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A New Class of Contra Continuous Functions via b-Open Sets in SETS

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Abstract: ***In this paper, we define and study a new class of functions named contra g^{*+}b-continuous and almost contra g^{*+}b-continuous functions in simple extended topological spaces (SETS) and investigate some of its basic properties and relations concerning the above newly introduced functions.***

Keywords: contra g^{*+}b-continuous, almost contra g^{*+}b-continuous, g^{*+}b-locally indiscrete

1. Introduction

In 1996, Dontchev [3] initiated the notion of contra continuous functions and an year later Dontchev, Ganster and Reilly [5] studied a new class of functions called regular set connected functions. Dontchev and Noiri [4], Jafari and Noiri [6,7] investigated the concepts of contra semi-continuous functions, contra pre-continuous functions and contra α-continuous functions between topological spaces respectively. Nasef [15] defined the so called contra b-continuous functions in topological spaces. A.A.Omari and M.S.M.Noorani [18] discussed the further properties of contra b-continuous functions and established the idea of almost contra b-continuous functions. Caldas, Jafari, Noiri and Simoes [2] proposed a new class of functions called generalized contra continuous (contra g-continuous) functions. New types of contra generalized continuity such as contra αg-continuity [6] and contra gc-continuity [4] have been introduced and investigated. Metin Akdag and Alkan Ozkan [14] introduced some of the fundamental properties of contra generalized b-continuous (contra gb-continuous) via the concept of gb-open sets. Thirumalaiswamy and Saranya [22], Vidhya and Parimelazhagan [25] devised and presented a new class of functions called contra g^*b-continuous and almost contra g^*b-continuous functions in topological spaces. The concept of extending a topology by a non-open set was proposed by Levine [11] in 1963. A simple extension of a topology τ is defined as $\tau(B) = \{(B\cap O) \cup O' / O, O' \in \tau\}$ by Levine. B.Kanchana and F.Nirmala Irudayam [9, 10] formulated the concept of g^{*+}b-closed sets and g^{*+}b-continuity in extended topological spaces. The purpose of this paper is to introduce the notion of contra g^{*+}b-continuous, almost contra g^{*+}b-continuous and study some of their properties.

Throughout this paper X, Y and Z (or (X,τ^+), (Y,σ^+) and (Z,η^+)) are simple extension topological space in which no separation axioms are assumed unless and otherwise stated. For any subset A of X, the interior of A is same as the interior in usual topology and the closure of A is newly defined in simple extension topological spaces.

2. Preliminaries

The following definitions are useful in the sequel.

Definition 2.1: A subset A of a topological space (X,τ) is called a,

(i) regular open set [21], if A = int(cl(A)) and a b-open set [1], if A ⊆ cl(int(A)) ∪ int(cl(A)).
(ii) generalized closed set (briefly g-closed)[12], if cl(A) ⊆ U, whenever A ⊆ U and U is open in X.
(iii) generalized b-closed set (briefly gb-closed) [17], if bcl(A) ⊆ U, whenever A ⊆ U and U is open in X.
(iv) g^*b-closed set [23], if bcl(A) ⊆ U, whenever A ⊆ U and U is g-open in X.

Definition 2.2: A subset A of a topological space (X,τ^+) is called a,

(i) regular^+ open set [8], if A = int(cl^+(A)).
(ii) b^+-open set [16], if A ⊆ cl^+(int(A)) ∪ int(cl^+(A)).
(iii) generalized^+ closed set (briefly g^+-closed) [9], if cl^+(A) ⊆ U, whenever A ⊆ U and U is open in X.
(iv) generalized b^+-closed set (briefly gb^+-closed) [9], if bcl^+(A) ⊆ U, whenever A ⊆ U and U is open in X.
(v) g^{*+}b-closed set [9], if bcl^+(A) ⊆ U, whenever A ⊆ U and U is g^+-open in X.

Definition 2.3: A function $f: X \to Y$ is called,

(i) gb-continuous [17], if $f^{-1}(V)$ is gb-closed in X for every closed set V of Y.
(ii) g^*b-continuous [24], if $f^{-1}(V)$ is g^*b-closed in X for every closed set V of Y.
(iii) contra continuous [3], if $f^{-1}(V)$ is closed in X for each open set V of Y.
(iv) contra pre-continuous [7], if $f^{-1}(V)$ is pre-closed in X for each open set V of Y.
(v) contra semi-continuous [4], if $f^{-1}(V)$ is semi-closed in X for each open set V of Y.
(vi) contra α-continuous [6], if $f^{-1}(V)$ is α-closed in X for every open set V of Y.

