

# Analysis of Cracks Starting from a Square Hole in a Rectangular Plate

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**Abstract:** Fracture Mechanics is the field of mechanics concerned with the study of the propagation of cracks in materials. Engineering structures are designed to withstand the loads they are expected to be subject to while in service. However, material imperfections which arise at the time of production or usage of the material are unavoidable, and hence must be taken into account. Stress intensity factor is a fracture parameter which decides the failure of the component. In this work, analysis of two symmetrical cracks emanating from a square hole in a rectangular plate with the pressure applied in the hole. Initially the plate is subjected to remote tensile loading & stress intensity factor is determined using finite element based software ANSYS. The value of stress intensity factor is compared with the estimated value which is determined using the boundary correction factor given in the literature. Then the loading condition is changed to pressure applied in the hole only. Further pressure is applied in the hole & also to the crack face & stress intensity factor is determined for every case. Variation of length of crack, pressure etc. is studied during this work.

**Keywords:** stress intensity factor, square hole, pressure, tensile load, crack

## 1. Introduction

Advanced structural components are being used in recent years in aircraft, automobiles, missile systems and space structures etc. Flaws and other discontinuities may cause the formation of small cracks, especially in members subjected to repeated loading. To fully utilize the potential of modern highly stressed structures, it is necessary to predict their behavior in the presence of flaws which frequently exist, more often due to manufacturing/fabrication defects or in-service damage. Stress intensity factor is one fracture parameter which when compared with toughness of the material gives details about the behavior of the component.

### Findings of literature review

- There are number of methods of analyzing cracked components.
- Finite element method is cheap & good method to analyze complex geometries.
- Very less work is done in case of cracks starting from a square hole.

### Objectives of the Work

- To prepare a finite element model of a rectangular plate with a square hole & cracks starting from hole.
- To validate the finite element model using the boundary correction factors available in the literatures by determining the Stress Intensity Factor.
- To determine Stress Intensity Factor for cracks starting from a square hole in a rectangular plate when pressure is applied inside the hole.
- To study the effect of crack length, pressure applied; mesh density & type of loading.

## 2. Problem Definition

A rectangular plate with a square hole & two symmetrical cracks emanating from the corners is analyzed during this work.

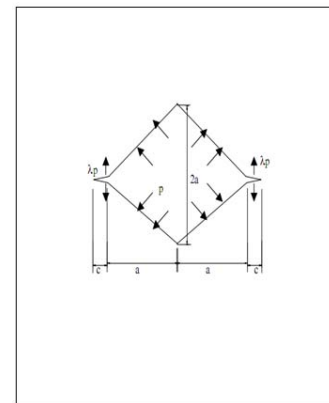


Figure 1: Rectangular plate with a square hole & two symmetrical cracks under pressure

## 3. Analytical Approach

We know that

Stress Intensity Factor =  $\sigma\sqrt{\pi c}$  x Boundary correction factor

But the boundary correction factors for this loading condition i.e. pressure applied in the hole are not available. So for the validation of the model, loading condition is taken as remote tensile load in a direction perpendicular to the cracks.

Boundary correction factor = 3.38 for  $c/a = 0.1$  .....[Ref. 10]

In the present case,  $c = 2.5$  mm &  $a = 25$  mm is taken.  
 $\sigma$  = Tensile load (in the form of Pressure) = 100 MPa  
S.I.F. =  $100 \times \sqrt{\pi \times 2.5} \times 3.38 = 947.43 \text{ MPa mm}^{1/2}$

This value of stress intensity factor will be compared with the value obtained by finite element analysis & model will be validated.

#### 4. Finite Element Analysis

The model of the plate is prepared in software ANSYS. The same software is used for processing & post-processing. The figure 2 shows meshed model of the plate with load condition & boundary condition. Here the model is meshed with 8 node quadrilateral element with the stress concentration defined at the crack tip. The model prepared is only quarter of the actual one. This is done by considering the symmetry about both the axis. Here remote tensile load is applied initially to validate the model. Input properties used are Young's Modulus =  $2.01 \times 10^5$  MPa & Poisson's ratio = 0.3

