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Poisson-Gamma Counting Process as a Survival Model under Size-Biased Sampling

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Abstract: ***In this paper a Size-Biased Poisson–Gamma distribution (SBPGD) has been proposed, which is obtained by compounding a Size-Biased Poisson distribution with a Size-Biased two parameter Gamma distribution. In this paper the pmf of the proposed distribution (SBPGD) is derived. The expressions for raw moments, central moments, coefficients of skewness and kurtosis have been derived. Survival and Hazard functions of proposed distribution are also obtained. The estimators of the parameters have been obtained by method of Moments as well as method of Maximum Likelihood.***

Keywords: Size-Biased Poisson Gamma distribution, maximum likelihood estimation, method of Moments, Survival function, Hazard function etc

1. Introduction

When an investigator records an observation from certain stochastic model (population), the recorded observation will not have the original distribution unless every observation is given an equal chance of being recorded. For example, suppose that original observation x_0 comes from a distribution with pmf/pdf $f_0(x_0)$ and that observation x is recorded according to a probability weighted by a weighting function w(x) > 0, then x actually comes from a distribution with pmf/pdf

$$f(x) = \frac{w(x)}{E[w(x_0)]} \; f_0(x) \qquad (1)$$

Rao (1965) introduced distribution of this type and called them weighted distribution. The weighted distribution with w(x) = x is called size-biased / length-biased distribution. Patil and Rao (1978) examined some general models leading to weighted distributions and showed how the weight w(x) = x occurs in a natural way in many sampling problems. A study of size-biased sampling and related form-invariant weighted distribution was done by Patil and Rao (1975).

In reliability/survival lifetime modeling, it is common to treat failure data as being continuous, implying some degree of precision in measurement. Too often in practice, however, failures are either noted at regular inspection intervals, occurs in a discrete process or are simply recorded in bins. In life testing experiments or survival time data, it is sometimes impossible or inconvenient to measure the life length on a continuous scale. Thus, it is essential to construct discrete lifetime models for discrete failure survival data, Lai (2013). Roy, D. (2004) have also discussed the properties of discrete Rayleigh distribution. Finite range discrete lifetime distributions are discussed by Lai, C.D. et al. (1995).

2. Size Biased Poisson-Gamma Distribution (SBPGD)

If X has a Poisson distribution with pmf

$$f(x/\lambda) = \frac{e^{-x}\lambda^x}{x!} \; ; \; x= 0, 1, 2, \ldots ; \lambda > 0. \qquad (2)$$

Let us consider a two parameter Gamma distribution with pdf is given by

$$g(x/\alpha, \beta) = \frac{\alpha^\beta}{\Gamma(\beta)} \; e^{-\alpha x} \; x^{\beta-1} \; ; \; \alpha, \beta > 0; \; 0 < x < \infty. \qquad (3)$$

Srivastava and Srivastava (2014) obtained the pmf of Poisson–Gamma Distribution (PGD) as

$$h(x; \alpha, \beta) = \frac{\Gamma(x+\beta)}{x!\,\Gamma(\beta)} \left(\frac{\alpha}{1+\alpha}\right)^\beta \left(\frac{1}{1+\alpha}\right)^x$$

$$\text{with Mean} = \int_{x=0}^{\infty} e^{tx} \; f(x) = \frac{\beta}{\alpha} \qquad (4)$$

Now pmf of Size-Biased Poisson Gamma (SBPG) distribution is

$$f(x; \alpha, \beta) = \frac{x\, h(x;\alpha,\beta)}{E(x)}$$

$$f(x; \alpha, \beta) = \frac{\alpha}{\beta} \frac{\Gamma(x+\beta)}{\Gamma x \; \Gamma\beta} \left(\frac{\alpha}{1+\alpha}\right)^\beta \left(\frac{1}{1+\alpha}\right)^x \; \alpha, \beta > 0 \;\; x = 1,2,\ldots \qquad (5)$$

