Complete and Complementary Domination Number in Interval Valued Fuzzy Graphs

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Abstract: ***An interval-valued fuzzy graph of a graph G = (A, B) of a graph G * = (V , E) is said to be complete if $\mu_B^-(xy) = min\{\mu_A^-(x), \mu_A^-(y)\}$ and $\mu_B^+(xy)$ =max{$\mu_A^+(x)$,$\mu_A^+(y)$} for all xy∈ E. Given a fuzzy graph ,choose v ∈ V(G) and put S ={v},For every v we have N(S) =V-S denoted by S′ is the complete dominating set. The minimum cardinality of a complete dominating set of interval valued fuzzy is called the complete domination number of G. we introduce complete and complementary domination number in interval valued fuzzy graphs and obtain some interesting results for this new parameter in interval valued fuzzy graphs.***

Keywords: Dominating set, Domination in interval valued fuzzy graph, Fuzzy graphs, complete domination in interval valued fuzzy graphs, Complementary domination in interval valued fuzzy graphs

1. Introduction

Zadeh [15] introduced the notion of interval-valued fuzzy sets as an extension of fuzzy sets which gives a more precise tool to model uncertainty in real life situations. Some recent work of Zadeh in connection with the importance of fuzzy logic may be found in [14]. Interval-valued fuzzy sets have been widely used in many areas of science and engineering, e g., in approximate reasoning medical diagnosis, multi valued logic, intelligent control, topological spaces etc .Hongmei and Lianhua introduced the definition of interval valued fuzzy graphs in [3]. Recently, Akram and Dudek[1] have studied several properties and operations on interval valued fuzzy graphs. In this paper, We analyze bounds on complete and Complementary dominating set of interval valued fuzzy graphs.

2. Preliminaries

Throughout this paper a fuzzy graph will denote a fuzzy graph without loops. First we collect some definitions to be used in this paper.

Definition 2.1
An interval- valued fuzzy set A on a set V is defined by

$$A = \{(x, [\mu_A^-(x), \mu_A^+(x)]) : x \in V\},$$

Where μ_A^- and μ_A^+ are fuzzy subsets of V such that $\mu_A^-(x) \le \mu_A^+(x)$ for all x ∈ V. If G * = (V , E) is a crisp fuzzy graph, then by an interval-valued fuzzy relation B on V we mean an interval-valued fuzzy set on E such that $\mu_B^-(xy) \le min\{\mu_A^-(x), \mu_A^-(y)\}$ and $\mu_B^+(xy) \le max\{\mu_A^+(x), \mu_A^+(y)\}$ for all xy∈ E and We write

$$B = \{xy, [\mu_B^-(xy), \mu_B^+(xy)] : xy \in E\}$$

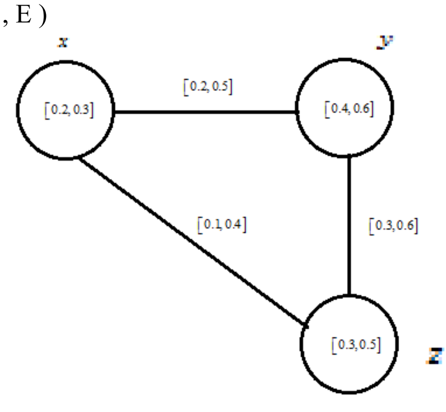
Definition 2. 2.
An interval-valued fuzzy graph of a graph G *=(V , E) is a pair G = (A, B), where A = [μ_A^- ,μ_A^+] is an interval-valued fuzzy set on V and B =[μ_B^- ,μ_B^+]] is an interval-valued fuzzy relation on V

Example 2.3.
Consider the graph G ∗ = (V , E), where V = { x, y , z } and E ={ xy , yz , zx}. Let A be an interval-valued fuzzy set on V and let B be an interval-valued fuzzy set on E ⊆ V × V defined by

$$A = \left\langle \left(\frac{x}{0.2}, \frac{y}{0.4}, \frac{z}{0.3} \right) \right\rangle, \left\langle \left(\frac{x}{0.3}, \frac{y}{0.6}, \frac{z}{0.5} \right) \right\rangle$$

$$B = \left\langle \left(\frac{xy}{0.2}, \frac{yz}{0.3}, \frac{zx}{0.1} \right) \right\rangle \left\langle \left(\frac{xy}{0.5}, \frac{yz}{0.6}, \frac{zx}{0.4} \right) \right\rangle$$

Then G = (A, B) is an interval-valued fuzzy graph of a graph G * = (V , E)



Definition 2.4.
The order p and size q of an interval-valued fuzzy graph of a graph G = (A, B) of a graph G * =(V , E) are defined to be

$$p = \sum_{v \in V} \frac{1 + \mu_A^+(v) - \mu_A^-(v)}{2}$$

And

$$q=\sum_{xy\in V}\frac{1+\mu_B^+(xy)-\mu_B^-(xy)}{2}$$

Definition 2. 5.