(vii) contra b-continuous [18], if $f^{-1}(V)$ is b-closed in X for every open set V of Y.
(viii) contra gb-continuous [14], if $f^{-1}(V)$ is gb-closed in X for every open set V of Y.
(ix) contra g^*b-continuous [22,25], if $f^{-1}(V)$ is g^*b-closed in X for every open set V of Y.
(x) almost continuous (almost contra-continuous) [20], if $f^{-1}(V)$ is open (closed) in X for each regular open set V of Y.
(xi) almost contra g^*b-continuous [22,25], if $f^{-1}(V)$ is g^*b-closed in X for every regular open set V of Y.
(xii) regular set connected [22], if $f^{-1}(V)$ is clopen in X for every regular open set V of Y.

Definition 2.4: A function $f:(X,\tau^+)\to(Y,\sigma^+)$ is called,
(i) gb^+-continuous [19], if $f^{-1}(V)$ is gb^+-closed in X for every closed set V of Y.
(ii) g^{*+}b-continuous [10], if $f^{-1}(V)$ is g^{*+}b-closed in X for every closed set V of Y.

Definition 2.5: A space (X,τ) is called a g^*b-locally indiscrete [22], if every g^*b-open set in it is closed.

3. Contra g^{*+}b-Continuous and Almost Contra g^{*+}b-Continuous Functions

In this section we promote the new idea of contra g^{*+}b-continuous functions and almost contra g^{*+}b-continuous functions in simple extended topological spaces.

Definition 3.1: A function $f:(X,\tau^+)\to(Y,\sigma^+)$ is called,
(i) contra$^+$-continuous if $f^{-1}(V)$ is closed in X for each open set V of Y.
(ii) contra pre^+-continuous if $f^{-1}(V)$ is pre^+-closed in X for each open set V of Y.
(iii) contra $semi^+$-continuous if $f^{-1}(V)$ is $semi^+$-closed in X for each open set V of Y.
(iv) contra α^+-continuous if $f^{-1}(V)$ is α^+-closed in X for every open set V of Y.
(v) contra b^+-continuous if $f^{-1}(V)$ is b^+-closed in X for every open set V of Y.
(vi) contra gb^+-continuous if $f^{-1}(V)$ is gb^+-closed in X for every open set V of Y.
(vii) almost$^+$ continuous (almost contra$^+$-continuous) if $f^{-1}(V)$ is open (closed) in X for each regular$^+$ open set V of Y.
(viii) almost contra pre^+-continuous if $f^{-1}(V)$ is pre^+-closed in X for each regular$^+$ open set V of Y.
(ix) almost contra $semi^+$-continuous if $f^{-1}(V)$ is $semi^+$-closed in X for each regular$^+$ open set V of Y.
(x) almost contra α^+-continuous if $f^{-1}(V)$ is α^+-closed in X for each regular$^+$ open set V of Y.
(xi) almost contra b^+-continuous if $f^{-1}(V)$ is b^+-closed in X for each regular$^+$ open set V of Y.
(xii) almost contra gb^+-continuous if $f^{-1}(V)$ is gb^+-closed in X for each regular$^+$ open set V of Y.

Definition 3.2: A function $f:(X,\tau^+)\to(Y,\sigma^+)$ is called contra g^{*+}b-continuous if $f^{-1}(V)$ is g^{*+}b-closed in X for every open set V of Y.

Example 3.3: Let X = Y = {a, b, c} with topologies $\tau=\{X,\emptyset,\{a,b\}\}, \tau^+=\{X,\emptyset,\{a\},\{a,b\}\}$ and $\sigma=\{Y,\emptyset,\{b,c\}\}, \sigma^+=\{Y,\emptyset,\{b\},\{b,c\}\}$. Let $f:X\to Y$ be the identity function. Then the function f is contra g^{*+}b-continuous.

Theorem 3.4: Every contra$^+$-continuous function is contra g^{*+}b-continuous but not conversely.
Proof: Let $f:X\to Y$ be contra$^+$-continuous. Let V be any open set in Y. Then the inverse image $f^{-1}(V)$ is closed in X. Since every closed set is g^{*+}b-closed, $f^{-1}(V)$ is g^{*+}b-closed in X. Therefore f is contra g^{*+}b-continuous.