Equation (5) may be expressed as

$$f(x; \alpha, \beta) = \frac{(x+\beta-1)!}{(x-1)!\beta!} \; \alpha^{(1+\beta)} \; (1+\alpha)^{-(x+\beta)} \;\; x =1,2,\ldots \; \alpha > 0, \; \beta = 1,2,\ldots \qquad (6)$$

which is the pmf of classical Negative Binomial Distribution (as generated by the number of independent trials necessary to obtain β occurrences of an event which has constant probability p $= \left(\frac{\alpha}{1+\alpha}\right)$ of occurrence at each trial, Johnson and Kotz (1969)).

$$f(x; \alpha, \beta) = \binom{x+\beta-1}{\beta} \; \alpha^{1+\beta} \; (1+\alpha)^{-(x+\beta)} \;\; x = 1,2,\ldots \; \alpha > 0, \; \beta =1,2,.. \qquad (7)$$

If β is not a non-negative integer, equation (6) may be termed as **'Psuedo'** Negative Binomial Distribution and it may be named as a **'Generalized'** Negative Binomial Distribution in the sense that β is either non-negative integer or not.(Srivastava & Srivastava, 2014).

The pmf of Size biased Poisson distribution is

$$f(x/\lambda) = \frac{e^{-\lambda}\,\lambda^{x-1}}{(x-1)!} \qquad (8)$$

The pmf of Size biased two parameter gamma distribution

$$g(x; \alpha, \beta) = \frac{\alpha^{1+\beta}\, e^{-\alpha x}\, x^\beta}{\beta \; \Gamma\beta} \qquad (9)$$

If parameter, λ of Size-Biased Poisson distribution follows the Size-Biased Gamma distribution then Compounding the size biased Poisson distribution with size biased two parameter Gamma distribution, we get the pmf of SBPGD

$$f(x;\alpha,\beta) = \int_0^\infty \frac{e^{-\lambda}\lambda^{x-1}}{(x-1)!}\,\frac{\alpha^{1+\beta}e^{-\lambda\alpha}\lambda^{\beta}}{\beta\,\Gamma\beta}\,d\lambda$$

$$= \frac{\alpha}{\beta}\,\frac{\Gamma(x+\beta)}{\Gamma x\,\Gamma\beta}\left(\frac{\alpha}{1+\alpha}\right)^{\beta}\left(\frac{1}{1+\alpha}\right)^{x} \quad \alpha,\beta>0 \quad x=1,2,\ldots \quad (10)$$

The pmf of (5) and (10) are same. Thus we see that we obtained that size biased PGD and size biased Poisson and size biased Gamma distributions are same.

Student (1907) used the Negative Binomial Distribution as an alternative to the Poisson distribution in describing counts on the plates of haemacytometer. The Negative Binomial Distribution was studied by Fisher (1941), Jeffreys (1941) and Anscombe (1950) under different parameterization. It has been shown to be the limiting form of Eggenberg and Polya"s urn model by Patil et al. (1984) and Gamma mixture of Poisson distribution by Greenwood and Yule (1920), addition of a set of correlated Poisson distributions by Martiz (1952). The Negative Binomial Distribution also arises out of a few stochastic processes as pointed by McKendrick (1914), Irwin (1941), Lundeberg (1940) and Kendall (1949). This distribution, being more flexible than Poison distribution, enjoys a plethora of applications. It can be used to model accident data, psychological data, economics data, consumer data, medical data, defense data and so on. Chandra and Roy (2012) proposed a continuous version of the Negative Binomial Distribution by considering a particular type of survival function. Adhikari and Srivastava (2013) discussed the Size-Biased Poisson-Lindley Distribution.