Let G = (A, B) be an interval-valued fuzzy graph of a graph on G * = (V , E) and S ⊆ V Then the cardinality of S is defined to be

$$\sum_{v\in V}\frac{1+\mu_A^+(v)-\mu_A^-(v)}{2}$$

Definition2.6.

An edge e = xy of an interval-valued fuzzy graph of G is called an effective edge if $\mu_B^-(xy)=\min\{\mu_A^-(x),\mu_A^-(y)\}$ and $\mu_B^+(xy)=\max\{\mu_A^+(x),\mu_A^+(y)\}$. In this case , the vertex x is called a neighbor of y conversely, N (x) = {y ∈ V: y is a neighbor of x} is called the neighborhood of x.

Example 2.7.

Consider the graph G * = (V , E), where V = { a,b,c,d} and E ={ ab,bc,cd,da}. Let A be aninterval- valued fuzzy set on V and let B be an interval-valued fuzzy set on E ⊆ V × V defined by

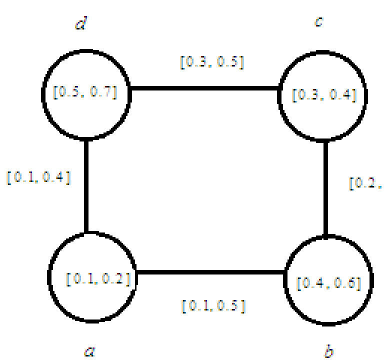
$$A=\left\langle\left(\frac{a}{0.1},\frac{b}{0.4},\frac{c}{0.3},\frac{d}{0.5}\right)\right\rangle,$$

$$\left\langle\left(\frac{a}{0.2},\frac{b}{0.6},\frac{c}{0.3},\frac{d}{0.7}\right)\right\rangle$$

$$B=\left\langle\left(\frac{ab}{0.1},\frac{bc}{0.2},\frac{cd}{0.3},\frac{da}{0.1}\right)\right\rangle,$$

$$\left\langle\left(\frac{ab}{0.5},\frac{bc}{0.6},\frac{cd}{0.5},\frac{da}{0.4}\right)\right\rangle$$

Then G = (A, B) is an interval-valued fuzzy graph of G *=(V, E)



In this example, ab and da are effective edges. Also, N (a) = { b, d }, N (b) = {a } ,N (d) = {a }, N (c) = φ (the empty set).

Definition 2.8.

Let G = (A, B) be an interval-valued fuzzy graph on V and x, y ∈ V . We say x dominates y if $\mu_B^-(xy)=\min\{\mu_A^-(x),\mu_A^-(y)\}$ and $\mu_B^+(xy)=\max\{\mu_A^+(x),\mu_A^+(y)\}$

A subset S of V is called a dominating set in G if for every v ∉S , there exists u ∈ S such that u dominates v .The minimum cardinality of a dominating set in G is called the domination number of G and is denoted by γ (G).

Remark 2.9.

(i) For any x, y ∈V , if x dominates y then y dominates x and as such domination is a symmetric relation.

(ii) $\mu_B^-(xy) < \min\{\mu_A^-(x),\mu_A^-(y)\}$ and $\mu_B^+(xy) < \max\{\mu_A^+(x),\mu_A^+(y)\}$ x, y ∈V, then the only dominating set in G is V.