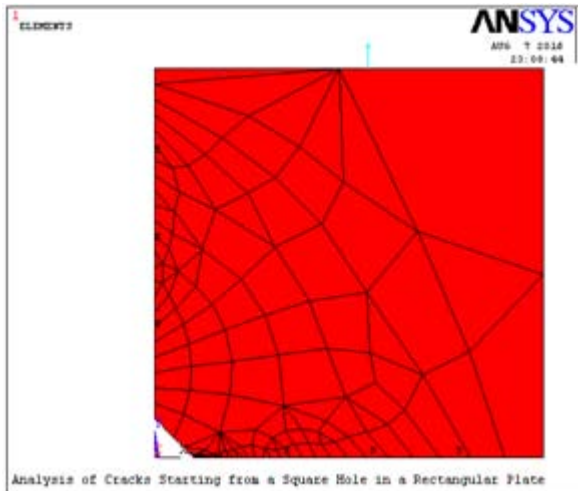


Figure 2: Meshed model with load condition & boundary condition

#### 5. Results & Discussions

##### Deformed shape

During the post-processing deformed shape of the geometry is observed. The same is shown in figure 3. It also shows un-deformed shape. Here the width of the plate is decreasing & height of the plate is increasing. Maximum deformation is 0.115906 mm.

**Von Mises stresses:** Von Mises stresses are also observed during the post-processing. Figure 4 shows Von Mises stresses. Here it is observed that the maximum stresses are appearing at the crack tip. The maximum Von Mises stresses are 1319 MPa.

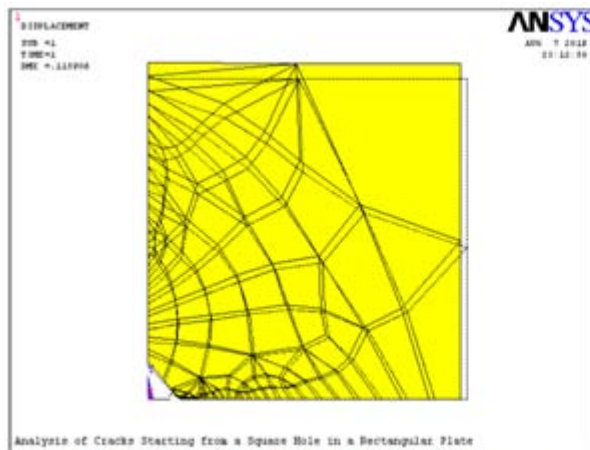


Figure 3: Deformed & Un-deformed shape

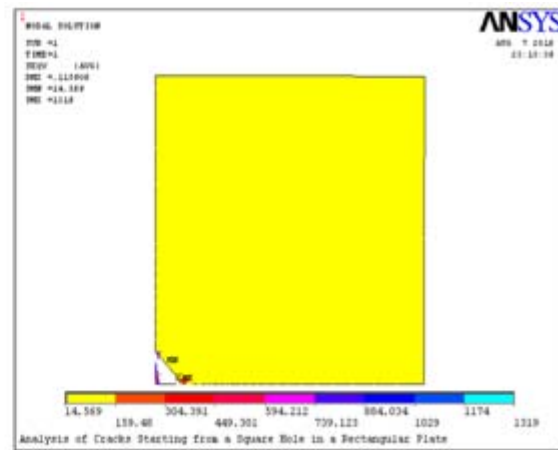


Figure 4: Von Mises stresses

##### Stress intensity factor

The stress intensity factor (S.I.F.) is determined by displacement extrapolation method.

$$S. I.F. = 976.19 \text{ MPa mm}^{1/2}$$

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KCALC Command
File
**** CALCULATE MIXED-MODE STRESS INTENSITY FACTORS ****
ASSUME PLANE STRAIN CONDITIONS
ASSUME A HALF-CRACK MODEL WITH SYMMETRY BOUNDARY CONDITIONS <USE 3 NODES>
EXTRAPOLATION PATH IS DEFINED BY NODES:      58      70      68
WITH NODE      58 AS THE CRACK-TIP NODE
USE MATERIAL PROPERTIES FOR MATERIAL NUMBER      1
EX = 0.21000E+06      NUXY = 0.30000      AT TEMP = 0.0000
**** KI = 976.19      ,      KII = 0.0000      ,      KIII = 0.0000      ****
    
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**Figure 5:** Stress Intensity Factor for tensile loading

**Comparison of results**

The boundary correction factors available in the literature are within +/- 1% limit. Using this boundary correction factor stress intensity factor value is estimated. Estimated stress intensity factor is 947.43 MPa mm<sup>1/2</sup>. Stress Intensity Factor obtained using Finite Element Analysis is 976.19 MPa mm<sup>1/2</sup>

$$\% \text{ Error} = \frac{SIF_{Estimated} - SIF_{FEA}}{SIF_{Estimated}} \times 100 = \frac{947.43 - 976.19}{947.43} \times 100 = -3.03$$