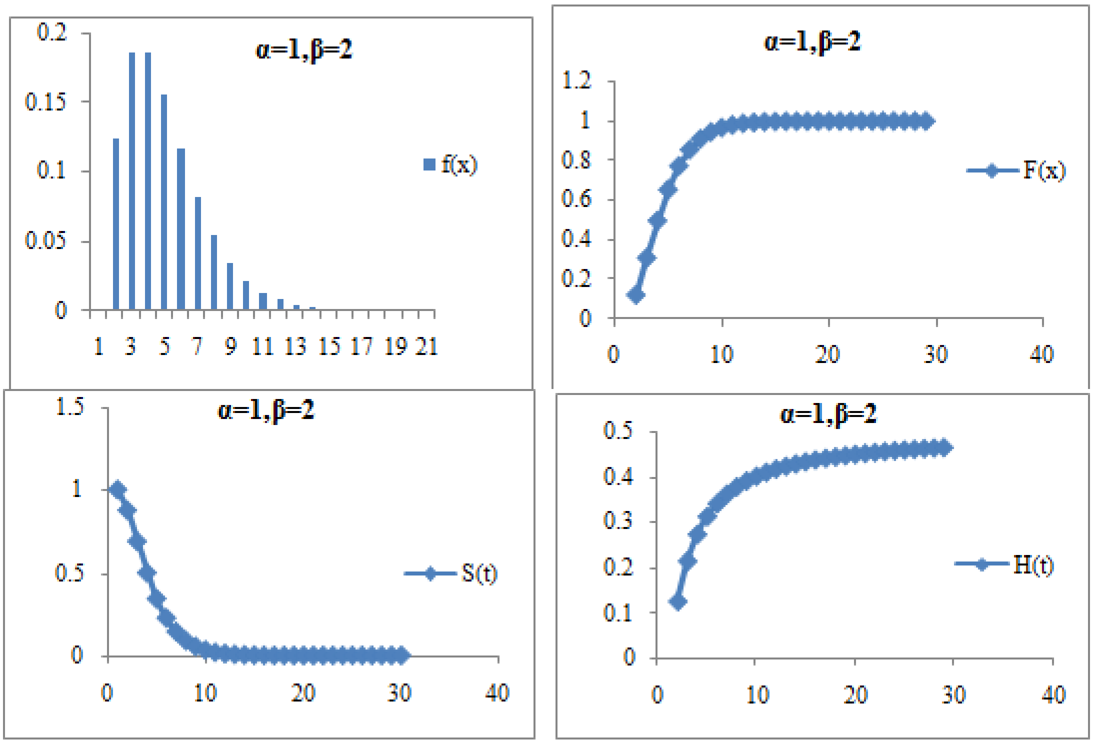

Figure 1: Showing of pmf, cdf, S(t) and H(t) of SBPGD (1.6)

3. Moments

The first four raw moments about origin and their corresponding central moments of Size Biased Poisson Gamma Distribution (SBPGD) are

$$\mu_1' = \frac{1+\beta}{\alpha}$$

$$\mu_2' = \frac{(1+\beta)(\alpha+\beta+2)}{\alpha^2}$$

$$\mu_3' = \frac{(1+\beta)(\alpha^2+\beta^2+6\alpha+5\beta+3\alpha\beta+6)}{\alpha^3}$$

$$\mu_4' = \frac{(1+\beta)(\alpha^3+\beta^3+14\alpha^2+36\alpha+30\alpha\beta+6\alpha\beta^2+7\alpha\beta^2+15\beta+9\beta^2+13)}{\alpha^4}$$

and their corresponding central moments are

$$\mu_2 = \frac{(1+\beta)(1+\alpha)}{\alpha^2}$$

$$\mu_3 = \frac{(1+\beta)(1+\alpha)(2+\alpha)}{\alpha^3}$$

$$\mu_4 = \frac{(1+\beta)(1+\alpha)[\alpha^2+3(3+\beta)(1+\alpha)]}{\alpha^4}$$

Thus the mean, variance, skewness, kurtosis and their coefficients are obtained

$$\text{Mean} = \mu_1' = \frac{1+\beta}{\alpha} \quad (11)$$

$$\text{Variance } (\mu_2) = \frac{(1+\beta)(1+\alpha)}{\alpha^2} \quad (12)$$

$$\beta_1 = \frac{\mu_3{}^2}{\mu_2{}^3} = \frac{(2+\alpha)^2}{(1+\beta)(1+\alpha)} \quad (13)$$

