$= \gcd(4i - 7, 4i - 1) = 1$

for $2 \le i \le n, i \equiv 0(mod\ 3)$

as these two numbers are odd and they are not the multiples of 3 and their difference is 6

$\gcd(f(u_i), f(u_{i+1})) = \gcd(4i - 3, 4i + 1) = 1$

for $2 \le i \le n - 1, i \not\equiv 0(mod\ 3)$

as these two numbers are odd and also their differences is 4

$\gcd(f(u_i), f(u_{i+1})) = \gcd(4i - 1, 4i + 1) = 1$

for $2 \le i \le n, i \equiv 0(mod\ 3)$

asthey are consecutive odd integer

$\gcd(f(u_i), f(w_i)) = \gcd(4i - 3, 4i - 2) = 1$

for $2 \le i \le n\ , i \not\equiv 0(mod\ 3)$

$\gcd(f(u_i), f(w_i)) = \gcd(4i - 1, 4i - 2) = 1$

for $2 \le i \le n, i \equiv 0(mod\ 3)$

as they are consecutive integer

$\gcd(f(u_i), f(x_i)) = \gcd(4i - 3, 4i - 1) = 1$

for $2 \le i \le n\ , i \not\equiv 0(mod\ 3)$

as these two numbers are odd and also their differences is 2

$\gcd(f(w_i), f(x_i)) = \gcd(4i - 2, 4i - 1) = 1$

for $1 \le i \le n\ , i \not\equiv 0(mod\ 3)$

$\gcd(f(w_i), f(x_i)) = \gcd(4i - 2, 4i - 3) = 1$

for $1 \le i \le n, i \equiv 0(mod\ 3)$

$\gcd(f(v_i), f(u_{i+1})) = \gcd(4i, 4(i+1) - 3)$

$= \gcd(4i, 4i + 1) = 1$

for $1 \le i \le n - 1, i + 1 \not\equiv 0(mod\ 3)$

$\gcd(f(u_i), f(v_i)) = \gcd(4i - 1, 4i) = 1$

for $2 \le i \le n, i \equiv 0(mod\ 3)$

as they are consecutive integer

$\gcd(f(v_i), f(u_{i+1})) = \gcd((4i, 4i + 3)) = 1$

for $1 \le i \le n - 1, i + 1 \equiv 0(mod\ 3)$

$\gcd(f(u_i), f(x_i)) = \gcd(4i - 1, 4i - 3) = 1$

for $2 \le i \le n\ , i \equiv 0(mod\ 3)$

$\gcd(f(u_i), f(v_i)) = \gcd(4i - 3, 4i) = 1$

for $2 \le i \le n\ , i \not\equiv 0(mod\ 3)$

as in these two numbers one is even and other is odd and their differences is 3 and they are not multiples of 3.

Thus f is a prime labeling.

Hence G is a prime graph.

Illustration 2.6

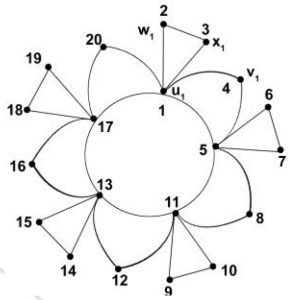

Figure 5: Prime labeling of duplication of every edge by a vertex in C_5^*

Theorem 2.7

The graph obtained by duplicating every rim edge by a vertex in Helm H_n is a prime graph.

Proof

Let $V(H_n) = \{c, u_i, w_i \ / 1 \le i \le n\}$

$E(H_n) = \{cu_i, u_iv_i \ / 1 \le i \le n\} \cup \{u_i u_{i+1} / 1 \le i \le n-1\} \cup \{u_nu_1\}$

Let G be the graph obtained by duplicating every rim edge by a vertex in Helm H_n and let the new vertices be $v_1, v_2, \dots, v_n$ by duplicating the edges $u_1u_2, u_2u_3, \dots . u_{n-1}u_n, u_nu_1$ respectively.

Then, $V(G) = \{c, u_i, v_i, w_i \ / 1 \le i \le n\}$

$E(G) = \{u_iu_{i+1} / 1 \le i \le n-1\} \cup \{v_i u_{i+1} / 1 \le i \le n - 1\} \cup \{cu_i, u_iw_i, u_iv_i \ / 1 \le i \le n\} \cup \{u_nu_1, v_nu_1\}$.

$|V(G)| = 3n + 1, |E(G)| = 5n$.

Define a labeling $f : V(G) \to \{1,2,3,\dots,3n+1\}$ as follows

Let $f(c) = 1, f(u_1) = 3. f(v_1) = 4$, and $f(w_1) = 2$.

$$f(u_i) = \begin{cases} 3i - 3, for\ 3 \le i \le n, i\ is\ odd \\ 3i - 1, for\ 2 \le i \le n, i\ is\ even \end{cases}$$

$$f(v_i) = \begin{cases} 3i + 1, for\ 1 \le i \le n, i\ is\ odd \\ 3i + 2, for\ 2 \le i \le n, i\ is\ even \end{cases}$$