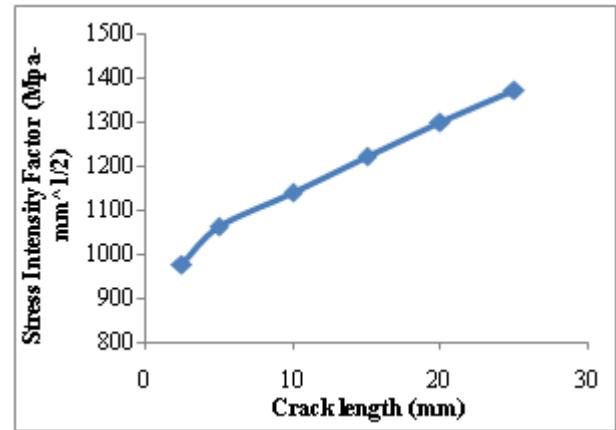
As the error is within limit, the same mesh density (element edge length) is continued for further study. In this way finite element model is validated.

**Effect of crack length on stress intensity factor**

Henceforth pressure is applied in the hole. Here crack length is varied & stress intensity factors are calculated. Different values of c/a ratio are obtained. The pressure applied is 100 MPa.

**Table 1:** Effect of crack length on Stress Intensity Factor under tensile loading

Crack length mm	C/a	Max. Von Mises Stress MPa	Max. Deformation mm	Stress Intensity Factor MPa- mm <sup>1/2</sup>
2.5	0.1	1319	0.115906	976.19
5	0.2	970.407	0.115605	1064.9
10	0.4	695.076	0.116850	1140
15	0.6	622.202	0.119206	1222.3
20	0.8	585.532	0.121892	1301.2
25	1	556.801	0.124802	1374.8



**Figure 6:** Variation of Stress Intensity Factor with crack length under remote tensile loading

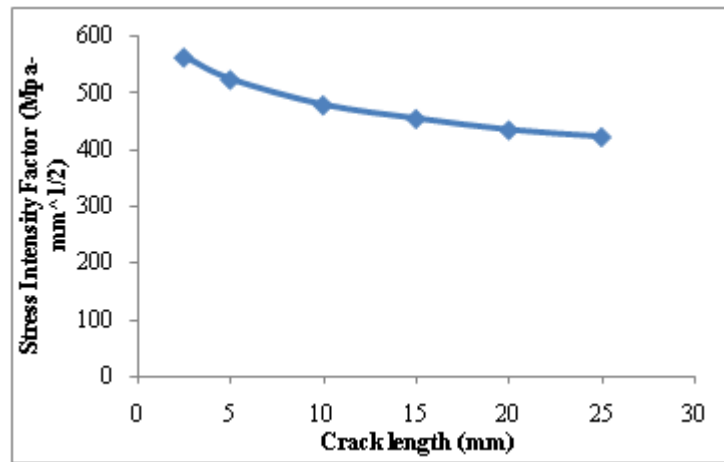
Stress Intensity Factor values increase as crack length increases almost linearly.

**Effect of crack length on stress intensity factor by applying pressure in the hole only**

Now pressure is applied in the square hole & Stress Intensity factors are determined. Applied Pressure = 100 MPa

**Table 2:** Effect of crack length on Stress Intensity Factor with pressure applied in the hole only

Crack length mm	C/a	Max. Von Mises Stress Mpa	Max. Deformation mm	Stress Intensity Factor Mpa- mm <sup>1/2</sup>
2.5	0.1	777.77	0.016522	563.84
5	0.2	485.137	0.017846	524.03
10	0.4	297.204	0.019881	478.74
15	0.6	234.301	0.021764	454.51
20	0.8	231.945	0.023454	434.39
25	1	201.373	0.025424	422.09