$$\beta_2 = \frac{\mu_4}{\mu_2{}^2} = \frac{\alpha^2+3(3+\beta)(1+\alpha)}{(1+\beta)(1+\alpha)} \quad (14)$$

$$\gamma_1 = \sqrt{\beta_1} = (2+\alpha)\sqrt{\frac{1}{(1+\beta)(1+\alpha)}} \quad (15)$$

$$\gamma_2 = \beta_2 - 3 = \frac{\alpha^2+6\,(1+\alpha)}{(1+\beta)(1+\alpha)} \quad (16)$$

4. Mode of SBPGD

$$P(x) = \frac{\alpha}{\beta}\,\frac{\Gamma(x+\beta)}{\Gamma x\,\Gamma\beta}\left(\frac{\alpha}{1+\alpha}\right)^{\beta}\left(\frac{1}{1+\alpha}\right)^{x} \quad x=1,2,\ldots\ldots \ \alpha,\beta>0 \ (17)$$

$$P(x-1) = \frac{\alpha}{\beta}\,\frac{\Gamma(x+\beta-1)}{\Gamma(x-1)\,\Gamma\beta}\left(\frac{\alpha}{1+\alpha}\right)^{\beta}\left(\frac{1}{1+\alpha}\right)^{x-1} \quad x=2,3,\ldots\ldots \ \alpha,\beta>0 \ (18)$$

$$\frac{P(x)}{P(x-1)} = \frac{(x+\beta-1)}{(x-1)(1+\alpha)} \quad (19)$$

Now, we have

$$(x+\beta-1) > (x-1)(1+\alpha) \text{ or } \frac{\beta+\alpha}{\alpha} > x \quad (20)$$

Case I: When $\frac{\beta+\alpha}{\alpha}$ is not an integer

Let us suppose that S is the integral part of $\frac{\beta+\alpha}{\alpha}$

So that

$$\frac{\beta+\alpha}{\alpha} = S + f,\ 0 < f < 1 \quad (21)$$

$$\frac{P(x)}{P(x-1)} = \frac{S+f}{x} = \begin{cases} > 1 \text{ if } x = 0,1,2,\dots,S \\ < 1 \text{ if } x = S+1, S+2,\dots \end{cases} \quad (22)$$

we get

P(0) < P(1) < P(2) <,…,< P(S-2) < P(S-1) < P(S) > P(S+1) >P(S+2) >… (23)

Which shows that P(S) is the maximum value and in this case this distribution is unimodal and the integral part of $\frac{\beta+\alpha}{\alpha}$ is the unique modal value.

Case II: When $\frac{\beta+\alpha}{\alpha}$ = k (say) is an integer

Here as in case I, we have

P(0) < P(1) < P(2) <,…,< P(k-2) < P(k-1) = P(k) > P(k+1) >P(k+2) >… (24)

In this case we have two maximum value as P(k - 1) and P(k) and thus the distribution is bimodal and two modal value and are at (k-1) and k that is $\left(\frac{\beta+\alpha}{\alpha} - 1\right)$ and $\left(\frac{\beta+\alpha}{\alpha}\right)$

Case III: if 0 < β ≤ 1, The mode always lies at zero

Case IV: if $\left(\frac{\beta+\alpha}{\alpha}\right) \le 1$, even then the mode will be zero irrespective the value of β

5. Method of Estimation

5.1. Maximum likelihood estimation (MLE)

Given a random sample $x_1, x_2,\dots, x_n$, of size n from the SBPG distribution with p.m.f. (5) is

$$f(x;\alpha,\beta) = \frac{\alpha}{\beta}\frac{\Gamma(x+\beta)}{\Gamma x\,\Gamma\beta}\,\alpha^{\beta}\,(1+\alpha)^{-(x+\beta)}\ ;\ \alpha,\beta > 0\ x = 1,2,\dots \quad (27)$$

