International Journal of Science and Research (IJSR)
ISSN (Online): 2319-7064
Index Copernicus Value (2013): 6.14 | Impact Factor (2015): 6.391

Some New Results on Absolute Difference of Cubic and Square Sum Labeling of a Class of Graphs

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Abstract: ***A graph labeling is an assignment of integers to the vertices or edges or both subject to certain conditions. In this paper, we introduce the new concept, an absolute difference of cubic and square sum labeling of a graph. The graph for which every edge label is the absolute difference of the sum of the cubes of the end vertices and the sum of the squares of the end vertices. It is also observed that the weights of the edges are found to be multiples of 2. Here we characterize few graphs for cubic and square sum labeling.***

Keywords: Graph labeling, cycle graph, fan graph square sum graphs, cubic graphs

1. Introduction

All graphs in this paper are finite and undirected. The symbol V(G) and E(G) denotes the vertex set and edge set of a graph G. The graph whose cardinality of the vertex set is called the order of G, denoted by p and the cardinality of the edge set is called the size of the graph G, denoted by q. A graph with p vertices and q edges is called a (p,q) graph.

A graph labeling is an assignment of integers to the vertices or edges. Some basic notations and definitions are taken from [2], [3] and [4]. Some basic concepts are taken from Frank Harary [2]. We introduced the new concept, an absolute difference of cubic and square sum labeling of a graph[1]. In this paper we investigated some new results on ADCSS labeling of product related graphs.

Definition: 1.1 [1]
Let G = (V(G), E(G)) be a graph. A graph G is said to be absolute difference of the sum of the cubes of the vertices and the sum of the squares of the vertices, if there exist a bijection

f : V(G) $\rightarrow$ {1,2,----------------,p} such that the induced function f^*_{adcss} : E(G) $\rightarrow$ multiples of 2 is given by $f^*_{adcss}(uv) = \left| f(u)^3 + f(v)^3 - (f(u)^2 + f(v)^2) \right|$ is injective.

Definition: 1.2
A graph in which every edge associates distinct values with multiples of 2 is called the sum of the cubes of the vertices and the sum of the squares of the vertices. Such a labeling is called an absolute difference of cubic and square sum labeling or an absolute difference css-labeling.

2. Main Results

Theorem 2.1
The cycle graph C_n is an ADCSS labeling.

Proof:
Let G = C_n and let v_1, v_2,----------------,v_n are the vertices of G.

Here $|V(G)| = n$ and $|E(G)| = n$

Define a function f : V $\rightarrow$ {1,2,3,--------,n } by

$f(v_i) = i$, i = 1,2,------, n.

For the vertex labeling f, the induced edge labeling f^*_{adcss} is defined as follows

$f^*_{adcss}(v_i\, v_{i+1}) = i^2(i-1) + (i+1)^2 i$, i = 1,2,---------------,n-1

$f^*_{adcss}(v_1\, v_n) = n^2(n-1)$

All edge values of G are distinct, which are multiples of 2.That is the edge values of G are in the form of an increasing order. Hence C_n admits absolute difference of css-labeling.

Theorem 2.2
The Fan graph F_n is is an ADCSS labeling.

Proof:
Let G = F_n and let v_1, v_2,----------------,v_{n+1} are the vertices of G.

Here $|V(G)| = n+1$ and $|E(G)| = 2n-1$

Define a function f : V $\rightarrow$ {1,2,3,--------,n+1} by

$f(v_i) = i$, i = 1,2,------, n+1.

For the vertex labeling f, the induced edge labeling f^*_{adcss} is defined as follows

$f^*_{adcss}(v_1\, v_i) = i^2(i-1)$, i = 2,----------------,n+1

$f^*_{adcss}(v_i\, v_{i+1}) = i^2(i-1) + (i+1)^2 i$, i = 2,----------------,n .

All edge values of G are distinct, which are multiples of 2.That is the edge values of G are in the form of an increasing order. Hence F_n admits absolute difference of css-labeling.

Theorem 2.3
The Wheel graph W_n is is an ADCSS labeling.

Proof:
Let G = W_n and let v_1, v_2,----------------,v_{n+1} are the vertices of G

Here $|V(G)| = n+1$ and $|E(G)| = 2n$

Define a function f : V $\rightarrow$ {1,2,3,--------,n+1 } by

$f(v_i) = i$, i = 1,2,------, n+1.

For the vertex labeling f, the induced edge labeling f^*_{adcss} is defined as follows

$f^*_{adcss}(v_2\, v_{n+1}) = (n+1)^2\, n + 4$

$f^*_{adcss}(v_i\, v_{i+1}) = i^2(i-1) + (i+1)^2 i$, i = 2,----------------,n .

$f^*_{adcss}(v_1\, v_i) = i^2(i-1)$, i = 2,----------------,n+1 .

All edge values of G are distinct, which are multiples of 2.That is the edge values of G are in the form of an increasing order. Hence W_n admits absolute difference of css-labeling.

