International Journal of Science and Research (IJSR)
ISSN (Online): 2319-7064
Index Copernicus Value (2013): 6.14 | Impact Factor (2015): 6.391

# Fuzzy Soft Semi Pre-Connected Properties in Fuzzy Soft Topological Spaces

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**Abstract:** ***In the present paper, we continue the study on fuzzy soft topological spaces and investigate the properties of fuzzy soft semi pre connected sets, fuzzy soft semi pre separated sets and fuzzy soft semi pre s-connected sets and have established several interesting properties supported by examples.***

**Keywords:** Fuzzy soft set, Fuzzy soft toplogicalspace, Fuzzysoft semi connected, Fuzzy soft pre connected

## 1. Introduction

Molodtsov[14] initiated the novel concept of soft set as a new mathematical tool for dealing with uncertainties in 1999. Maji et al[12] introduced the concept of Fuzzy Soft Set and some properties regarding fuzzy soft union , intersection, complement of a fuzzy soft set Demorgan Law etc. These results were further revised and improved by Ahmad and Kharal[3]. In 2011, Neog et al.[15] put forward some more propositions regarding fuzzy soft set theory and they introduced the notions of union and intersection of two fuzzy soft sets over two fuzzy soft classes.Tanay et al.[19] introduced the definition of fuzzy soft topology in a subset of the initial universe set while Roy and Samanta[17] gave the definition of fuzzy soft topology over the initial universe set. Kharal and Ahmad [3]have been introduced the concepts of fuzzy soft bijective mapping, fuzzy soft identity mapping and fuzzy soft continuous mapping. BanashreeBora[4] studied some propositions and examples related to fuzzy soft continuous mapping Some fuzzy soft topological properties based on fuzzy semi (resp. β-)open soft sets, were introduced in [1,8,9,10]. Abd El-latif et al.[2,9] introduced the concept of Fuzzy soft semi connected and Fuzzy soft pre connected properties in fuzzy soft topological spaces.In the present paper, we continue the study on fuzzy soft topological spaces and investigate the properties of fuzzy soft semi pre connected sets, fuzzy soft semi pre separated sets and fuzzy soft semi pre s-connected sets and have established several interesting properties supported by examples.

## 2. Preliminaries

**Definition 2.1[20]**

A fuzzy set $A$ in $U$ is a set of ordered pairs $A=\{(x,\mu_A(x)) : x\in U\}$ where $\mu_A(x):U\to[0,1]=I$ is a mapping and $\mu_A(x)\ (or\ A(x))$ states the grade of belongness of $x$ in A. The family of all fuzzy sets in $U$ is denoted by $I^U$.

**Definition 2.2[4]**

A pair $(F,A)$ is called a fuzzy soft set over $U$ where $F:A\to\tilde{P}(\mathrm{U})$ is a mapping from A into $\tilde{P}(U)$

**Definition 2.3[4]**

Let $A\subseteq E$. Then the mapping $F_A:E\to\tilde{P}(U)$, defined by $F_A(e)=\mu^e F_A$ (a fuzzy subset of $U$ ) is called fuzzy soft set over $(U,E)$, where $\mu^e F_A=\overline{0}$ if $e\in E-A$ and $\mu^e F_A\neq\overline{0}$ if $e\in A$. The set of all fuzzy soft set over $(U,E)$ is denoted by $FS(U,E)$.

**Definition 2.4[2]**

Let $(X,\Im,E)$ be a fuzzy soft topological space. A fuzzy soft set $f_A$ over $X$ is said to be fuzzy closed soft set in $X$, if its relative complement $f_A^{/}$ is fuzzy open soft set . We denote the set of all fuzzy closed soft sets by $FCS(X,\Im,E)$ or $FCS(X)$.

**Definition 2.5[2]**

Let $(X,\Im,E)$ be a fuzzy soft topological space and $f_A\in FSS(X)$ .The fuzzy soft closure of $f_A$, denoted by $Fcl(f_A)$ is the intersection of all fuzzy closed soft super sets of $f_A$ . That implies $Fcl(f_A)=\cap\{h_D : h_D$ is fuzzy closed soft sets and $f_A\subseteq h_D\}$. The fuzzy soft interior of $g_B$ , denoted by $F\operatorname{int}(f_A)$ is the fuzzy soft union of all fuzzy open soft subsets of $f_A$. That implies $F\operatorname{int}(g_B)=\cup\{h_D : h_D$ is fuzzy open soft set and $h_D\subseteq g_B\}$.

**Definition 2.6[4]**

Let $(U,E,\Im)$ and $(\mathrm{U}^*,E^*,\Im^*)$ be fuzzy soft topological spaces. Let $\rho:\mathrm{U}\to U^*$ and $\psi:E\to E^*$ be mappings and $f\ =(\rho,\psi):(\mathrm{U},\mathrm{E})\to(\mathrm{U}^*,\mathrm{E}^*)$ be a fuzzy soft mapping. Then $f = (\rho, \psi)$ is said to be fuzzy soft continuous if the inverse image under $f = (\rho, \psi)$ of any $G_B \in (\Im^*)$ is a fuzzy soft set $F_A \in \Im$, that is $f^{-1}(G_B) \in \Im$ whenever $G_B \in (\Im^*)$.