Example 3.5: Let X = Y = {a, b, c} with topologies $\tau=\{X,\emptyset,\{a,b\}\}, \tau^+=\{X,\emptyset,\{a\},\{a,b\}\}$ and $\sigma=\{Y,\emptyset,\{b\}\}, \sigma^+=\{Y,\emptyset,\{b\},\{a,c\}\}$. Let $f:X\to Y$ be the identity function. Then the function f is contra g^{*+}b-continuous but not contra$^+$-continuous.

Theorem 3.6: Every contra pre^+-continuous function is contra g^{*+}b-continuous but not conversely.
Proof: Let $f:X\to Y$ be contra pre^+-continuous. Let V be any open set in Y. Then the inverse image $f^{-1}(V)$ is pre^+-closed in X. Since every pre^+-closed set is g^{*+}b-closed, $f^{-1}(V)$ is g^{*+}b-closed in X. Therefore f is contra g^{*+}b-continuous.

Example 3.7: Let X = Y = {a, b, c} with topologies $\tau=\{X,\emptyset,\{a\},\{a,b\}\}$, $\tau^+=\{X,\emptyset,\{a\},\{b\},\{a,b\}\}$ and $\sigma=\{Y,\emptyset,\{a\}\}, \sigma^+=\{Y,\emptyset,\{a\},\{b,c\}\}$. Let $f:X\to Y$ be the identity function. Then the function f is contra g^{*+}b-continuous but not contra pre^+-continuous.

Theorem 3.8: Every contra $semi^+$-continuous function is contra g^{*+}b-continuous but not conversely.
Proof: Let $f:X\to Y$ be contra $semi^+$-continuous. Let V be any open set in Y. By the property of contra $semi^+$-continuity we have the inverse image $f^{-1}(V)$ to be $semi^+$-closed in X. But we know that every $semi^+$-closed set is g^{*+}b-closed. Hence $f^{-1}(V)$ is g^{*+}b-closed in X. Therefore f is contra g^{*+}b-continuous.

Example 3.9: Let X = Y = {a, b, c} with topologies $\tau=\{X,\emptyset,\{a,b\}\}, \tau^+=\{X,\emptyset,\{a\},\{a,b\}\}$ and

$\sigma = \{Y, \emptyset, \{a\}\}, \sigma^+ = \{Y, \emptyset, \{a\}, \{a, c\}\}$. Let $f: X \to Y$ be the identity function. Then the function f is contra $g^{*+}b$-continuous but not contra semi$^+$-continuous.

Theorem 3.10: Every contra α^+-continuous function is contra $g^{*+}b$-continuous but not conversely.

Proof: Let $f: X \to Y$ be contra α^+-continuous. Let V be any open set in Y. Then the inverse image $f^{-1}(V)$ is α^+-closed in X. Since every α^+-closed set is $g^{*+}b$-closed, $f^{-1}(V)$ is $g^{*+}b$-closed in X. Therefore f is contra $g^{*+}b$-continuous.

Example 3.11: Let X = Y = {a, b, c} with topologies $\tau = \{X, \emptyset, \{a, b\}\}, \tau^+ = \{X, \emptyset, \{a\}, \{a, b\}\}$ and $\sigma = \{Y, \emptyset, \{b, c\}\}, \sigma^+ = \{Y, \emptyset, \{b\}, \{b, c\}\}$. Define a function $f: X \to Y$ by f(a) = b, f(b) = a, f(c) = c. Then the function f is contra $g^{*+}b$-continuous but not contra α^+-continuous.

Theorem 3.12: Every contra b^+-continuous function is contra $g^{*+}b$-continuous but not conversely.

Proof: Let $f: X \to Y$ be contra b^+-continuous. Let V be any open set in Y. By the property of contra b^+-continuity we have the inverse image $f^{-1}(V)$ to be b^+-closed in X. But we know that every b^+-closed set is $g^{*+}b$-closed. Hence $f^{-1}(V)$ is $g^{*+}b$-closed in X. Therefore f is contra $g^{*+}b$-continuous.