Definition 2.1
A kayak paddle is described as two cycles, C_r and C_s that are joined together by a path of length l. These graphs are denoted by KP(r.s,l).

Theorem 2.4
The graph kayak paddle is is an ADCSS labeling.

Proof:
Let G = KP(r,s,l) and let v_1, v_2,----------------,$v_{r+s+l-2}$ are the vertices of G.
Here $|V(G)| = r+s+l-2$ and $|E(G)| = r+s+l-1$
Define a function f : V → {1,2,3,--------,r+s+l-2 } by
$f(v_i) = i$, i = 1,2,------, r+s+l-2 .
For the vertex labeling f, the induced edge labeling f^*_{adcss} is defined as follows
$f^*_{adcss}(v_1\ v_r) = = r^2(r-1)$
$f^*_{adcss}(v_i\ v_{i+1}) = i^2(i-1) + (i+1)^2 i$, i = 1,2,---------, r+s+l-3.
$f^*_{adcss}(v_{r+l-1}\ v_{r+s+l-2}) = (r+l-1)^2(r+l-2) + (r+s+l-2)^2(r+s+l-3)$
All edge values of G are distinct, which are multiples of 2.That is the edge values of G are in the form of an increasing order. Hence G admits absolute difference of css-labeling.

Definition 2.2
To get a web graph, we take a closed helm graph, remove the central vertex, and attach a pendant edge to each vertex of the n-cycle. We could continue this iterative process of placing an edge between pendant vertices, and then attaching new pendant edges to each vertex of the n – cycle to get more and more cycles. We denote a generalized web graph as W(t,n), where 't' is the number of n – cycles.

Theorem 2.5
The Web graph W(t.n) is is an ADCSS labeling.

Proof:
Let G =W(t,n) and let v_1, v_2,----------------,$v_{(t+1)n}$ are the vertices of G.
Here $|V(G)| = (t+1)n$ and $|E(G)| = 2nt$
Define a function f : V → {1,2,3,--------,(t+1)n } by
$f(v_i) = i$, i = 1,2,------, (t+1)n .
For the vertex labeling f, the induced edge labeling f^*_{adcss} is defined as follows
$f^*_{adcss}(v_{jn+i}\ v_{jn+i+1}) = (jn+i)^2(jn+i-1) + (jn+i+1)^2(jn+i)$,
j = 0,1,2,--------t-1.
i = 1,2,----------,n-1
$f^*_{adcss}(v_{in+1}\ v_{(i+1)n}) = (in+1)^2(in) + \{(i+1)n\}^2\{(i+1)n-1\}$,
j = 0,1,2,--------t-1.
$f^*_{adcss}(v_{in+j}\ v_{(i+1)n+j}) = (in+j)^2(in+j-1) + \{(i+1)n+j\}^2\{(i+1)n+j-1\}$,
j = 1,2,--------,n.
i = 1,2,--------,t-1

All edge values of G are distinct, which are multiples of 2.That is the edge values of G are in the form of an increasing order. Hence W(t.n) admits absolute difference of css-labeling.

Theorem 2.6
The graph obtained from the fan $F_n = P_n + K_1$ by inserting one vertex between every two consecutive vertices of the path P_n is is an ADCSS labeling.

Proof:
Let G be the graph with vertices v_1, v_2,----------------,v_{2n} .
Here $|V(G)| = 2n$ and $|E(G)| = 3n-2$
Define a function f : V → {1,2,3,--------,2n } by
$f(v_i) = i$, i = 1,2,------, 2n .
For the vertex labeling f, the induced edge labeling f^*_{adcss} is defined as follows
$f^*_{adcss}(v_i\ v_{i+1}) = i^2(i-1) + (i+1)^2 i$, i = 2,3,-----------,2n-1.
$f^*_{adcss}(v_1\ v_{2i}) = (2i)^2\ (2i-1)$, i = 1,2,---------------,n
All edge values of G are distinct, which are multiples of 2.That is the edge values of G are in the form of an increasing order. Hence G admits absolute difference of css-labeling.

Definition 2.3
The mC_n – snake is a graph obtained from m copies of C_n by identifying the vertex $V_{(k+2)j}$ in the j^{th} copy of a vertex $V_{1(j+1)}$ in the (j+1)th copy, when n = 2k+1 and identifying the vertex $V_{(k+1)j}$ in the j^{th} copy of a vertex $V_{1(j+1)}$ in the (j+1)th copy, when n = 2k.

Theorem 2.7
The mC_n – snake is is an ADCSS labeling.