**Theorem 2.7[2]**
Let $FSS(X)_E$ and $FSS(Y)_K$ be two families of fuzzy soft sets. For the fuzzy soft function $f_{pu} : FSS(X)_E \to FSS(Y)_K$ the following statements hold,

(a) $f_{pu}^{-1}((g,B)^{-}) = (f_{pu}^{-1}(g,B))^{-1} \forall (g,B) \in FSS(Y)_K$.

(b) $f_{pu}(f_{pu}^{-1}((g,B))) \subseteq (g,B) \forall (g,B) \in FSS(Y)_K$. If $f_{pu}$ is surjective , then the equality holds.

(c) $(f,A) \subseteq f_{pu}^{-1}(f_{pu}((f,A))) \forall (f,A) \in FSS(X)_E$. If $f_{pu}$ is injective , then the equality holds.

(d) $f_{pu}(\tilde{0}_E) = \tilde{0}_K$, $f_{pu}(\tilde{1}) \subseteq \tilde{1}_K$. If $f_{pu}$ is surjective, then the equality holds.

(e) $f_{pu}^{-1}(\tilde{1}_K) = \tilde{1}_E$ and $f_{pu}^{-1}(\tilde{0}_K) = \tilde{0}_E$.

(f) If $(f,A) \subseteq (g,A)$, then $f_{pu}(f,A) \subseteq f_{pu}(g,A)$.

(g) If $(f,B) \subseteq (g,B)$, then $f_{pu}^{-1}(f,B) \subseteq f_{pu}^{-1}(g,B) \forall (f,B),(g,B) \in FSS(Y)_K$.

(h) $f_{pu}^{-1}(\cup_{j\in J}(f,B)_j) = \cup_{j\in J} f_{pu}^{-1}(f,B)_j$ and $f_{pu}^{-1}(\cap_{j\in J}(f,B)_j) = \cap_{j\in J} f_{pu}^{-1}(f,B)_j, \forall (f,B)_j \in FSS(Y)_K$.

(i) $f_{pu}(\cup_{j\in J}(f,A)_j) = \cup_{j\in J} f_{pu}(f,A)_j$ and $f_{pu}(\cap_{j\in J}(f,A)_j) = \cap_{j\in J} f_{pu}(f,A)_j \forall (f,A)_j \in FSS(X)_E$. If $f_{pu}$ is injective , then the equality holds.

**Definition 2.8[2]**
Let $(X, \Im, E)$ be a fuzzy soft topological space. A fuzzy soft separation of $\tilde{1}_E$ is a pair of non null proper fuzzy open soft sets $g_B, h_C$ such that $g_B \cap h_C = \tilde{0}_E$ and $\tilde{1}_E = g_B \cup h_C$.

**Definition 2.9[2]**
A fuzzy soft topological space $(X, \Im, E)$ is said to be fuzzy soft connected if and only if there is no fuzzy soft separations of $\tilde{X}$ .Otherwise $(X, \Im, E)$ is said to be fuzzy soft disconnected space.

**Definition 2.10[2]**
Let $(X, \tau, E)$ be a soft topological space . A soft semi separation on X is a pair of non null proper semi open soft sets $F_A, G_B$ such that $F_A \cap G_B = \phi$ and $X = F_A \cup G_B$ .

**Definition 2.11[2]**
A soft topological space $(X, \tau, E)$ is said to be soft semi connected if and only if there is no soft semi separations of X. Otherwise $(X, \tau, E)$ is said to be soft semi disconnected space.

**Definition 2.12[2]**
Let $(X, \Im, E)$ be a fuzzy soft topological space and $f_A \in FSS(X)_E$. If $f_A \subseteq F\,\mathrm{int}(Fcl(f_A))$ then $f_A$ is called fuzzy pre open soft set. We denote the set of all fuzzy pre open soft sets by $FPOS(X, \Im, E)$, or $FPOS(X)$ and the set of all fuzzy pre closed soft sets by $FPCS(X, \Im, E)$, or $FPCS(X)$.

**Definition 2.13[2]**
Let $(X, \Im, E)$ be a fuzzy soft topological space and $f_A \in FSS(X)_E$ and $f_e \in FSS(X)_E$. Then

(1) $f_e$ is called fuzzy pre interior soft point of $f_A$ if $\exists g_B \in FPOS(X)$ such that $f_e \in g_B \subseteq f_A$. The set of all fuzzy pre interior soft points of $f_A$ is called the fuzzy pre soft interior of $f_A$ and is denoted by $FP\,\mathrm{int}(f_A)$ consequently , $FP\,\mathrm{int}(f_A) = \cup (g_B : g_B \subseteq f_A, g_B \in FPOS(X)\}$.

(2) $f_e$ is called fuzzy pre closure soft points of $f_A$ is called the fuzzy pre soft closure $f_A$ and is denoted by $FPcl(f_A)$ consequently , $FPcl(f_A) = \cap \{h_D : h_D \in FPOS(X), f_A \subseteq h_D\}$.

**Definition 2.14[2]**
Let $(X, \Im, E)$ be a fuzzy soft topological space . A fuzzy soft pre separation on $\tilde{1}_E$ is a pair of non null proper fuzzy pre open soft sets $f_A, g_B$ such that $f_A \cap g_B = \tilde{0}_E$ and $\tilde{1}_E = f_A \cup g_B$.