Example 3.13: Let X = Y = {a, b, c, d} with topologies $\tau = \{X, \emptyset, \{a\}, \{a, d\}, \{c, d\}, \{a, c, d\}\}$, $\tau^+ = \{X, \emptyset, \{a\}, \{d\}, \{a, d\}, \{c, d\}, \{a, c, d\}\}$ and $\sigma = \{Y, \emptyset, \{a\}, \{b, c\}, \{a, b, c\}, \{a, b, d\}\}$, $\sigma^+ = \{Y, \emptyset, \{a\}, \{b\}, \{a, b\}, \{b, c\}, \{a, b, c\}, \{a, b, d\}\}$

Let $f: X \to Y$ be the identity function. Then the function f is contra $g^{*+}b$-continuous but not contra b^+-continuous.

Theorem 3.14: Every contra $g^{*+}b$-continuous function is contra gb^+-continuous but not conversely.

Proof: Let $f: X \to Y$ be contra $g^{*+}b$-continuous. Let V be any open set in Y. Then the inverse image $f^{-1}(V)$ is $g^{*+}b$-closed in X. Since every $g^{*+}b$-closed set is gb^+-closed, $f^{-1}(V)$ is gb^+-closed in X. Therefore f is contra gb^+-continuous.

Example 3.15: Let X = Y = {a, b, c} with topologies $\tau = \{X, \emptyset, \{a\}\}, \tau^+ = \{X, \emptyset, \{a\}, \{a, c\}\}$ and $\sigma = \{Y, \emptyset, \{a, b\}\}, \sigma^+ = \{Y, \emptyset, \{b\}, \{a, b\}\}$. Let $f: X \to Y$ be the identity function. Then the function f is contra gb^+-continuous but not contra $g^{*+}b$-continuous.

Theorem 3.16:

(i) Every contra$^+$-continuous function is contra semi$^+$-continuous function.

(ii) Every contra$^+$-continuous function is contra pre$^+$-continuous function.

(iii) Every contra semi$^+$-continuous function is contra gb^+-continuous function.

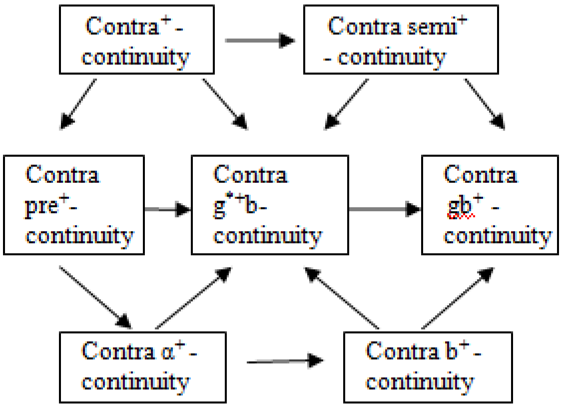
(iv) Every contra pre$^+$-continuous function is contra α^+-continuous function.

(v) Every contra α^+-continuous function is contra b^+-continuous function.

(vi) Every contra b^+-continuous function is contra gb^+-continuous function.

Proof: The proof is obvious.

Remark 3.17: As a summation of the concepts the above theorems we propose are in the following diagrammatic representation.



Remark 3.18: The following two examples will show that the concept of $g^{*+}b$-continuous and contra $g^{*+}b$-continuous are independent from each other.

Example 3.19: Let X = Y = {a, b, c} with topologies $\tau = \{X, \emptyset, \{a, b\}\}, \tau^+ = \{X, \emptyset, \{a\}, \{a, b\}\}$ and $\sigma = \{Y, \emptyset, \{b, c\}\}, \sigma^+ = \{Y, \emptyset, \{c\}, \{b, c\}\}$. Let $f: X \to Y$ be the identity function. Then the function f is contra $g^{*+}b$-continuous but not $g^{*+}b$-continuous.

Example 3.20: Let X = Y = {a, b, c} with topologies $\tau = \{X, \emptyset, \{a, b\}\}, \tau^+ = \{X, \emptyset, \{a\}, \{a, b\}\}$ and $\sigma = \{Y, \emptyset, \{a, b\}\}, \sigma^+ = \{Y, \emptyset, \{b\}, \{a, b\}\}$. Let $f: X \to Y$ be the identity function. Then the function f is $g^{*+}b$-continuous but not contra $g^{*+}b$-continuous.