The likelihood function will be

$$P(x_i;\alpha,\beta) = \prod_{i=1}^{n} f(x_i;\ \alpha,\beta) \quad (28)$$

The log likelihood becomes

$$L = \sum_{i=1}^{n}\log\frac{\Gamma(\sum_{i=1}^{n}x_i)+\beta)}{\Gamma(\sum_{i=1}^{n}x_i)\,\Gamma\beta} + n(1+\beta)\log\alpha - n\beta\log(1+\alpha) - \log(1+\alpha)\,(\sum_{i=1}^{n}x_i) \quad (29)$$

Here we get, the likelihood equations as

$$\frac{\delta logL}{\delta\alpha} = \frac{n(1+\beta)}{\alpha} - \frac{n\beta}{(1+\alpha)} - \frac{(\sum_{i=1}^{n}x_i)}{(1+\alpha)} = 0 \quad (30)$$

and

$$\frac{\delta logL}{\delta\beta} = \sum_{i=1}^{n}\frac{\delta}{\delta\beta}\left(\log\frac{\Gamma(\sum_{i=1}^{n}x_i)+\beta)}{\Gamma(\sum_{i=1}^{n}x_i)\,\Gamma\beta}\right) + n\log\left[\frac{\alpha}{(1+\alpha)}\right] - \frac{n}{\beta} = 0 \quad (31)$$

which leads to

$$\alpha = \frac{n(1+\beta)}{(\sum_{i=1}^{n}x_i) - n} \quad (32)$$

Putting this value of α in (31), we get

$$\sum_{i=1}^{n}\frac{\delta}{\delta\beta}\left(\log\frac{\Gamma(\sum_{i=1}^{n}x_i)+\beta)}{\Gamma(\sum_{i=1}^{n}x_i)\,\Gamma\beta}\right) + n\log\left[\frac{n(\beta+1)}{(\sum_{i=1}^{n}x_i)+n\beta}\right] - \frac{n}{\beta} = 0 \quad (33)$$

Here we are unable to get a direct solution for β. This can be solved by the trial and error or any other iterative method.

5.2. Method of Moments

The pmf of PSBGD are given as

$$g(x;\ \alpha,\beta) = \frac{\alpha}{\beta}\frac{\Gamma(x+\beta)}{\Gamma x\,\Gamma\beta}\left(\frac{\alpha}{1+\alpha}\right)^{\beta}\left(\frac{1}{1+\alpha}\right)^{x}\ x = 1,2,\dots\dots;\ \alpha,\beta > 0 \quad (34)$$

we have

$$\mu_1{}' = \frac{1+\beta}{\alpha} \quad (35)$$

$$\mu_2{}' = \frac{(1+\beta)(\alpha+\beta+2)}{\alpha^2} \quad (36)$$

From (35) we have,

$$\mu_1{}' = \bar{x} = \frac{1+\beta}{\alpha}$$

$$\hat{\alpha} = \frac{1+\beta}{\bar{x}} \quad (37)$$

Now we can write

$$\mu_2{}' = \frac{\sum f_i x_i^2}{n} = \frac{(1+\beta)(\alpha+\beta+2)}{\alpha^2}$$

Then

$$\hat{\beta} = \frac{2\bar{x}^2 + \bar{x} - \frac{\sum f_i x_i^2}{n}}{\frac{\sum f_i x_i^2}{n} - \bar{x} - \bar{x}^2} \quad (38)$$

6. Survival Function

Let F(k) be the cdf and f(k) is pmf of X. The survival function is given by

$$S(k) = 1 - F(k) = \Pr\{X > k\} = \sum_{j=k+1}^{\infty} f(j),\ k = 1,2,\dots \quad (39)$$

with S(0) = 1. S may be defined over the whole non-negative real line by

$$S(t) = S(k) \text{ for } 0 \le k \le t < k+1,\ k = 1,2,3,\dots \quad (40)$$

Where t ∈ [0, ∞). Here S (t) is a right continuous function. According to Lai (2013) our case is k =0,1,2…, that will be obtained by Y = X-1, and thus S(0-) = 1 and S(0) = pr (X = 0).