**Definition 2.15[2]**
A fuzzy soft topological space $(X, \Im, E)$ is said to be fuzzy soft pre connected if and only if there is no fuzzy soft pre separations of $\tilde{1}_E$ . Otherwise, $(X, \Im, E)$ is said to be fuzzy soft pre disconnected space.

## 3. Fuzzy Soft Semi Pre- Connectedness

**Definition 3.1**
Let $(U,\Im,E)$ be a fuzzy soft topological space . A fuzzy soft semi pre separation on $\tilde{1}_E$ is a pair of non null proper fuzzy semi pre open soft sets $f_A, g_B$ such that $f_A \cap g_B = \tilde{0}_E$ and $\tilde{1}_E = f_A \cup g_B$.

**Definition 3.2**
A fuzzy soft topological space $(U,\Im,E)$ is said to be fuzzy soft semi pre connected if and only if there is no fuzzy soft semi pre separations of $\tilde{1}_E$ . Otherwise, $(U,\Im,E)$ is said to be fuzzy soft semi pre disconnected space.

**EXAMPLE 3.3**
1) The discrete fuzzy soft topological space of more than one member is always fuzzy disconnected.
2) The indiscrete fuzzy soft topological space is always fuzzy soft connected.

**Definition 3.4**

A fuzzy soft subspace $(V,\Im_V,E)$ of fuzzy soft topological space $(U,\Im,E)$ is said to be fuzzy semi pre open soft subspace if $h_E^V \in FSPOS(X)$, where $h_E^V$ is fuzzy soft subspace of $(U,\Im,E)$.

**Theorem 3.5**
Let $(V,\Im_V,E)$ be a fuzzy soft semi pre connected subspace of fuzzy soft topological space $(U,\Im,E)$ such that $h_E^V \cap g_A \in FSPOS(X), g_A \in FSPOS(X)$ . If $\tilde{1}_E$ has a fuzzy soft semi pre separations $f_A, g_B$. Then, either $h_E^V \subseteq f_A$ ,or $h_E^V \subseteq g_B$.

**Proof.**Let $f_A, g_B$ be fuzzy soft semi pre separation on $\tilde{1}_E$. By hypothesis , $f_A \cap h_E^V \in FSPOS(X)$, $g_B \cap h_E^V \in FSPOS(X)$ and $[g_B \cap h_E^V] \cup [f_A \cap h_E^V] = h_E^V$ . Since $h_E^V$ is fuzzy soft semi pre connected.Then , either $g_B \cap h_E^V = \tilde{0}_E$ or $f_A \cap h_E^V = \tilde{0}_E$. Therefore, either $h_E^V \subseteq f_A$ ,or $h_E^V \subseteq g_B$.

**Example 3.6**
Let $(U,\Im,E)$ be fuzzy soft topological spaces. Let $U = \{x,y,z\}$ and $E = \{e_1, e_2, e_3\}$ and $(U,E)$ classes of fuzzy soft sets. Let

$f_A = \{(e_1,\{x/0, y/0, z/0\}),(e_2,\{x/0, y/0, z/0\}),(e_3,\{x/0, y/0, z/0\})\}$

and $g_B = \{(e_1,\{x/1, y/1, z/1\}),(e_2,\{x/1, y/1, z/1\}),(e_3,\{x/1, y/1, z/1\})\}$ be fuzzy soft semi pre separation on $\tilde{1}_E$ and fuzzy soft subspace $h_E^V =$
$\{(e_1,\{x/0.1, y/03, z/0.5\}),(e_2,\{x/0.2, y/0.4, z/0.3\}),(e_3,\{x/0.4, y/0.1, z/0.2\})\}$ By hypothesis ,

$f_A \cap h_E^V = \{(e_1,\{x/0, y/0, z/0\}),(e_2,\{x/0, y/0, z/0\}),(e_3,\{x/0, y/0, z/0\})\} \cap$
$\{(e_1,\{x/0.1, y/03, z/0.5\}),(e_2,\{x/0.2, y/0.4, z/0.3\}),(e_3,\{x/0.4, y/0.1, z/0.2\})\} \in FSPOS(X)$,

$g_B \cap h_E^V = \{(e_1,\{x/1, y/1, z/1\}),(e_2,\{x/1, y/1, z/1\}),(e_3,\{x/1, y/1, z/1\})\} \cap$
$\{(e_1,\{x/0.1, y/03, z/0.5\}),(e_2,\{x/0.2, y/0.4, z/0.3\}),(e_3,\{x/0.4, y/0.1, z/0.2\})\} \in FSPOS(X)$,

and $[g_B \cap h_E^V] \cup [f_A \cap h_E^V] = h_E^V$. Since $h_E^V$ is fuzzy soft semi pre connected .Then , either $g_B \cap h_E^V = \tilde{0}_E$ or $f_A \cap h_E^V = \tilde{0}_E$. Therefore, either $h_E^V \subseteq f_A$ ,or $h_E^V \subseteq g_B$.

**Theorem 3.7**
If $(U,\Im_2,E)$ is a fuzzy soft semi pre connected space and $\Im_1$ is fuzzy soft coarser than $\Im_2$ , then $(U,\Im_1,E)$ is also a fuzzy soft semi pre connected.