Theorem 3.21: If $f: X \to Y$ is contra $g^{*+}b$-continuous map and $g: Y \to Z$ is continuous map, then their composition $g \circ f: X \to Z$ is contra $g^{*+}b$-continuous.

Proof: Let U be any open set in Z. Since $g: Y \to Z$ is continuous, $g^{-1}(U)$ is open in Y. Since $f: X \to Y$ is contra $g^{*+}b$-continuous, $f^{-1}(g^{-1}(U))$ is $g^{*+}b$-closed in X. Hence $(g \circ f)^{-1}$(U) is $g^{*+}b$-closed in X. Thus $g \circ f$ is contra $g^{*+}b$-continuous.

Theorem 3.22: If $f: X \to Y$ is $g^{*+}b$-irresolute map and $g: Y \to Z$ is $g^{*+}b$-continuous map, then their composition $g \circ f: X \to Z$ is contra $g^{*+}b$-continuous.

Proof: Let U be any open set in Z. Then $g^{-1}(U)$ is $g^{*+}b$-closed in Y, because $g: Y \to Z$ is contra $g^{*+}b$-continuous. Since $f: X \to Y$ is $g^{*+}b$-irresolute,

$f^{-1}(g^{-1}(U))$ = $(g \circ f)^{-1}$(U) is g^{*+}b-closed in X. Thus $g \circ f$ is contra g^{*+}b-continuous.

Remark 3.23: The composition of two contra g^{*+}b-continuous maps need not be a contra g^{*+}b-continuous map as seen from the following example.

Example 3.24: Let X = Y = Z = {a, b, c} with topologies $\tau = \{X, \emptyset, \{a,b\}\}$, $\tau^+ = \{X, \emptyset, \{a\}, \{a,b\}\}$; $\sigma = \{Y, \emptyset, \{b\}\}$, $\sigma^+ = \{Y, \emptyset, \{b\}, \{b,c\}\}$ and $\eta = \{Z, \emptyset, \{a,c\}\}$, $\eta^+ = \{Z, \emptyset, \{a\}, \{a,c\}\}$. Let $f: (X,\tau^+) \to (Y,\sigma^+)$ and $g: (Y,\sigma^+) \to (Z,\eta^+)$ be the identity map. Both f and g are contra g^{*+}b-continuous but their composition $g \circ f : (X,\tau^+) \to (Z,\eta^+)$ is not a contra g^{*+}b-continuous. Since for the open set {a,c} in Z, $(g \circ f)^{-1}(\{a,c\})$ = {a,b} is not g^{*+}b-closed in X and hence $g \circ f$ is not contra g^{*+}b-continuous.

Definition 3.25: A space (X, τ^+) is called a g^{*+}b-locally indiscrete if every g^{*+}b-open set in it is closed.

Example 3.26: Let X = {a, b} with the topology $\tau = \{X, \emptyset, \{a\}\}$, $\tau^+ = \{X, \emptyset, \{a\}, \{b\}\}$. Then (X, τ^+) g^{*+}b-locally indiscrete space.

Theorem 3.27: If a function $f: X \to Y$ is g^{*+}b-continuous and (X, τ^+) is g^{*+}b-locally indiscrete, then f is contra$^+$-continuous.

Proof: Let V be any open set in Y. Then the inverse image $f^{-1}(V)$ is g^{*+}b-open in X as f is g^{*+}b-continuous. Since (X, τ^+) is g^{*+}b-locally indiscrete $f^{-1}(V)$ is closed in X. Hence f is contra$^+$-continuous.

Theorem 3.28: If a function $f: X \to Y$ is contra g^{*+}b-continuous and X is the space where "every g^{*+}b-closed set is closed", then f is contra$^+$-continuous.

Proof: Let V be any open set in Y. Then the inverse image $f^{-1}(V)$ is g^{*+}b-closed in X as f is contra g^{*+}b-continuous. By hypothesis, $f^{-1}(V)$ is closed in X. Hence f is contra$^+$-continuous.

Theorem 3.29: If a function $f: X \to Y$ is g^{*+}b-irresolute map with Y as g^{*+}b-locally indiscrete space and $g : Y \to Z$ is contra g^{*+}b-continuous map, then their composition $g \circ f: X \to Z$ is contra g^{*+}b-continuous.