$f(w_i) = 3i, for\ 2 \le i \le n$

Since $f(c) = 1$

$\gcd(f(c), f(u_i)) = 1, for\ 1 \le i \le n$

$\gcd(f(u_n), f(u_1)) = \gcd(3n - 2, 3) = 1$ if n is odd

$\gcd(f(u_n), f(u_1)) = \gcd(3n - 1, 3) = 1$ if n is even

$\gcd(f(v_n), f(u_1)) = \gcd(3n + 1, 3) = 1$ if n is odd

$\gcd(f(v_n), f(u_1)) = \gcd(3n + 2, 3) = 1$ if n is even

Clearly,

$\gcd(f(u_i), f(u_{i+1})) = \gcd(3i - 2, 3(i+1) - 1)$

$= \gcd(3i - 2, 3i + 2) = 1$

for $3 \le i \le n$, i is odd

$\gcd(f(u_i), f(u_{i+1})) = \gcd(3i - 1, 3(i+1) - 2)$

$= \gcd(3i - 1, 3i + 1) = 1$

for $2 \le i \le n$, i is even

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as these two numbers are odd and their differences are 4 and 2 respectively.
$\gcd(f(u_i), f(v_i))= \gcd(3i-2, 3i+1)=1$
for $3\leq i \leq n$, i is odd
.$\gcd(f(u_i), f(v_i))= \gcd(3i-1, 3i+2)=1$
for $2\leq i \leq n$, i is even
as in these two numbers one is even and other is odd and their differences is 3 and they are not multiples of 3.
$\gcd(f(v_i), f(u_{i+1}))= \gcd(3i+1, 3i+2)= 1$
for $3\leq i \leq n-1$, i is odd
$\gcd(f(v_i), f(u_{i+1}))= \gcd(3i+2, 3i+1)= 1$
for $2\leq i \leq n-1$, i is even
$\gcd(f(u_i), f(w_i))= \gcd(3i-1, 3i)= 1$
for $2\leq i \leq n$, i is even
as they are consecutive integer.
$\gcd(f(u_i), f(w_i))= \gcd(3i-2, 3i)= 1$
for $3\leq i \leq n$, i is odd
Thus f is a prime labeling.
Hence G is a prime graph.

Illustration 2.7

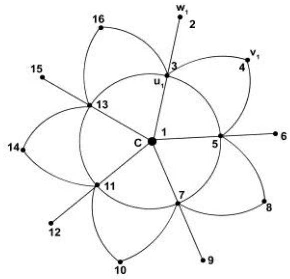

Figure 6: Prime labeling of duplication of every edge by a rim edge by a H_5

Theorem 2.8
The graph obtained by duplicating every rim and pendant edge by a vertex in Helm H_n is a prime graph. if $4n+1 \not\equiv 0 (mod\ 3)$.

Proof
Let $V(H_n) = \{c, u_i, w_i \ / 1 \leq i \leq n\}$
$E(H_n)=\{cu_i, u_iw_i \ /1 \leq i \leq n\} \cup \{u_i u_{i+1} / \quad 1 \leq i \leq n-1\} \cup \{u_n u_1\}$
Let G be the graph obtained by duplicating every rim edge and pendant edge by a vertex in H_n and let the new vertices be $v_1, v_2, \ldots, v_n$ and $x_1, x_2, \ldots, x_n$ by duplicating the edges $u_1u_2, u_2u_3, \ldots . u_{n-1}u_n, u_nu_1$ and $u_1w_1, u_2w_2, \ldots . u_nw_n$ respectively.

$$V(G) = \{c, u_i, v_i, w_i, x_i \ / \ 1 \leq i \leq n\}$$

$E(G) = \{cu_i, u_iw_i, u_ix_i, u_iv_i, w_ix_i / 1 \leq i \leq n\} \cup \{u_i u_{i+1}, v_iu_{i+1} / 1 \leq i \leq n-1\} \cup \{u_nu_1, v_nu_1\}$.

$$|V(G)| = 4n+1, |E(G)| = 7n.$$

Define a labeling $f : V(G) \rightarrow \{1,2,3, \ldots, 4n+1\}$ as follows
Let $f(c) = 1, f(u_1) = 4n+1$.
$f(u_i) = 4i-3, for\ 2 \leq i \leq n, i \not\equiv 0 (mod\ 3)$
$f(w_i) = 4i-2, for\ 1 \leq i \leq n$
$f(x_i) = 4i-1, for\ 1 \leq i \leq n, i \not\equiv 0 (mod\ 3)$
$f(v_i) = 4i, for\ 1 \leq i \leq n,$
$f(u_i) = 4i-1, for\ 1 \leq i \leq n, i \equiv 0 (mod\ 3)$
$f(x_i) = 4i-3\ for\ 1 \leq i \leq n, i \equiv 0 (mod\ 3)$
Thus f admits a prime labeling.
Hence G is a prime graph.

Illustration 2.8

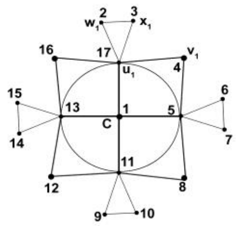

Figure 7: Prime labeling of duplication of every rim and pendent edge by a vertex**in H_4**

3. Conclusion

Labeled graph is the topic of current due to its diversified application. We investigate Eight new results on prime labeling. It is an effort to relate the prime labeling and some graph operations. This approach is novel as it provides prime labeling for the larger graph resulted due to certain graph operation on a given graph.

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