**Proof.** Let $f_A, g_B$ be fuzzy soft semi pre separation on $(U,\Im_1,E)$. Then, $f_A, g_B \in \Im_1$. Since $\Im_1 \subseteq \Im_2$. $f_A, g_B \in \Im_2$ such that $f_A, g_B$ is fuzzy soft semi pre separation on $(U,\Im_2,E)$, which is a contradiction with the fuzzy soft semi pre connectedness of $(U,\Im_2,E)$. Hence $(U,\Im_1,E)$ is fuzzy soft semi pre connected.

**Remark 3.8**

The converse of theorem 3.7 is not true in general, as shown in the following example.

**Example 3.9**

Let $U=\{x,y,z\}$ and $E=\{e_1,e_2,e_3\}$ and $(U,E)$ classes of fuzzy soft sets.

Let $\Im_1$ be the indiscrete fuzzy soft topology, then $\Im_1$ is fuzzy soft semi pre connected, on the other hand, let

$\Im_2=\{\tilde{1}_E,\tilde{0}_E,f_A,g_A,k_B,h_B,s_E,v_E\}$ where $f_A,g_A,k_B,h_B,s_E,v_E$ fuzzy soft sets are over $U$ defined as follows.

$$f_A=\{(e_1,\{x/0,y/0,z/0\}),(e_2,\{x/0,y/0,z/0\}),(e_3,\{x/0,y/0,z/0\})\}$$

$$k_B=\{(e_1,\{x/1,y/1,z/1\}),(e_2,\{x/1,y/1,z/1\}),(e_3,\{x/1,y/1,z/1\})\},$$

$$g_A=\{(e_1,\{x/0.2,y/0.5,z/0\}),(e_2,\{x/0.2,y/0.7,z/0.4\}),(e_3,\{x/0.3,y/0.8,z/0.9\})\},$$

$$h_B=\{(e_1,\{x/0.5,y/0.2,z/0.3\}),(e_2,\{x/1,y/0.8,z/0.3\}),(e_3,\{x/0.5,y/0.2,z/0.3\})\},$$

$$s_E=\{(e_1,\{x/0.2,y/0.5,z/0.8\}),(e_2,\{x/0.1,y/0.6,z/0.7\}),(e_3,\{x/1,y/1,z/1\})\},$$

$$v_E=\{(e_1,\{x/1,y/1,z/1\}),(e_2,\{x/0.5,y/0,z/0.3\}),(e_3,\{x/1,y/0.8,z/0.3\})\}$$

Then $\Im_2$ defines a fuzzy soft topology on $U$ such that $\Im_1\subseteq\Im_2$. Now, $f_A$ and $k_B$ are fuzzy semi pre open soft sets in which form a fuzzy soft semi pre separation of $(U,\Im_2,E)$ where $f_A\cap k_B=\tilde{0}_E$ and $\tilde{1}_E=f_A\cup k_B$. Hence, $(U,\Im_2,E)$ is fuzzy soft semi pre disconnected.

**Theorem 3.10**

A fuzzy soft subspace $(V,\Im_V,E)$ of fuzzy soft semi pre disconnectedness space $(U,\Im,E)$ is fuzzy soft semi pre disconnected if $h_E^V\cap g_A\in FSPOS(X)\,\forall g_A\in FSPOS(X)$.

**Proof.**

Let $(V,\Im_V,E)$ be fuzzy soft semi pre connected space. Since $(U,\Im,E)$ is fuzzy soft semi pre disconnected. Then, there exist fuzzy soft semi pre separation $f_A,g_B$ on $(U,\Im,E)$. By hypothesis, $f_A\cap h_E^V\in FSPOS(X),g_B\cap h_E^V\in FSPOS(X)$ and $[g_B\cap h_E^V]\cup[f_A\cap h_E^V]=h_E^V$, which is a contradiction with the fuzzy soft semi pre connectedness of $(V,\Im_V,E)$. Therefore $(V,\Im_V,E)$ is fuzzy soft semi pre disconnected.

**Remark 3.11**

A fuzzy soft semi pre disconnectedness property is not hereditary property in general, as in the following example.

**Example 3.12**

In Example 3.9 let $V=\{x,y\}\subseteq U$ .We consider the fuzzy soft set $h_E^V$ over $(V,E)$ defined as follows:

$$h_E^V=\{(e_1,\{x/1,y/1,z/0\}),\{(e_2,\{x/1,y/1,z/0)\},e_3,\{x/1,y/1,z/0\})\},$$

Then we find $\Im_V$ as follows, $\Im_V=\{h_E^V\cap z_E:z_E\in\Im\}$ where $h_E^V\cap\tilde{0}_E=\tilde{0}_E,h_E^V\cap\tilde{1}_E=h_E^V,h_E^V\cap f_A=h_C$, where

$$h_C=\{(e_1,\{x/0,y/0,z/0\}),(e_2,\{x/0,y/0,z/0\}),(e_3,\{x/0,y/0,z/0\})\},$$

$h_E^V\cap g_A=h_W$ where

$$h_W=\{(e_1,\{x/0.2,y/0.5,z/0\}),(e_2,\{x/0.2,y/0.7,z/0\}),(e_3,\{x/0.3,y/0.8,z/0\})\},$$