Proof:
Let G = mC_n – snake and let v_1, v_2,----------------,v_{mn-m+1} are the vertices of G.
Here $|V(G)| = mn-m+1$ and $|E(G)| = mn$
Define a function f : V → {1,2,3,--------,mn-m+1 } by
$f(v_i) = i$, i = 1,2,------, mn-m+1 .
For the vertex labeling f, the induced edge labeling f^*_{adcss} is defined as follows
$f^*_{adcss}(v_i\ v_{i+1}) = i^2(i-1) + (i+1)^2 i$
i = 1,2,3,-----------------------,n-2
i = n,n+1,-----------------------,2n-3
i = 2n-1,2n,-----------------------,3n-4

= (m-1)n-(m-2), ---------------, mn-m-1
$f^*_{adcss}(v_{1+(i-1)(n-1)}\ v_{n+(i-1)(n-1)+\frac{n-1}{2}}) = \{1+(i-1)(n-1)\}^2\ \{(i-1)(n-1) + \{n+(i-1)(n-1)+\frac{n-1}{2}\}^2\{n+(i-1)(n-1)+\frac{n-1}{2}-1\}$
i = 1,2,--------------,m-1, when n is odd

$f^*_{adcss}(v_{1+(i-1)(n-1)}\ v_{n+(i-1)(n-1)+\frac{n-2}{2}}) = \{1+(i-1)(n-1)\}^2\ (i-1)(n-1) + \{(i-1)(n-1)+\frac{n-2}{2}\}^2\{n+(i-1)(n-1)+\frac{n-2}{2}-1\}$
i = 1,2,--------------,m-1, when n is even
$f^*_{adcss}(v_{i(n-1)}\ v_{n+(i-1)(n-1)+\frac{n-1}{2}}) = \{i(n-1)\}^2\ \{i(n-1)-1\} + \{(i-1)(n-1)+\frac{n-1}{2}\}^2\{n+(i-1)(n-1)+\frac{n-1}{2}-1\}$
i = 1,2,--------------,m-1, when n is odd

$f^*_{adcss}(v_{i(n-1)}\, v_{n+(i-1)(n-1)+\frac{n-2}{2}}) = \{i(n-1)\}^2 \{i(n-1)-1\} + \{n+(i-1)(n-1)+\frac{n-2}{2}\}^2\{n+(i-1)(n-1)+\frac{n-2}{2}-1\}$

i = 1,2,--------------,m-1, when n is even

$f^*_{adcss}(v_{mn-m+1} v_{mn-m-n+2}) = (mn-m+1)^2(mn-m) + (mn-m-n+2)^2(mn-m-n+1)$

$f^*_{adcss}(v_{mn-m+1} v_{mn-m}) = (mn-m+1)^2(mn-m) + (mn-m)^2(mn-m-1)$

All edge values of G are distinct, which are multiples of 2.That is the edge values of G are in the form of an increasing order. Hence mC_n – snake admits absolute difference of css-labeling.

Theorem 2.8

The quadrilateral snake $S_{4,n}$ obtained from the path $P_n(u_0,u_1,u_2,$-----------------$,u_n)$ by replacing edge u_iu_{i+1} by the cycle $(u_iu_{i+1}v_{i+1}w_{i+1}u_i)$ $1 \leq i \leq n$ is is an ADCSS labeling.

Proof:

Let G = $S_{4,n}$ and let v_1, v_2,----------------,v_{3n+1} are the vertices of G.

Here $|V(G)|$ = 3n-2 and $|E(G)|$ = 4n-4

Define a function f : V → {1,2,3,--------,3n+1 } by

$f(v_i) = i$, i = 1,2,------, 3n + 1 .

For the vertex labeling f, the induced edge labeling f^*_{adcss} is defined as follows

$f^*_{adcss}(v_i\, v_{i+1}) = i^2(i-1) + (i+1)^2 i$, i = 2,3,-----------,3n.

$f^*_{adcss}(v_i\, v_{i+3}) = i^2(i-1) + (i+3)^2 (i+2)$,

i = 1, 4,7,---------,3n-2

All edge values of G are distinct, which are multiples of 2.That is the edge values of G are in the form of an increasing order. Hence $S_{4,n}$ admits absolute difference of css-labeling.

Theorem 2.9

The graph $2m\Delta_k - snake$ is is an ADCSS labeling .

Proof:

Let G = $2m\Delta_k - snake$ and let v_1, v_2,----------------,$v_{(2m+1)k+1}$ are the vertices of G.

Here $|V(G)|$ = (2m+1)k+1 and $|E(G)|$ = (4m+1)k

Define a function f : V → {1,2,3,--------,(2m+1)k+1 } by

$f(v_i) = i$, i = 1,2,------, (2m+1)k+1 .

For the vertex labeling f, the induced edge labeling f^*_{adcss} is defined as follows

$f^*_{adcss}(v_{jk+i}\, v_{2km+i}) = (jk+i)^2(jk+i-1)+(2km+i)^2(2km+i-1)$

j = 0,1,2,---------------,2m-1

i = 1,2,3,---------------,k

$f^*_{adcss}(v_{jk+i}\, v_{2km+i+1}) = (jk+i)^2(jk+i-1) + (2km+i+1)^2(2km+i)$

j = 0,1,2,---------------,2m-1

i = 1,2,3,---------------,k

All edge values of G are distinct, which are multiples of 2.That is the edge values of G are in the form of an increasing order. Hence $2m\Delta_k - snake$ admits absolute difference of css-labeling.

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