Remark 2.10

Since {v } is a dominating set of K $_{\mu A}$ for each v∈ V , we have

$$(i)\gamma(k_{\mu A})=\min_{v\in V}\frac{1+\mu_A^+(v)-\mu_A^-(v)}{2}$$

$$(ii)\gamma(\overline{k_{\mu A}})=p$$

$$(iii)\gamma(k_{\mu A^1,\mu A^2})=\min_{v\in V}\frac{1+\mu_A^+(v)-\mu_A^-(v)}{2}+\min_{w\in V}\frac{1+\mu_A^+(w)-\mu_A^-(w)}{2}$$

Example 2.11

Consider the graph G * = (V , E), where V = { w,x,y,z} and E ={ wx,xy,yz,zw,wy}. Let A be an interval- valued fuzzy set on V and let B be an interval-valued fuzzy set on E ⊆ V × V defined by

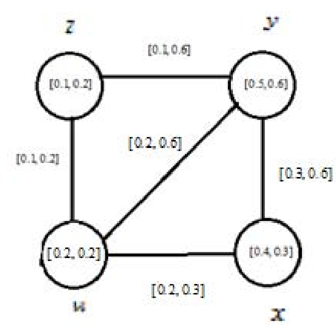
$$A=\left\langle\left(\frac{w}{0.2},\frac{x}{0.3},\frac{y}{0.5},\frac{z}{0.1}\right)\right\rangle$$

$$\left\langle\left(\frac{w}{0.2},\frac{x}{0.3},\frac{y}{0.6},\frac{z}{0.2}\right)\right\rangle$$

$$B=\left\langle\left(\frac{wx}{0.2},\frac{xy}{0.3},\frac{yz}{0.1},\frac{wy}{0.2},\frac{zw}{0.1}\right)\right\rangle,$$

$$\left\langle\left(\frac{wx}{0.3},\frac{xy}{0.6},\frac{yz}{0.6},\frac{wy}{0.6},\frac{zw}{0.2}\right)\right\rangle$$

Then G = (A, B) is an interval-valued fuzzy graph of G * = (V , E).



Since {y} is a dominating set of G for each y∈ V and We have the domination number of interval- valued fuzzy graph G,

$$\gamma(G) = \min_{y\in V} \frac{1+\mu_A^+(y)-\mu_A^-(y)}{2} = \frac{1+0.6-0.5}{2} = 0.55$$

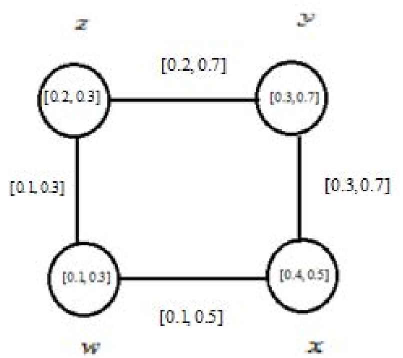
Example 2.12

Consider the graph G * = (V , E), where V = { w,x,y,z} and E ={ wx,xy,yz,zw }. Let A be an interval- valued fuzzy set on V and let B be an interval-valued fuzzy set on E ⊆ V × V defined by

$$A = \left\langle \left(\frac{w}{0.1}, \frac{x}{0.4}, \frac{y}{0.3}, \frac{z}{0.2} \right) \right\rangle, \left\langle \left(\frac{w}{0.3}, \frac{x}{0.5}, \frac{y}{0.7}, \frac{z}{0.3} \right) \right\rangle$$

$$B = \left\langle \left(\frac{wx}{0.1}, \frac{xy}{0.3}, \frac{yz}{0.2}, \frac{zw}{0.1} \right) \right\rangle, \left\langle \left(\frac{wx}{0.5}, \frac{xy}{0.7}, \frac{yz}{0.7}, \frac{zw}{0.3} \right) \right\rangle$$

Then G = (A, B) is an interval-valued fuzzy graph of G * = (V , E).



Since {w,y} is a minimal dominating set of G for each w,y∈ V and We have the domination number of interval- valued fuzzy graph G,

$$\gamma(G) = \min_{w\in V} \frac{1+\mu_A^+(w)-\mu_A^-(w)}{2} + \min_{y\in V} \frac{1+\mu_A^+(y)-\mu_A^-(y)}{2}$$

$$= \frac{1+0.3-0.1}{2} + \frac{1+0.7-0.3}{2} = 1.3$$

Definition 2.13

A set S of vertices of an interval- valued fuzzy graph G is said to be independent if $\mu_B^-(xy) < \min\{\mu_A^-(x), \mu_A^-(y)\}$ and $\mu_B^+(xy) < \max\{\mu_A^+(x), \mu_A^+(y)\}$ for all x,y ∈ V

3. Complete Domination Number in Interval Valued Fuzzy Graphs

In the section, we introduce complete domination number in interval valued fuzzy graphs and obtain some interesting results for this new parameter in interval valued fuzzy graphs

Definition 3.1

A fuzzy graph in which there exists an edge between every pair of vertices is called a complete fuzzy graph

Definition 3.2.

