

Influence of Increasing Live Load in Stress on Hyperbolic Paraboloid Shell Roof

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Abstract: *Hyperbolic paraboloid shell roofs are thin shell roof type. Thin shell roofs derives their strength through shape rather than mass. These are the most important member of anticlastic group from the builder's point of view. Hyperbolic paraboloid is generated by sweeping a convex parabola along a concave parabola. Great cantilevers and spans are possible with hyperbolic paraboloid shell roofs by reason of its doubly curved shape. Using the assumed floor plan, manual design for the structure is done based on membrane theory, and the size of structural components namely shell roof, edge beam and tie beam under the given loads are computed. The structure is modelled using CATIA V5 software for assigning materials and loading on the structure is done using ANSYS Workbench and the analysis is done using ANSYS 15. The analysis of saddle type hyperbolic paraboloid shell roof is carried out to study the membrane stresses (N_x, N_y and N_{xy}). The influence of live load on the membrane stresses at critical point has been studied by varying live load from 0.5 kN/m^2 to 2.5 kN/m^2 . When the live load increases, membrane stresses (N_x, N_y and N_{xy}) increases at corner and saddle points.*

Keywords: Shell roof, Hyperbolic paraboloid, Anticlastic group Membrane stresses, Membrane theory

1. Introduction

Shell forms which are becoming increasingly popular today are not a new type of construction. A hyperbolic paraboloid is an infinite surface in three dimensions with hyperbolic and parabolic cross-sections. Hyperbolic paraboloid shell roofs will be especially useful in areas subject to earth quakes. There are different types of hyperbolic paraboloid shell roofs include umbrella roof, inverted umbrella roof, saddle-type hyperbolic paraboloid shell roof, gabled hyperbolic paraboloid shell roof, hipped hyperbolic paraboloid shell roof etc. The one doubly curved shell that cuts costs through easier forming is the hyperbolic paraboloid. The use of reinforced concrete in the hyperbolic paraboloid offers the same advantages inherent to all shells of this material such as lightness, incombustibility, economy of materials, security against impact, and little sensitiveness to foundation settlement. Shells of this type have been used for entrance canopies, churches, footings, warehouses roofs, gas stations, dwellings, factories, bowling lanes, and many other buildings. The simple beauty and many advantages of the hyperbolic paraboloid mark it as a structure which will be progressively utilized in the future.

The hyperbolic paraboloid shell benefits construction because of its straight line generators. The membrane theory for this shell predicted that the convex parabolas would be in compression while the concave parabolas would be in tension of an equal magnitude. In the membrane analysis, it is assumed that the shell carry loads by in-plane stress resultant s and usually only deep doubly curved shells behave like membranes. Thus, this shape would be desirable since it would be less apt to buckle than a form with a single curvature. In this paper the behaviour of a saddle type hyperbolic paraboloid shell roof under increasing live load is studied using ANSYS 15 software.

2. Membrane theory analysis and design

As per IS 2210-1988 (Clause 8.2.3.1), only deep doubly curved shells behave like membranes and it is only for such shells that a membrane analysis is generally adequate for design. Shells may be considered deep if the rise to span ratio is greater than or equal to $1/5$. Figure 1 shows the parabolic arches which are perpendicular to each other and with which the hyper shell surface can be assumed to be made of. We may assume that these arches take loads on the shells. One will be in tension and the other in compression. If we reverse the theorem that pure shear produces pure tension and compression, the resultant of these forces can be taken as pure shear in the shell proper. We have to provide steel for the tension along the line of tensile arch. But as it is inconvenient, we provide equal steel in both directions of the edge member.

Shear = Horizontal thrust of each arch
$$\frac{w}{2} \left(\frac{ab}{h} \right) = \frac{w}{2c} = \frac{w}{2(\text{warp})} = \frac{wR}{2} \text{ or } \frac{wab}{2h} \quad (1)$$

where $\frac{h}{ab}$ is called the warp and the reciprocal $\frac{ab}{h}$ is the radius of curvature R .

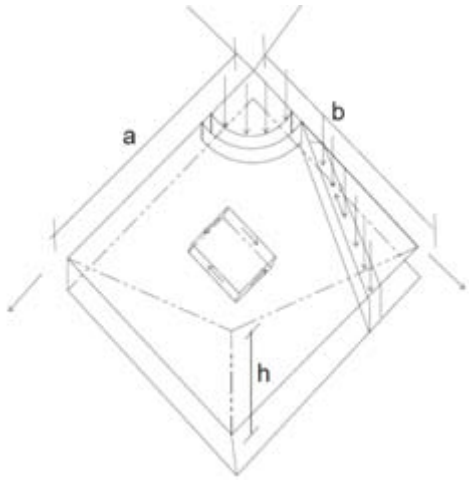


Figure 1: Forces acting in a hyper shell

Equation (1) gives the tensile or compressive thrust induced in the shell by a uniform load on the shell. The shell needs to be reinforced only for this force. In reality, since the slope of the surface (h/ab) steepens near column supports, the load cannot be considered as uniform, but this departure for a shallow shell can be tolerated. The designer should be aware of this aspect.