7. Classical Hazard rate function:

Let hazard (failure) rate function h(k) defined as

$$h(k) = \Pr(X = k \mid X \ge k) = \frac{\Pr(X=k)}{\Pr(X \ge k)} = \frac{f(k)}{S(k-1)} \quad (41)$$

provided

Pr(X ≥ i) = 0. It may be expressed as

$h(x) = \frac{S(k-1) - S(k)}{S(k-1)}$ (42) Equation (42) may be considered as the classical discrete hazard rate function. For convenience, we may simply refer it as the hazard rate function without the prefix „classical‟. (Lai, (2013))

8. Necessary and Sufficient Conditions (Lai, (2013))

A sequence {h(k), k ≥ 1} is a discrete hazard rate if and only if

a. For all k < m, h(k) < 1 and h(m) = 1.The distribution is then defined over {1,2,…,m}, or
b. For all k ∈ N^+ = {1,2,…}, 0 ≤ h(k) ≤ 1 and $\sum_{i=1}^{\infty} h(i) = \infty$. The distribution is defined over k ∈ N^+ in this case (Shaked et al. (1995)).

It is easily verified that Hazard Rate obtained by (42) for the distribution (5).

9. Application

9.1 consider a hypothetical data set (Source unknown)

x	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20	21
f	3	5	7	11	13	18	17	16	15	14	13	12	10	8	7	6	5	4	3	2	1

Let us fit (5) using the method of moments. Here we get sample mean $\bar{x}$ = 9.358, n = 190 and $\frac{\sum f_i x_i^2}{n}$ = 107.590. Solving (37) and (38) we have, $\hat{\alpha}$ = 0.878 and $\hat{\beta}$ = 7.215 Using these estimators, we obtain the expected frequencies as shown in Table-1

Table 1: Chi-Square Goodness-of-fit test for the proposed model SBPGD (1.5)

x	O_i	E_i	$(O_i - E_i)^2$	$\frac{(O_i - E_i)^2}{E_i}$
1	3	1	4	4
2	5	2	9	4.5
3	7	4	9	2.25
4	11	7	16	2.285714286
5	13	11	4	0.363636363
6	18	14	16	1.142857143
7	17	16	1	0.0625
8	16	18	4	0.222222222
9	15	18	9	0.5
10	14	17	9	0.529411764
11	13	16	9	0.5625
12	12	14	4	0.285714285
13	10	12	4	0.333333333
14	8	10	4	0.4
15	7	8	1	0.125
16	6	6	0	0
17	5	5	0	0
18 19 20 21	4 3 2 = 10 1	4 3 2 = 11 2	1	0.090909090
	190	190		χ^2 =17.65379849

Interpretation and conclusion

The calculated value of Chi-Square is equal to 17.65379849. The tabulated value of Chi-Square at 15 d.f. at 5 % level of significance is 24.996. From the results it is obvious that the estimated value of Chi-Square is less than the tabulated value of Chi-Square. So we can say that our proposed distribution is a good fit.

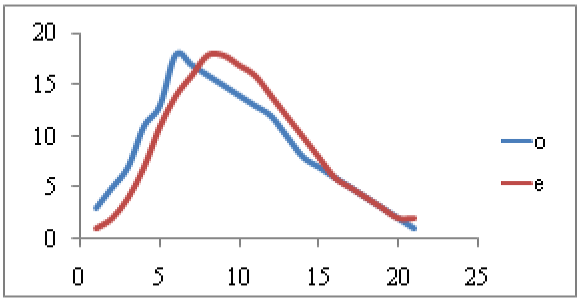

Figure 2: Observed and Fitted Frequency curves for table 1

9.2 When we choose β as integral part, so we have β = 7 from the above data set

Let us fit (5) using the method of moments. Here we get sample mean $\bar{x}$ = 9.358, n = 190 and $\frac{\sum f_i x_i^2}{n}$ = 107.590 Solving (37) and (38) we have, $\hat{\alpha}$ = 0.855 and $\hat{\beta}$ = 7 Using these estimators, we obtain the expected frequencies as shown in Table-2