Proof: Let U be any closed set in Z. Since $g: Y \to Z$ is contra g^{*+}b-continuous, $g^{-1}(U)$ is g^{*+}b- open in Y. Since Y is g^{*+}b-locally indiscrete, $g^{-1}(U)$ is closed in Y. Hence $g^{-1}(U)$ is g^{*+}b-closed set in Y. Since $f: X \to Y$ is g^{*+}b-irresolute, $f^{-1}(g^{-1}(U))$ = $(g \circ f)^{-1}$(U) is g^{*+}b-closed in X. Thus $g \circ f$ is contra g^{*+}b-continuous.

Definition 3.30: A function $f: (X,\tau^+) \to (Y,\sigma^+)$ is called almost contra g^{*+}b-continuous if $f^{-1}(V)$ is g^{*+}b-closed in X for every regular$^+$ open set V of Y.

Example 3.31: Let X = Y = {a, b, c} with topologies $\tau = \{X, \emptyset, \{b\}\}, \tau^+ = \{X, \emptyset, \{b\}, \{b,c\}\}$ and $\sigma = \{Y, \emptyset, \{a\}, \{a,b\}\}, \sigma^+ = \{Y, \emptyset, \{a\}, \{b\}, \{a,b\}\}$. Let $f: X \to Y$ be the identity function. Then the function f is almost contra g^{*+}b-continuous.

Theorem 3.32: Every contra g^{*+}b-continuous function is almost contra g^{*+}b-continuous but not conversely.

Proof: Since every regular$^+$ open set is open set, the proof follows.

Example 3.33: Let X = Y = {a, b, c} with topologies $\tau = \{X, \emptyset, \{a,b\}\}, \tau^+ = \{X, \emptyset, \{a\}, \{a,b\}\}$ and $\sigma = \{Y, \emptyset, \{a\}, \{a,b\}\}, \sigma^+ = \{Y, \emptyset, \{a\}, \{b\}, \{a,b\}\}$. Let $f: X \to Y$ be the identity function. Then the function f is almost contra g^{*+}b-continuous but not contra g^{*+}b-continuous.

Theorem 3.34:

(i) Every almost contra$^+$-continuous function is almost contra g^{*+}b-continuous function.

(ii) Every almost contra pre$^+$-continuous function is almost contra g^{*+}b-continuous function.

(iii) Every almost contra semi$^+$-continuous function is almost contra g^{*+}b-continuous function.

(iv) Every almost contra α$^+$-continuous function is almost contra g^{*+}b-continuous function.

(v) Every almost contra b$^+$-continuous function is almost contra g^{*+}b-continuous function.

(vi) Every almost contra g^{*+}b-continuous function is almost contra gb$^+$-continuous function.

Proof: The proof is obvious.

Remark 3.35: Converse of the above statements is not true as shown in the following example.

Example 3.36:

(i) Let X = Y = {a, b, c} with topologies $\tau = \{X, \emptyset, \{a,b\}\}, \tau^+ = \{X, \emptyset, \{a\}, \{a,b\}\}$ and $\sigma = \{Y, \emptyset, \{a\}, \{a,c\}\}$, $\sigma^+ = \{Y, \emptyset, \{a\}, \{c\}, \{a,c\}\}$. Let $f: X \to Y$ be the identity function. Then the function f is almost contra g^{*+}b-continuous but not almost contra$^+$-continuous.

(ii) Let X = Y = {a, b, c} with topologies $\tau = \{X, \emptyset, \{a\}, \{a,b\}\}$, $\tau^+ = \{X, \emptyset, \{a\}, \{b\}, \{a,b\}\}$ and $\sigma = \{Y, \emptyset, \{b\}, \{b,c\}\}$, $\sigma^+ = \{Y, \emptyset, \{b\}, \{c\}, \{b,c\}\}$. Define a function $f: X \to Y$ by $f(a) = b, f(b) = a, f(c) = c$. Then the function f is almost contra g^{*+}b-continuous but not almost contra pre$^+$-continuous.

(iii) Let X = Y = {a, b, c} with topologies $\tau = \{X, \emptyset, \{a,b\}\}, \tau^+ = \{X, \emptyset, \{a\}, \{a,b\}\}$ and $\sigma = \{Y, \emptyset, \{a\}, \{a,c\}\}$, $\sigma^+ = \{Y, \emptyset, \{a\}, \{c\}, \{a,c\}\}$. Let $f: X \to Y$ be the identity function. Then the function f is almost contra g^{*+}b-continuous but not almost contra semi$^+$-continuous.