$h_E^V\cap k_B=h_R$, where

$$h_R=\{(e_1,\{x/1,y/1,z/0\}),(e_2,\{x/1,y/1,z/0\}),(e_3,\{x/1,y/1,z/0\})\},$$

$h_E^V\cap h_B=h_r$, where

$$h_r = \{(e_1, \{x/0.5, y/0.2, z/0\}), (e_2, \{x/0.1, y/0.6, z/0\}), (e_3, \{x/1, y/1, z/0\})\}$$

$h_E^V \cap s_E = h_p$, where
$h_p = \{(e_1, \{x/0.2, y/0.5, z/0\}, (e_2, \{x/0.1, y/0.6, z/0\}), (e_3, \{x/1, y/1, z/0\})\}$,

Thus the collection $\Im_V = \{h_E^V \cap z_E : z_E \in \Im\}$ is a fuzzy soft topology on $(Y, E)$ in which there is no fuzzy soft semi pre separation on $(V, \Im_V, E)$. Therefore $(V, \Im_V, E)$ is fuzzy soft semi pre connected at the time that $(U, \Im, E)$ is fuzzy soft semi pre disconnected as shown in Example 3.9.

**Definition 3.13**
Let $(U, \Im_1, E), (V, \Im_2, K)$ be fuzzy soft topological spaces and $f_{pu} : FSS(U)_E \to FSS(V)_K$ be a soft function. Then, $f_{pu}$ is called;

(1) Fuzzy semi pre continuous soft if $f_{pu}^{-1}(g_B) \in FSPOS(U) \forall g_B \in \Im_2$.

(2) Fuzzy semi pre open soft if $f_{pu}(g_A) \in FSPOS(V) \forall g_A \in \Im_1$.

(3) Fuzzy semi pre closed soft if $f_{pu}(f_A) \in FSPCS(V) \forall f_A \in \Im_1$.

(4) Fuzzy semi pre irresolute soft if $f_{pu}^{-1}(g_B) \in FSPOS(U) \forall g_B \in FSPOS(V)$.

(5) Fuzzy semi pre irresolute open soft if $f_{pu}(g_A) \in FSPOS(V) \forall g_A \in FSPOS(U)$.

(6) Fuzzy semi pre irresolute closed soft if $f_{pu}(f_A) \in FSPCS(V) \forall f_A \in FSPCS(V)$.

**Theorem 3.14**
Let $(U_1, \Im_1, E)$ and $(U_2, \Im_2, K)$ be fuzzy soft topological spaces and $f_{pu} : (U_1, \Im_1, E) \to (U_2, \Im_2, K)$ be a fuzzy semi pre irresolute subjective soft function. If $(U_1, \Im_1, E)$ is fuzzy soft semi pre connected, then $(U_2, \Im_2, K)$ is also a fuzzy soft semi pre connected.

Proof. Let $(U_2, \Im_2, K)$ be a fuzzy soft semi pre disconnected space .Then , there exist $f_A, g_B$ pair of non null proper fuzzy semi pre open soft subsets of $\tilde{1}_K$ such that $f_A \cap g_B = \tilde{0}_K$ and $\tilde{1}_K = f_A \cup g_B$. Since $f_{pu}$ is fuzzy semi pre irresolute soft function, then $f_{pu}^{-1}(f_A), f_{pu}^{-1}(g_B)$ are pair of non null proper fuzzy semi pre open soft subsets of $\tilde{1}_E$ such that

$$f_{pu}^{-1}(f_A) \cap f_{pu}^{-1}(g_B) = f_{pu}^{-1}(f_A \cap g_B) = f_{pu}^{-1}(\tilde{0}_K) = \tilde{0}_E$$

and

$$f_{pu}^{-1}(f_A) \cup f_{pu}^{-1}(g_B) = f_{pu}^{-1}(f_A \cup g_B) = f_{pu}^{-1}(\tilde{1}_K) = \tilde{1}_E$$

from Theorem 2.7 . This means that $f_{pu}^{-1}(f_A), f_{pu}^{-1}(g_B)$ forms a fuzzy soft semi pre separation of $\tilde{1}_E$ , which is a contradiction with the fuzzy soft semi pre connectedness of $(U_1, \Im_1, E)$ . Therefore, $(U_2, \Im_2, K)$ is fuzzy soft semi pre connected.

## 4. Fuzzy Soft Semi Pre s-Connected Spaces

**Definition 4.1**
A non null fuzzy soft subsets $f_A, g_B$ of fuzzy soft topological space $(U, \Im, E)$ are said to be fuzzy soft semi pre separated sets if

$$FSPcl(f_A) \cap g_B = FSPcl(g_B) \cap f_A = \tilde{0}_E .$$

**Theorem 4.2**
Let $f_A \subseteq g_B$, $h_C \subseteq k_D$ and $g_B, k_D$, are fuzzy soft semi pre separated subsets of fuzzy soft topological space $(U, \Im, E)$ . Then, $f_A, h_C$ are fuzzy soft semi pre separated sets.

**Proof:** Let $f_A \subseteq g_B$, then $FSPcl(f_A) \subseteq FSPcl(g_B)$. It follows that,

$$FSPcl(f_A) \cap h_C \subseteq FSPcl(f_A) \cap k_D \subseteq FSPcl(g_B) \cap k_D = \tilde{0}_E.$$

Also, since $h_C \subseteq k_D$. Then $FSPcl(h_C) \subseteq FSPcl(k_D)$.

Hence, $f_A \cap FSPcl(h_C) \subseteq FSPcl(k_D) \cap g_B = \tilde{0}_E$ .