Table 2: Chi-Square Goodness-of-fit test for the proposed model SBPGD (1.6)

x	O_i	E_i	$(O_i - E_i)^2$	$\frac{(O_i - E_i)^2}{E_i}$
1	3	1	4	4
2	5	2	9	4.5
3	7	4	9	2.25
4	11	7	16	2.285714286
5	13	11	4	0.363636363
6	18	14	16	1.142857143
7	17	17	0	0
8	16	17	1	0.0588235529
9	15	18	9	0.5
10	14	18	16	0.8888888888
11	13	16	9	0.5625
12	12	14	4	0.285714285
13	10	12	4	0.333333333
14	8	10	4	0.4
15	7	8	1	0.125
16	6	6	0	0
17	5	5	0	0
18 19 20 21	4 3 2 = 10 1	4 3 2 = 10 1	0	0
	190	190		χ^2 =17.69646785

Interpretation and conclusion

The calculated value of Chi-Square is equal to 17. 69646785. The tabulated value of Chi-Square with 15 d.f. at 5 % level of significance is 24.996. From the results it is obvious that the estimated value of Chi-Square is less than the tabulated value of Chi-Square. So we can say that our proposed distribution is a good fit.

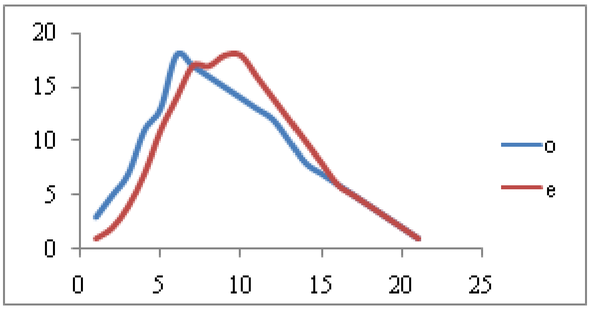

Figure 3: Observed and Fitted Frequency curves for table 2

9.3 Consider a hypothetical data set (Source unknown)

X	1	2	3	4	5	6	7	8	9	10	11	12	13	14
F	3	5	7	11	13	15	17	18	19	19	16	15	14	12
X	15	16	17	18	19	20	21	22	23	24	25			
F	11	10	9	8	7	6	5	4	3	2	1			

Let us fit (5) using the method of moments. Here we get sample mean $\bar{x} = 10.42$, n = 250 and $\frac{\sum f_i x_i^2}{n} = 152.02$

Solving (37) and (38) we have, $\hat{\alpha} = 0.3155$ and $\hat{\beta} = 2.288$

Using these estimators, we obtain the expected frequencies as shown in Table-2

Table 3: Chi-Square Goodness-of-fit test for the proposed model SBPGD (1.5)

X	O_i	E_i	$(O_i - E_i)^2$	$\frac{(O_i - E_i)^2}{E_i}$
1	3	2	1	0.5
2	5	6	1	0.166666667
3	7	9	4	0.444444444
4	11	12	1	0.083333333
5	13	15	4	0.266666667
6	15	17	4	0.235294117
7	17	18	1	0.055555555
8	18	18	0	0
9	19	18	1	0.055555555
10	19	17	4	0.235294117
11	16	16	0	0
12	15	15	0	0
13	14	14	0	0
14	12	11	1	0.090909090
15	11	10	1	0.1
16	10	9	1	0.111111111
17	9	8	1	0.125
18	8	7	1	0.142857142
19	7	6	1	0.166666667
20	6	6	0	0
21	5	4	1	0.25
22 23 24 25	4 3 2 = 10 1	4 3 3 = 12 2	4	0.33333333
Total	250	250		χ^2 =3.19573541

Interpretation and conclusion

The calculated value of Chi-Square is equal to 3.19573541. The tabulated value of Chi-Square with 19 d.f. at 5 % level of significance is 30.144. From the results it is obvious that the calculated value of Chi-Square is less than the tabulated value of Chi-Square. So we can say that our proposed distribution is a good fitt.