**Figure 7:** Variation of Stress Intensity Factor with crack length with pressure applied in the hole only

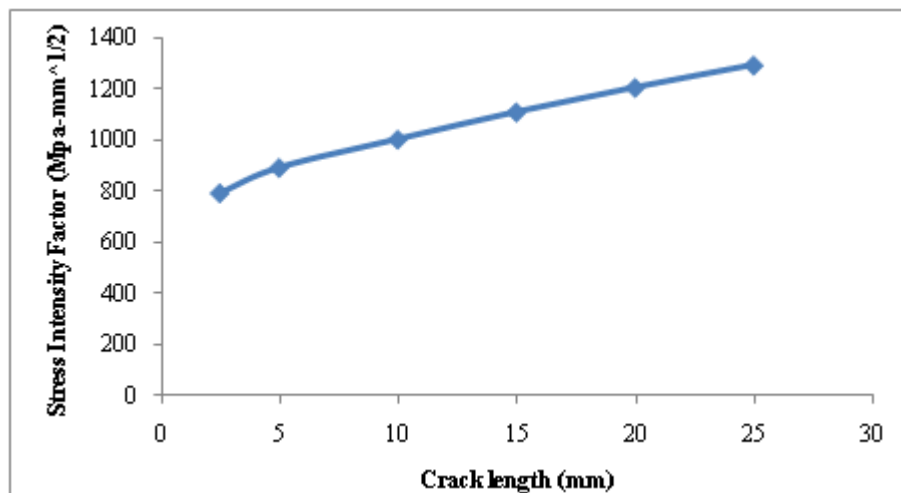
The values of Stress Intensity Factor go on decreasing with increase in crack length.

**Effect of crack length on stress intensity factor by applying pressure in the hole and crack face**

Applying pressure at both crack & hole 100 MPa, Stress Intensity Factor values are determined.

**Table 3:** Effect of crack length on Stress Intensity Factor with pressure applied in the hole & crack face

Crack length mm	C/a	Max. Von Mises Stress MPa	Max. Deformation mm	Stress Intensity Factor MPa- mm <sup>1/2</sup>
2.5	0.1	1045	0.016833	793.34
5	0.2	787.41	0.018879	893.39
10	0.4	579.699	0.023102	1004.7
15	0.6	527.77	0.027921	1112.1
20	0.8	500.684	0.033460	1208.4
25	1	478.031	0.038641	1296.5



**Figure 8:** Variation of Stress Intensity Factor with crack length with pressure applied in the hole & crack face

Stress Intensity Factor increases almost linearly with increase in crack length.

**Effect of pressure applied in the hole only on the stress intensity factor**

Effect of increasing pressure in the hole only is considered for crack length = 25 mm

**Table 4:** Effect of increasing pressure on Stress Intensity Factor

Load (Pressure) MPa	Von Mises Stress MPa	Deformation mm	Stress Intensity Factor MPa- mm <sup>1/2</sup>
100	201.373	0.025424	422.09
110	221.51	0.027966	464.30
120	241.647	0.030508	506.51
130	261.785	0.033051	548.72
140	281.922	0.035593	590.93
150	302.059	0.038135	633.14

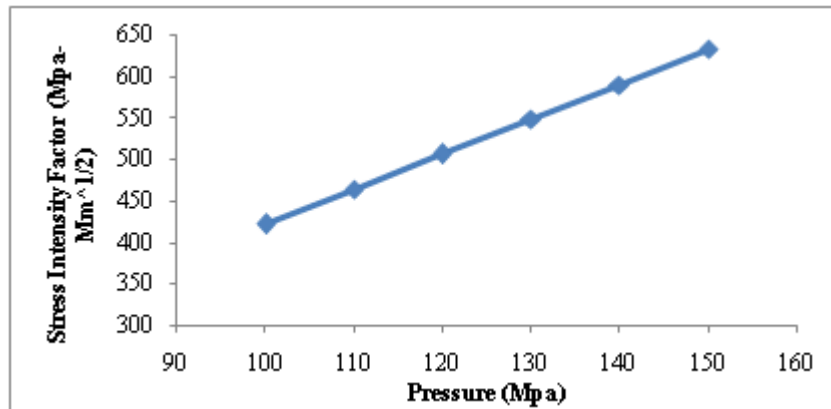


Figure 9: Variation of Stress Intensity Factor with variation in pressure

Stress Intensity Factor increases linearly with increase in pressure.

## 6. Conclusions

In this dissertation work, analysis of two symmetrical cracks emanating from a square hole is done. Following are the conclusions.

- In case of remote tensile loading, as crack length increases, Stress Intensity Factor & maximum deformation increases. But maximum Von Mises stress decreases.
- In case of pressure applied in the square hole, as crack length increases, Stress Intensity Factor & maximum Von Mises stress decreases. But maximum deformation increases.
- Values of Stress Intensity Factor are much more in case of remote tensile loading as compared to those with pressure applied inside the hole.
- Stress intensity Factor increases due to the application of the pressure to the crack face along with the pressure in the hole when compared with those with only pressure applied in the hole. But the values of Stress Intensity Factor are smaller than those obtained by applying remote tensile loading.
- With increase in pressure, all parameters i.e. maximum Von Mises stress, maximum deformation, Stress Intensity Factor increase linearly.

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