The pure tension and compression produces only shear proper as shown below.

$$\text{For a H P shell } N_x = N_y = 0 \text{ and } N_{xy} = \frac{wab}{2h} \quad (2)$$

The entire load is taken by the shell by pure shear, which indirectly produces equal compression and tension of the same magnitude in the edge member is the summation of all the shear forces. Saddle type hyperbolic shell roof is designed as per membrane theory.

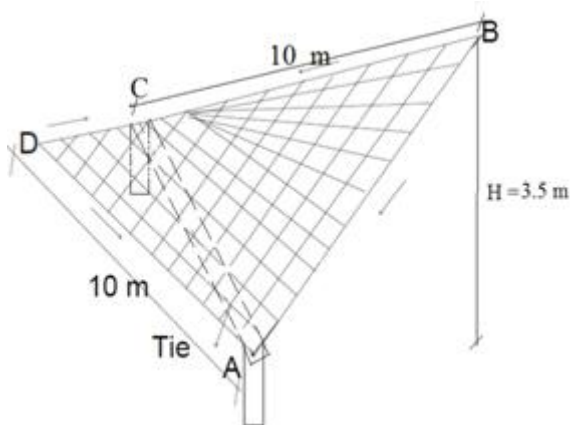


Figure 2: Layout of saddle type hyperbolic paraboloid shell roof

The general dimensions of the shell roof are 10 m by 10 m of floor plan with 3.5 m rise .075 m thickness. Normal stress and shear stress are obtained as 32.857 kN/m. The geometry of saddle type hyperbolic paraboloid shell roof includes edge beam and tie beam. The edge beam having a width of 300 mm and depth of 225 mm. The tie beam having a width of 500 mm and depth of 800 mm.

3. Modelling and Finite Element Analysis

Due to the difficulty of modelling of HP shell roof in ANSYS 15, the structure is modelled in CATIA V5. Model is converted into stp file for assigning materials and loading in ANSYS WORKBENCH. After assigning of materials and loading to structure an input file is created and export it to ANSYS 15 to analyse the structure. The model in CATIA V5 is shown in Figure 3. Assigning of materials and loading to the structure is done in ANSYS WORKBENCH. The density of concrete is 2400 kg/m^3 , characteristic compressive strength of concrete is 20 N/mm^2 , the modulus of elasticity is estimated as 22360 N/mm^2 .

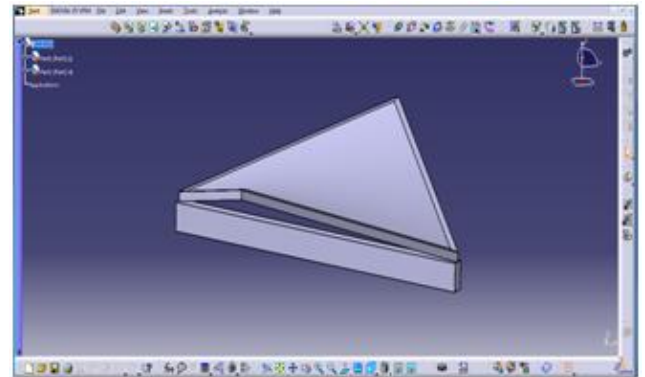


Figure 3: Isometric view of structure

The loading of model in ANSYS WORKBENCH is shown in Figure 4. A live of $.5 \text{ kN/m}^2$ is applied on roof and fixed support is provided on the both ends of the tie beam.