Thus, $f_A, h_C$ are fuzzy soft semi pre separated sets.

**Theorem 4.3**
Twofuzzy semi pre closed soft subsets of fuzzy soft topological space $(U, \Im, E)$ are fuzzy soft semi pre separated sets if and only if they are disjoint.

**Proof.**
Let $f_A, g_B$ are fuzzy soft semi pre separated sets. Then $FSPcl(g_B) \cap f_A = g_B \cap FSPclf_A = \tilde{0}_E$ . Since $f_A, g_B$ are fuzzy semi pre closed soft sets. Then $f_A \cap g_B = \tilde{0}_E$ .Conversely, let $f_A, g_B$ are disjoint fuzzy semi pre closed soft sets. Then ,$g_B \cap FSPcl(f_A) = f_A \cap g_B = \tilde{0}_E$ and

$FSPcl(g_B) \cap f_A = f_A \cap g_B = 0_E$. It follows that $f_A, g_B$ are fuzzy soft semi pre separated sets.

**Definition 4.4**
A fuzzy soft topological space $(U, \Im, E)$ is said to be fuzzy soft semi pre s-connected if and only if $1_E$ can not expressed as the fuzzy soft union of two fuzzy soft semi pre separated sets in $(U, \Im, E)$.

**Theorem 4.5**
Let $(Z, \Im_z, E)$ be a fuzzy soft subspace of fuzzy soft topological space $(U, \Im, E)$ and $f_A, g_B \subseteq Z_E \subseteq 1_E$. Then $f_A$ and $g_B$ are fuzzy soft semi pre separated on $\Im_z$ if and only if $f_{A_1}, g_{B_1}$ are Fuzzy soft semi pre separated on $\Im$, where $\Im_z$ is the fuzzy soft subspace for $Z_E$.

**Proof.**
Suppose that $f_A$ and $g_B$ are fuzzy soft semi pre separated on $\Im_z \Leftrightarrow FSPcl_{\Im_z}(f_A) \cap g_B = 0_E$ and $f_A \cap FSPcl_{\Im_S}(g_B) = 0_E \Leftrightarrow [FSPcl_{\Im}(f_A) \cap Z_E] \cap g_B = FSPcl_{\Im}(f_A) \cap g_B = 0_E$ and

$[FSPcl_{\Im}(g_B) \cap Z_E] \cap f_A = FSPcl_{\Im}(g_B) \cap f_A = 0_E$

$\Leftrightarrow$ $f_A$ and $g_B$ are fuzzy soft semi pre separated on $\Im$.

**Theorem 4.6**
Let $Z_E$ be a fuzzy soft subset of fuzzy soft topological space $(U, \Im, E)$ .Then $Z_E$ is fuzzy soft pre s-connected w.r.t $(U, \Im, E)$ if and only if $Z_E$ is fuzzy soft semi pre s-connected w.r.t $(Z, \Im_z, E)$.

**Proof.** Suppose that $Z_E$ is not fuzzy soft semi pre s-connected w.r.t $(Z, \Im_Z, E)$ .Then $Z_E = f_{1A} \cup f_{2B}$ where $f_{1A}$ and $f_{2B}$ are fuzzy soft semi pre separated on $\Im_Z$ Let $(Z, \Im_z, E)$ be a fuzzy soft subspace of fuzzy soft topological space $(U, \Im, E)$ and $f_A, g_B \subseteq Z_E \subseteq 1_E$. Then $f_A$ and $g_B$ are fuzzy soft semi pre separated on $\Im_z$ if and only if $f_{A_1}, g_{B_1}$ are Fuzzy soft semi pre separated on $\Im$, where $\Im_z$ is the fuzzy soft subspace for $Z_E$. Therefore $Z_E$ is not fuzzy soft semi pre s-connected w.r.t $(U, \Im, E)$.

**Theorem 4.7**
Let $(Z, \Im_z, E)$ be a fuzzy soft semi pre s-connected subspace of fuzzy soft topological space $(U, \Im, E)$ and $f_A$, $g_B$ be fuzzy soft semi pre separated of $1_E$ with $Z_E \subseteq f_A \cup g_B$. Then either $Z_E \subseteq f_A$, or $Z_E \subseteq g_B$.

**Proof.** Let $Z_E \subseteq f_A \cup g_B$ for some fuzzy soft semi pre separated subsets $f_A$, $g_B$ of $1_E$ .Since $Z_E = (Z_E \cap f_A) \cup (Z_E \cap g_B)$. Then $(Z_E \cap f_A) \cap FSPcl_{\Im}(Z_E \cap g_B) \subseteq (f_A \cap FSPcl_{\Im} g_B) = 0_E$. Also

$FSPcl_{\Im}(Z_E \cap f_A) \cap (Z_E \cap g_B) \subseteq FSPcl_{\Im}(f_A) \cap g_B = 0_E$

.Since $(Z, \Im_z, E)$ is fuzzy soft semi pre s-connected. Thus, either $Z_E \cap f_A = 0_E$ or $Z_E \cap g_B = 0_E$. It follows that , $Z_E = Z_E \cap f_A$ or $Z_E = Z_E \cap g_B$. This implies that , $Z_E \subseteq f_A$ or $Z_E \subseteq g_B$.

