Maximum Entropy Discrete Univariate Probability Distribution using Six Kapur’s Measure of Entropy

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Abstract: In the present paper we consider a discrete random variate which takes a finite number of values 1, 2, 3, ..., n and find the maximum entropy probability distribution under certain conditions.

Keywords: Kapur’s measure, Probability Distribution, variate

1. Introduction

We shall apply Lagrange’s method of undermined multipliers to maximize Kapur’s entropy \( K_1, K_2, K_3, K_4, K_5, K_6 \) subject to one or more conditions of the type:

\[ \sum_{i=1}^{n} P_i = 1 \text{ and } \sum_{i=1}^{m} i P_i = M \]

Shannon [2] has defined measure of entropy as

\[ K = -\frac{\log P_i}{f} \]

This measure of entropy has been generalized by Kapur [1] in following manner.

\[ \propto \frac{\gamma}{\beta} \]

This measure of entropy has been generalized by Kapur [1] in the following manner.

\( \alpha > 0, \gamma > 0, M > 0, \beta > 0, \alpha + \beta - 1 > 0 \)

\[ f(1) = 0 \text{ and } f'(1) = 1 \]

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\[ K[\log P_1 - P_1 + \log P_2 - P_2 + \ldots \ldots \ldots \ldots \log P_n - P_n] \]

\[ n^{1/n} \cdot n^{1/n} \ldots \ldots \ldots \ldots n^{1/n} \]

\[ K[\alpha^n] - 1 \]

\[ L = K[\log P_1 + \log P_2 + \ldots \ldots \ldots \ldots \log P_n] + \left( \sum_{i=1}^{n} P_i - 1 \right) \]

\[ f(1) = f'(1) = 1 \]

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\[ K_4 = \left( \sum_{i=1}^{n} P_i l_n P_i \right)^{\gamma} \]

\[ K_5 = \left( \sum_{i=1}^{n} P_i l_n P_i \right)^{\gamma} \]

\[ K_6 = \left( \sum_{i=1}^{n} P_i l_n P_i \right)^{\gamma} \]

\[ L = \sum_{i=1}^{n} P_i + \lambda \left( \sum_{i=1}^{n} P_i - 1 \right) \]

Maximizing this, \( \frac{\partial L}{\partial P_1} = \frac{\partial L}{\partial P_2} = \ldots = 0 \) we get

\[ \lambda \frac{\alpha}{\beta} = \frac{1}{\gamma} \]

\[ \gamma = \frac{n}{\alpha} \]

\[ \therefore p_i = \frac{[yf(\alpha)]^{\frac{1}{\gamma}} - 1}{\gamma} \]

3.1 Kapur [1] obtained the following result

If Shannon’s measure \( \sum_{i=1}^{n} p_i l_n p_i \) is maximized Subject to

\[ \sum_{i=1}^{n} p_i - 1 \]

Then

\[ p_1 = p_2 = p_3 = \ldots \ldots p_n = \frac{1}{n} \]

3. We will discuss the case in which the discrete variate takes only a finite set of values. Let these values are 1, 2, 3, ..., n

2. Kapur [1, Chapter 2] in his famous treatise discussed various Probability Distribution for entropy, being a natural entropy under (a) No constraint and (b) when arithmetic mean alone is prescribed. The natural question arises what happens if Shannon’s measure of entropy is replaced by \( K_1, K_2, K_3, K_4, K_5 \)? In this paper we have obtained analogous result. Kapur’s results in chapter 2 becomes the particular case of own results.

Volume 5 Issue 8, August 2016

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or
\[ p_1 = p_2 = p_3 = \ldots = \frac{1}{n} \ldots \]  
(12)
Therefore the variate follows the uniform distribution

Proof for \( K_2 \) Our problem is to maximize
\[ \frac{\prod_{i=1}^{n} p_i^{1-r} r!}{(1-r)^n} \]
subject to \( \sum_{i=1}^{n} p_i = 1 \)
The Lagrangian is
\[ L \equiv 1 - \left( \sum_{i=1}^{n} p_i^{1-r} r! \right) + \gamma \left( \sum_{i=1}^{n} p_i - 1 \right) \]

Maximizing this,
\[ \frac{\partial L}{\partial p_i} = \frac{\partial L}{\partial p_2} = \ldots = \frac{\partial L}{\partial p_n} = 0 \]
we get,
\[ p_1 = p_2 = p_3 = \ldots = p_i = \frac{1}{n} \ldots \]  
(13)

Therefore the variate follows the uniform distribution

similar technique adopted for \( K_3, K_4 \& K_5 \).

Kapur [4] obtained following result

Let the prescribed arithmetic mean be \( m(1 < m < n) \) then if \( -\sum_{i=1}^{n} p_i l_a p_i \) is maximized subject to \( \sum_{i=1}^{n} p_i = 1 \) and \( \sum_{i=1}^{n} l_p i = m \) gives
\[ p_i = ab^i i = 1, 2, \ldots, \ldots, n \]  
(14)
where \( a \) and \( b \) are determined by using the constraints
\[ \sum_{i=1}^{n} b^i i = 1 \ldots \ldots \]  
(15)
and \( a \sum_{i=1}^{n} i b^i = m \ldots \ldots \)  
(16)

In this note we replace Shannon’s measure of entropy by \( K_5 \).

We obtain following result
If \( (\sum_{i=1}^{n} l_p i) M \) is maximized subject to \( \sum_{i=1}^{n} l_p i = m \)
then \( p_i = AB^i i = 1 \ldots \ldots \)  
(17)
where \( A \) and \( B \) are obtained by
\[ A \sum_{i=1}^{n} B^i i = 1 \ldots \ldots \]  
(18)
and \( A \sum_{i=1}^{n} i B^i = m \ldots \ldots \)  
(19)

We get
\[ M(-\sum_{i=1}^{n} p_i l_a p_i) M^{-1} - \gamma - \mu m = 0 \]

Now under given condition \( \sum_{i=1}^{n} p_i = 1 \) and \( \sum_{i=1}^{n} i p_i = m \)
\[ A \sum_{i=1}^{n} B^i i = 1 \]
and \( \sum_{i=1}^{n} i B^i = m \)
This establishes the result

References


Author Profile

Hareesh R. Triwedi, is an Associate Professor in Department of Mathematics completed his M.Sc and Ph.D in Mathematics, the work area is Maximum Entropy Principle, published and presented number of papers in Conferences and seminar working at D B Science College since last 32 years, Life member ISITA. (Indian Society for Information Theory and Applications)