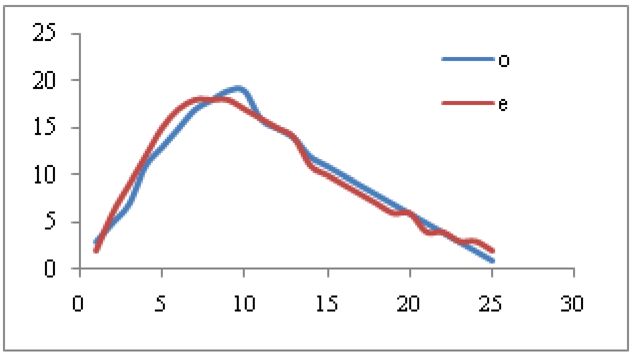

Figure 4: Observed and Fitted Frequency curves for table 3

9.4 When we choose β as integral part so we have $\hat{\beta} = 2$ from the above data set

Let us fit 5) using the method of moments. Here we get sample mean $\bar{x} = 10.42$, n = 250 and $\frac{\sum f_i x_i^2}{n} = 152.02$

Solving (37) and (38) we have, $\hat{\alpha} = 0.3155$ and $\hat{\beta} = 2$ ($\hat{\beta} = 2.288$)

Using these estimators, we obtain the expected frequencies as shown in Table-4

Table 4: Chi-Square Goodness-of-fit test for the proposed model SBPGD (1.6)

X	O_i	E_i	$(O_i - E_i)^2$	$\frac{(O_i - E_i)^2}{E_i}$
1	3	4	1	0.25
2	5	8	9	1.125
3	7	12	25	2.083
4	11	15	16	1.067
5	13	18	25	1.389
6	15	18	9	0.5
7	17	19	4	0.211
8	18	19	1	0.053
9	19	17	4	0.235
10	19	16	9	0.5625
11	16	15	1	0.0667
12	15	13	4	0.3077
13	14	12	4	0.3333
14	12	11	1	0.0909
15	11	9	4	0.4444
16	10	8	4	0.5
17	9	7	4	0.5714
18	8	6	4	0.6667
19	7	5	4	0.8
20	6	4	4	1
21	5	4	1	0.25
22 23 24 25	4 3 2 = 10 1	3 3 2 = 10 2	1	0.
	250	250		χ^2 =12.5066

Interpretation and conclusion

The calculated value of Chi-Square is equal to 12.5975. The tabulated value of Chi-Square with 19 d.f. at 5 % level of significance is 30.144. From the results it is obvious that the calculated value of Chi-Square is less than the tabulated value of Chi-Square. So we can say that our proposed distribution is a good fit.

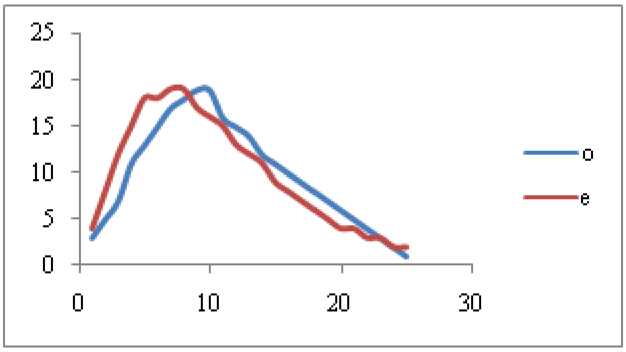

Figure 5: Observed and Fitted Frequency curves for table 4

10. Conclusion

In view of the above discussions, we conclude that Size-Biased Poisson Gamma Distribution (SBPGD) may be used as a discrete survival model with better accuracy.

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