**Theorem 4.8**
Let $(Z, \Im_Z, N)$ and $(Y, \Im_Y, M)$ be fuzzy soft semi pre s-connected subspaces of fuzzy soft topological space $(U, \Im, E)$ such that none of them is fuzzy soft semi pre separated. Then, $Z_N \cup Y_M$ is fuzzy soft semi pre s-connected.

**Proof.** Let $(Z, \Im_Z, N)$ and $(Y, \Im_Y, M)$ be fuzzy soft semi pre s-connected subspaces of $1_E$ such that $Z_N \cup Y_M$ is not fuzzy soft semi pre s-connected .Then there exist two non null fuzzy soft semi pre separated sets $k_D$ and $h_C$ of $1_E$ such that $Z_N \cup Y_M = k_D \cap h_C$ . Since $Z_N, Y_M$ are fuzzy soft semi pre s-connected , $Z_N, Y_M \subseteq$ $Z_N \cup f_A = k_D \cup h_C$ .By theorem 4.7 ,either $Z_N \subseteq k_D or Z_N \subseteq h_C$, also either $Y_M \subseteq k_D or Y_M \subseteq h_C$. If $Z_N \subseteq k_D or Z_N \subseteq h_C$ . Then $Z_N \cap h_C \subseteq k_D \cap h_C = 0_E$ or $Z_N \cap k_D = 0_E$ . Therefore

$[Z_N \cup Y_M] \cap k_D = [Z_N \cap k_D] \cup [Y_M \cup k_D] = [Y_M \cap k_D] \cup 0_E = Y_M \cap k_D = Y_M$ since $Y_M \subseteq k_D$. Similarly if $Y_M \subseteq k_D$ or $Y_M \subseteq h_C$ .we get $[Z_N \cup Y_M] \cap h_C = Z_N$.

Now $[(Z_N \cup Y_M) \cap h_C] \cap FSPcl[(Z_N \cup Y_M) \cap k_D] \subseteq [(Z_N \cup Y_M) \cap h_C] \cap [FSPcl[(Z_N \cup Y_M) \cap FSPcl(k_D)]$

$=[Z_N \cup Y_M] \cap [h_C \cap FSPcl(k_D)] = 0_E$ and

$FSPcl[(Z_N \cup Y_M) \cap h_C] \cap [(Z_N \cup Y_M) \cap k_D] \subseteq [FSPcl[(Z_N \cup Y_M) \cap FSPcl(h_C)] \cap [(Z_N \cup Y_M) \cap k_D] = [Z_N \cup Y_M] \cap [FSPcl(h_C) \cap k_D] = 0_E$

.It follows that , $[Z_N \cup Y_M] \cap k_D = Z_N$ and $[Z_N \cup Y_M] \cap h_C = Y_M$ are fuzzy soft semi pre separated , which is a contradiction .Hence $Z_N \cup Y_M$ is fuzzy soft semi pre s-connected.

**Theorem 4.9**

Let $(Z, \Im_Z, N)$ be a fuzzy soft semi pre s-connected subspace of fuzzy soft topological space $(U, \Im, E)$ and $S_M \in SS(X)_E$. If $z_N \subseteq S_M \subseteq FSPcl(z_N)$. Then, $(S, \Im_S, M)$ is fuzzy soft semi pre s-connected subspace of $(U, \Im, E)$.

**Proof**

Suppose that $(S, \Im_S, M)$ is not fuzzy soft semi pre s-connected subspace of $(U, \Im, E)$ .Then there exist fuzzy soft semi pre separated sets $f_A$ and $g_B$ on $\Im$ such that $S_M = f_A \cup g_B$. So, we have $z_N$ is fuzzy soft semi pre s-connected subset of fuzzy soft pre s-disconnected space .By theorem 4.7, either $z_N \subseteq f_A$ or $z_N \subseteq g_B$. If $z_N \subseteq f_A$ .Then , $FSPcl(z_N) \subseteq FSPcl(f_A)$ . It follows that $FSPcl(z_N) \cap g_B \subseteq FSPcl(f_A) \cap g_B = 0_E$ .Hence , $g_B = FSPcl(z_N) \cap g_B = 0_E$. which is a contradiction . If $z_N \subseteq g_B$..By a similar way, we can get $f_A = 0_E$ , which is a contradiction. Hence $(S, \Im_S, M)$ is is fuzzy soft semi pre s-connected subspace of $(U, \Im, E)$.

**Corollary 4.10**

If $(Z, \Im_Z, N)$ is fuzzy soft semi pre s-connected subspace of fuzzy soft topological space $(U, \Im, E)$. Then, $FSPcl(z_N)$ is fuzzy soft semi pre s-connected.

**Proof.** It is obvious from Thorem 4.9.

**Theorem 4.11**

If for all pair of distinct fuzzy soft point $f_e, g_e$, there exist a fuzzy soft semi pre s-connected set $z_N \subseteq 1_E$ with $f_e, g_e \in z_N$, then $1_E$ is fuzzy soft semi pre s-connected.