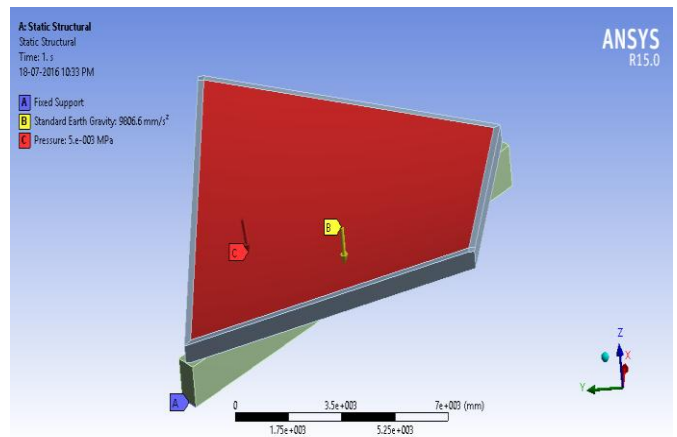


Figure 4: Loads and supports

After loading an input file is created and export it to ANSYS 15 for analysing the structure. The ANSYS model used SURF 157 to model shell roof and SOLID 186 to model edge beam and tie beam. SOLID 186 is a higher order 3-D 20-node solid element that exhibits quadratic displacement behavior. The element supports plasticity, hyperelasticity, creep, stress stiffening, large deflection, and large strain capabilities. SURF 157 is used for various load and surface effect applications in 3-D structural analyses. The element is defined by four to eight nodes and the material properties.

4. Results and Discussions

The normal stress and shear stress diagram when a live load of 0.5 kN/m² is shown below

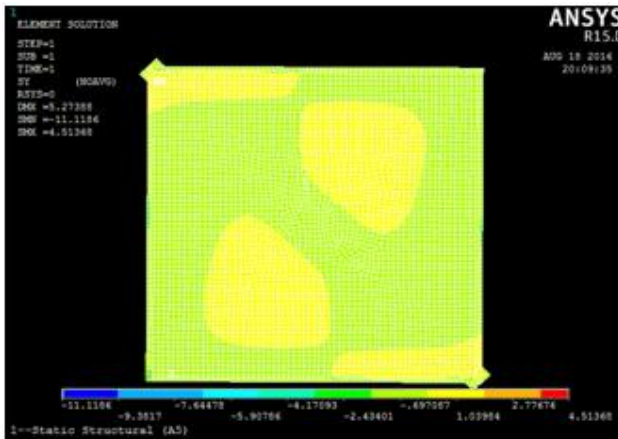


Figure 5: Normal stress (Nx) diagram

From Figure 5 it is found that maximum value of normal stress in x direction is 10.78 N/mm² and minimum value of normal stress in x direction is 4.78 N/mm². It is observed the value of stress is maximum at the supports.

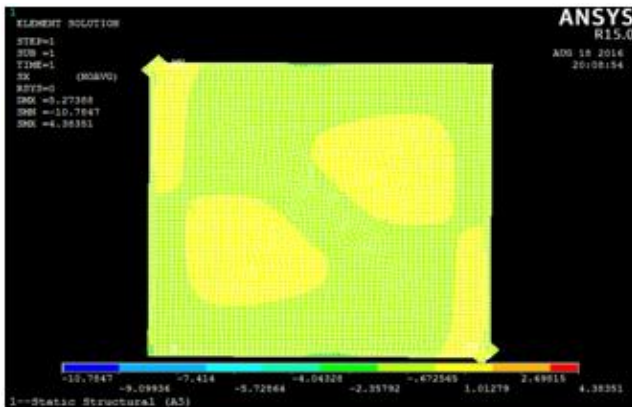


Figure6: Normal stress (Ny) diagram

From Figure 6 it is found that maximum value of normal stress in y direction is 11.11 N/mm² and minimum value of normal stress in y direction is 4.51 N/mm².

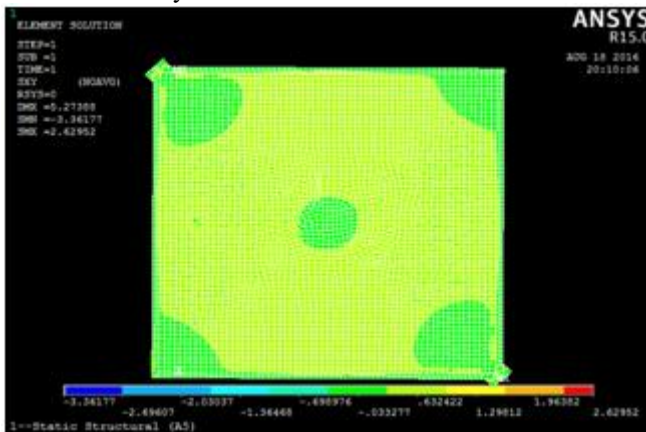


Figure 7: Shear stress(Nxy) diagram

From Figure 7 it is found that maximum value of shear stress in y direction is 3.17 N/mm² and minimum value of shear

stress in y direction is 2.83 N/mm². Table 1 shows the values of membrane stress at three locations namely corner, saddle and crown Point.