An interval-valued fuzzy graph of a graph G = (A, B) of a graph G * = (V , E) is said to be complete if $\mu_B^-(xy) = \min\{\mu_A^-(x), \mu_A^-(y)\}$ and $\mu_B^+(xy) = \max\{\mu_A^+(x), \mu_A^+(y)\}$ for all xy∈ E and is denoted by $K_{\mu A}$

Definition 3.3.

An interval-valued fuzzy graph of a graph G = (A, B) of a graph G * = (V , E) is said to be bipartile if the vertex set V can be partitioned into two nonempty sets V_1 and V_2 such that $\mu_B^-(xy) = 0$ and $\mu_B^+(xy) = 0$ if x,y∈V_1 or x,y∈ V_2. Further if $\mu_B^+(xy) = \max\{\mu_A^+(x), \mu_A^+(y)\}$ and $\mu_B^-(xy) = \min\{\mu_A^-(x), \mu_A^-(y)\}$ for all x ∈V_1 and y ∈ V_2 then G is called a complete bipartile graph and is denoted by $k_{\mu_A^-, \mu_A^+}$ where μ_A^- and μ_A^+ are restrictions of μ_A^- and μ_A^+ on V_1 and V_2 respectively.

Definition 3.4

Given a fuzzy graph G, choose v ∈ V (G) and put S ={v},For every v we have N(S) =V-S denoted by S′ is the complete dominating set of interval valued fuzzy graph. The minimum cardinality of a complete dominating set of interval valued fuzzy is called the complete domination number of G.

Theorem 3.5

Let G be a complete interval valued fuzzy graph without isolated vertices, If S is a complete minimal dominating set, then V-S is a complete dominating set.

Proof

Let S be complete minimal dominating set of G. Suppose V-S is a not complete dominating set. Then there exists a vertex u∈S such that u is a not dominated by any one vertex in V-S. Since G has no isolated vertices u is a strong neighbor of at least one vertex in S-{u}.Then S-{u} is a complete dominating set, which contradicts the minimality of S. Thus every vertex in S is a strong neighbor of at least one vertex in V-S. Hence V-S is a complete dominating set.

Theorem 3.6

An independent set is a maximal independent set of an complete interval valued fuzzy graph G if and only if it is independent and dominating set.

Proof

Let S be a maximal independent set of G. Thus for every v ∈ V - S, the set S ∪ {v} is not independent. So, for every vertex v ∈ V – S there is a vertex u ∈ S such that u is dominated by v. Thus S is a dominating set. Hence S is independent and dominating. Conversely, let S be independent and dominating. If possible, suppose S is not maximal independent. Then there exists v ∈ V - S such that the set S ∪ {v} is independent. Then no vertex in S is dominated by v. Thus D cannot be a dominating set, which is a contradiction. Hence S must be a maximal independent set.

Theorem 3.7
In complete interval valued fuzzy graph G, every maximal independent set is a minimal dominating set.

Proof
Let S b e a maximal independent set in G. By Theorem 3.6, S is a dominating set. Suppose S be not a minimal dominating set. Then there exists at least one vertex v ∈ S for which S -{v } is a dominating set. But if S - {v} dominates V - (S - {v}), then at least one vertex in S -{v } must dominate v. This contradicts the fact that S is an independent s et of G. Hence S must be a minimal dominating set.

Theorem 3.8
Suppose S and S′ are not complete interval valued dominating sets of fuzzy graph G then there exists at least one v ϵ G such that N (S) ∩N (S′)= ∅

Proof
Obviously, by the definition of the complete interval valued dominating set.

Theorem 3.9
Suppose S and S′ are complete interval valued dominating subsets of G such that
N (S)∩N (S′) ≠∅

Proof
Obviously, by the definition of the complete interval valued dominating set

4. Complementary Domination Number in Interval Valued Fuzzy Graphs

In the section, We introduce Complementary domination in interval valued fuzzy graph.