**Proof.**

Suppose that $1_E$ is fuzzy soft semi pre s-disconnected .Then $1_E = f_A \cup g_B$, where $f_A, g_B$ are Fuzzy soft semi pre separated sets .It follows $f_A \cap g_B = 0_E$ ,So, $\exists f_e \in f_A$ and $g_e \in g_B$. Since $f_A \cap g_B = 0_E$ .Then $f_e, g_e$, are distinct fuzzy soft point in $1_E$. By hypothesis there exists a fuzzy soft semi pre s-connected set $z_N$ such that $f_e, g_e \in z_N \subseteq 1_E$ and $f_e, g_e \in z_N$. Moreover we have $z_N$ is fuzzy soft semi pre s-connected subset of a fuzzy soft semi pre s-disconnected space. It follows by Theorem 4.7, either $z_N \subseteq f_A$ or $z_N \subseteq g_B$ and both cases is a contradiction with the hypothesis. Therefore $1_E$ is fuzzy soft semi pre s-connected.

**Theorem 4.12**

Let $\{(Z_j, \Im_{z_j}, N) : j \in J\}$ be a non null family of fuzzy soft semi s-connected subspaces of fuzzy soft topological space $(U, \Im, E)$. $\cap_{j \in J}(z_j, N) \neq 0_E$, then $(\cup_{j \in J} Z_j, \Im_{\cup_{j \in J} z_j}, N)$ is also a fuzzy soft semi pre s-connected fuzzy subspace of $(U, \Im, E)$.

**Proof .**

Suppose that $(Z, \Im_z, N) = (\cup_{j \in J} Z_j, \Im_{\cup_{j \in J} z_j}, N)$ is fuzzy soft semi pre s-disconnected .Then $z_N = f_A \cup g_B$ for some fuzzy soft semi pre separated subsets $f_A, g_B$ of $1_E$. Since $\cap_{j \in J}(z_j, N) \neq 0_E$, Then $\exists f_e \in \cap_{j \in J}(z, N)_j$. It follows that $f_e \in Z_N$. So either $f_e \in f_A$ or $f_e \in g_B$. Suppose that $f_e \in f_A$. Since $f_e \in (Z, N)_j \forall j \in J$ and $(z, N)_j \subseteq z_N$. So, we have $(z, N)_j$ is fuzzy soft semi pre s-connected subset of fuzzy soft semi pre s-disconnected set $Z_N$ .By Theorem 4.7 either $(z, N)_j \subseteq f_A$.or $(z, N)_j \subseteq g_B \forall j \in J$. If $(z, N)_j \subseteq f_A \forall j \in J$.Then $Z_N \subseteq f_A$. This implies that $g_B = 0_E$ , which is a contradiction . Therefore $(Z, \Im_z, N) = (\cup_{j \in J} Z_{j,} \Im_{\cup_{j \in J} z_j}, N)$ is fuzzy soft semi pre s-disconnected.

**Theorem 4.13**

Let $(U_1, \Im_1, E)$ and $(U_2, \Im_2, K)$ be fuzzy soft topological spaces and $f_{pu} : (U_1, \Im_1, E) \to (U_2, \Im_2, K)$ be a fuzzy semi pre irresolute surjective soft function.If $(U_1, \Im_1, E)$ is fuzzy soft semi pre s- connected, then $(U_2, \Im_2, K)$ is also a fuzzy soft semi pre s- connected.

**Proof**

Let $(U_2, \Im_2, K)$ be a fuzzy soft semi pre disconnected space .Then , there exist $f_A, g_B$ pair of non null proper fuzzy semi pre open soft subsets of $\tilde{1}_K$ such that $\tilde{1}_K = f_A \cup g_B$.

$FSPcl(f_A) \cap g_B = FSPcl(g_B) \cap f_A = \tilde{0}_E$. Since $f_{pu}$ is fuzzy semi pre irresolute soft function , then $f_{pu}^{-1}(f_A), f_{pu}^{-1}(g_B)$ are pair of non null proper fuzzy semi pre open soft subsets of $\tilde{1}_E$ such that

$$FSPcl(f_{pu}^{-1}(f_A)) \cap f_{pu}^{-1}(g_{\mathbf{B}}) \subseteq f_{pu}^{-1}(FSPcl(f_A)) \cap f_{pu}^{-1}(g_{\mathbf{B}})$$

$$= f_{pu}^{-1}(f_A \cap g_B) = f_{pu}^{-1}(\tilde{0}_K) = \tilde{0}_E ,$$

$$f_{pu}^{-1}(f_A) \cap FSPcl(f_{pu}^{-1}(g_{\mathbf{B}})) \subseteq f_{pu}^{-1}(f_A) \cap f_{pu}^{-1}(FSPcl(g_{\mathbf{B}}))$$

$$= f_{pu}^{-1}(f_A \cap g_{\mathbf{B}}) = f_{pu}^{-1}(\tilde{0}_k) = \tilde{0}_E$$

and

$$f_{pu}^{-1}(f_A) \cup f_{pu}^{-1}(g_B) = f_{pu}^{-1}(f_A \cup g_B) = f_{pu}^{-1}(\tilde{1}_K) = \tilde{1}_E$$

from Theorem 2.7 and [[10] Theorem 4.2].

This means that $f_{pu}^{-1}(f_A), f_{pu}^{-1}(g_B)$ are pair of non null proper fuzzy semi pre open soft subsets of $\tilde{1}_E$ ,

Which is a contradiction of the fuzzy soft semi pre s-connectedness of $(U_1, \Im_1, E)$

Therefore $(U_2, \Im_2, K)$ is fuzzy soft semi pre s- connected..

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