Table 1: Values of stresses

Location	Shear Stress, Nxy (Mpa)	Normal Stress, Nx (Mpa)	Normal Stress, Ny (Mpa)
Saddle point	.293	.182	.184
Corner point	.244	.844	.754
Crown point	.189	.228	.229

The live load on the shell roof is increased from .5 kN/m² to 2.5 kN/m².

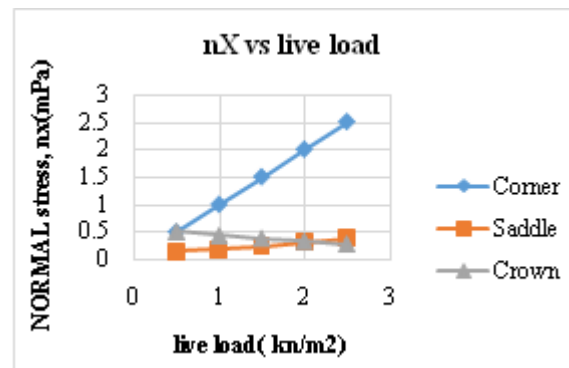


Figure 8: Normal stress (Nx) vs live load graph

From Figure 8 it is observed that, when the live load increases normal stress in x direction at corner and saddle are found to be increasing, it is quite obvious that the stresses at corner must increase as supports are provided at corner. Normal stress in x direction at crown is found to be decreasing. when the live load is 2kN/m² normal stress in x direction at saddle and crown found to be equal.

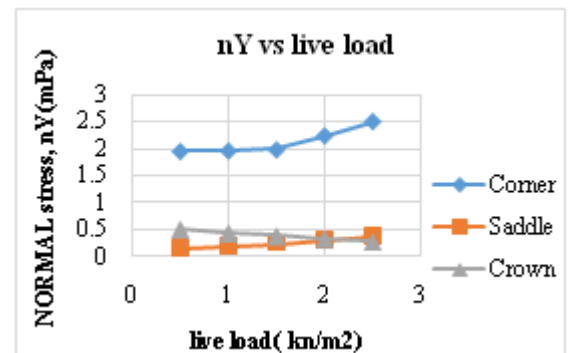


Figure 9: Normal stress (Ny) vs live load graph

From Figure 9 it is observed that, when the live load increases normal stress in y direction at corner and saddle are found to be increasing, it is quite obvious that the stresses at corner must increase as supports are provided at corner. Normal stress in y direction at crown is found to be decreasing. When the live load is 2 kN/m² normal stress in y direction at saddle and crown found to be equal.

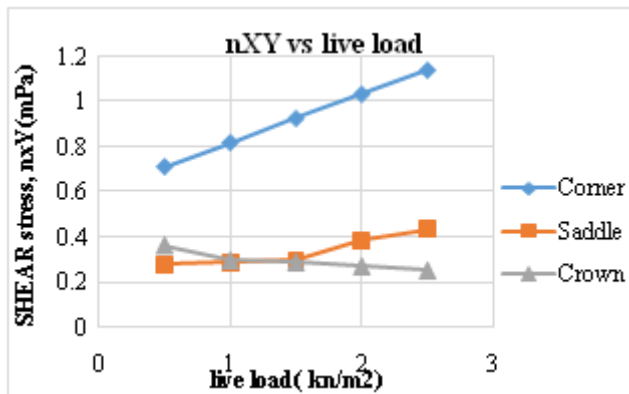


Figure 10: Normal stress (Nx) vs live load graph

From Figure 9]10it is observed that, when the live load increases shear stress in xy direction at corner and saddle are found to be increasing, it is quite obvious that the stresses at corner must increase as supports are provided at corner. Shear stress in xy direction at crown is found to be decreasing. When the live load is 1.5kN/m²shear stress in xy direction at saddle and crown found to be equal.

5. Conclusions

Hyperbolic paraboloid shells are usually doubly curved, thus avoiding the tendency for bending moments to occur. This shows that shapes of this, type, can, be utilized when building with materials which have a low tensile strength. In this study the variation of membrane stresses (Nx,Ny and Nxy) at three locations namely corner ,saddle and crown point of the hyperbolic paraboloid shell roof is studied. The live load on the shell roof is increased from .5 kN/m² to 2.5 kN/m². When the live load increases membrane stresses (Nx,Ny and Nxy) at corner and saddle are found to be increasing ,it is quite obvious that the stresses at corner must increaseas supports are provided at corner. Membrane stresses (Nx,Ny and Nxy) at crown is found to be decreasing .When the load is 1.5 kN/m² shear stress(Nxy) at saddle and crown found to be equal. When the load is 2 kN/m² normal stress(Nx and Ny) at saddle and crown found to be equal.

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