Definition 4.1
The complement of an interval-valued fuzzy graph of a graph G of a graph G * = (V, E) is the interval-valued fuzzy graph of a graph $\overline{G}$ = ($\overline{A}$, $\overline{B}$),where $\overline{A}=[\mu_A^{+},\mu_A^{-}]$ and $\overline{B}=[\mu_B^{-},\mu_B^{+}]$

is defined by $\overline{\mu_B^-}(xy)=\min\{\mu_A^-(x),\mu_A^-(y)\}$

$\overline{\mu_B^+}(xy)=\max\{\mu_A^+(x),\mu_A^+(y)\}$ for all xy∈ E

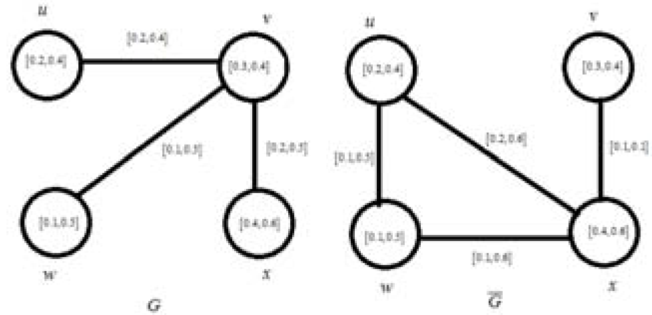
Example 4.2
Consider the graph G * = (V , E), where V = { u,v,w,x } and E ={ uv,vw,vx}. Let A be an interval- valued fuzzy set on V and let B be an interval-valued fuzzy set on E ⊆ V × V defined by

$$A=\left\langle\left(\frac{u}{0.2},\frac{v}{0.3},\frac{w}{0.1},\frac{x}{0.4}\right)\right\rangle,\left\langle\left(\frac{u}{0.4},\frac{v}{0.4},\frac{w}{0.5},\frac{x}{0.6}\right)\right\rangle$$

$$B=\left\langle\left(\frac{uv}{0.4},\frac{vw}{0.5},\frac{vx}{0.6}\right)\right\rangle,\left\langle\left(\frac{uv}{0.4},\frac{vw}{0.5},\frac{vx}{0.6}\right)\right\rangle$$

Then G = (A, B) is of G * = (V , E).

An interval-valued fuzzy graph



Definition 4.3
Let $\overline{G}$ = ($\overline{A}$, $\overline{B}$), be a Complementary interval-valued fuzzy graph G on V and x, y ∈ V, We say x dominates y if

$$\overline{\mu_B^-}(xy)=\min\{\mu_A^-(x),\mu_A^-(y)\}$$

$$\overline{\mu_B^+}(xy)=\max\{\mu_A^+(x),\mu_A^+(y)\}$$

A subset S of V is called a dominating set in $\overline{G}$ if for every v ∉S , there exists u ∈ S such that u dominates v .The minimum cardinality
of a dominating set in G is called the domination number of G and is denoted by γ ($\overline{G}$).

Theorem 4.4 [6]
For any interval-valued fuzzy graph γ (G)+ γ ($\overline{G}$) ≤ 2Pwhere γ ($\overline{G}$) is the domination number of $\overline{G}$ and the equality holds if and only if $0 < \mu_B^-(xy) < \min\{\mu_A^-(x),\mu_A^-(y)\}$ and $0 < \mu_B^+(xy) < \max\{\mu_A^+(x),\mu_A^+(y)\}$ for all x,y ∈ V

Theorem 4.5
Let $\overline{G}$ be a Complementary interval-valued fuzzy graph G without isolated vertices, If S is a Complementary minimal dominating set, then V-S is Complementary dominating set.

Proof
Obviously, by the definition of the Complementary interval valued dominating set.

Theorem 4.6
For the fuzzy graph G, $\overline{G}$ ⊆ CD(G) [Common minimal dominating fuzzy graph G] further,
$\overline{G}$ = CD(G) if and only if every minimal dominating set of G is independent.

Proof
If (x,y) ∈E(G), then extend {x,y} to maximal independent set S of vertices in G. since s is a minimal dominating set of G , we obtain $\overline{G}$ ⊆CD(G).

Suppose $\overline{G}$ = CD(G). It implies that two vertices in the minimal dominating set S are not adjacent in G. Thus S is independent.

Conversely, Suppose every minimal dominating set of G is independent. Then two vertices adjacent in G cannot be adjacent in CD(G).Thus CD(G) $\subseteq \overline{G}$ and since $\overline{G} \subseteq$ CD(G), We see that $\overline{G}$ = CD(G).

5. Conclusion

Complete and Complementary domination in interval valued fuzzy graph is defined. Theorems related to this concept are derived and the relation between domination number in interval valued fuzzy graph and Complete, Complementary domination in interval valued fuzzy graphs are established.

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