(iv) Let X = Y = {a, b, c} with topologies $\tau = \{X, \emptyset, \{a,b\}\}, \tau^+ = \{X, \emptyset, \{a\}, \{a,b\}\}$ and $\sigma = \{Y, \emptyset, \{a\}, \{a,c\}\}$, $\sigma^+ = \{Y, \emptyset, \{a\}, \{c\}, \{a,c\}\}$. Let $f: X \to Y$ be the identity function. Then the function f is almost contra g^{*+}b-continuous but not almost contra α^+-continuous.

(v) Let X = Y = {a, b, c, d} with topologies $\tau = \{X, \emptyset, \{a\}, \{a,d\}, \{c,d\}, \{a,c,d\}\}$, $\tau^+ = \{X, \emptyset, \{a\}, \{d\}, \{a,d\}, \{c,d\}, \{a,c,d\}\}$ and $\sigma = \{Y, \emptyset, \{c\}, \{a,c\}, \{a,b,d\}\}$, $\sigma^+ = \{Y, \emptyset, \{c\}, \{d\}, \{a,c\}, \{c,d\}, \{a,c,d\}, \{a,b,d\}\}$. Let $f: X \to Y$ be the identity function. Then the function f is almost contra g^{*+}b-continuous but not almost contra b^+-continuous.

(vi) Let X = Y = {a, b, c} with topologies $\tau = \{X, \emptyset, \{a\}\}, \tau^+ = \{X, \emptyset, \{a\}, \{a,c\}\}$ and $\sigma = \{Y, \emptyset, \{a,b\}\}$, $\sigma^+ = \{Y, \emptyset, \{c\}, \{a,b\}\}$. Let $f: X \to Y$ be the identity function. Then the function f is almost contra gb^+-continuous but not almost contra g^{*+}b-continuous.

Theorem 3.37: If a map $f: X \to Y$ from a topological space X into a topological space Y, then the following statements are equivalent:

(a) f is almost contra g^{*+}b-continuous.

(b) for every regular$^+$ closed set F of Y $f^{-1}(F)$ is g^{*+}b- open in X.

Proof:

(a) ⇒ (b) : Let F be a regular$^+$ closed set in Y, then Y – F is a regular$^+$ open set in Y. By (a), $f^{-1}(Y - F) = X - f^{-1}(F)$ is g^{*+}b-closed set in X. This implies $f^{-1}(F)$ is g^{*+}b-open set in X. Therefore (b) holds.

(b) ⇒ (a) : Let G be a regular$^+$ open set of Y. Then Y – G is a regular$^+$ closed set in Y. By (b), $f^{-1}(Y - G)$ is g^{*+}b-open set in X. This implies $X - f^{-1}(G)$ is g^{*+}b-open set in X, which implies $f^{-1}(G)$ is g^{*+}b-closed set in X. Therefore (a) holds.

Definition 3.38: A function $f: (X, \tau^+) \to (Y, \sigma^+)$ is said to be regular$^+$ set connected if $f^{-1}(V)$ is clopen in X for every regular$^+$ open set V of Y.

Theorem 3.39: If a function $f: X \to Y$ is almost contra g^{*+}b-continuous and almost$^+$ continuous together with X is the space where "every g^{*+}b-closed set is closed", then f is regular$^+$ set connected.

Proof: Let V be a regular$^+$ open set in Y. Since f is almost contra g^{*+}b-continuous and almost$^+$ continuous, $f^{-1}(V)$ is g^{*+}b-closed and open in X. By hypothesis, $f^{-1}(V)$ is clopen in X. Therefore f is regular$^+$ set connected.

References

[1] D.Andrijevic, "On b-open sets," Mat. Vesink, 48, pp.59-64, 1996.
[2] M.Caldas, S.Jafari, T.Noiri and M.Simoes, "A new generalization of contra-continuity via Levine's g-closed sets," Chaos, Solitons and Fractals, 32, pp.1597-1603, 2007.
[3] J.Dontchev, "Contra-continuous functions and strongly S-closed spaces," Int. J. Math. Sci, 19 (2), pp.303-310, 1996.
[4] J.Dontchev and T.Noiri, "On contra semi-continuous functions," Mathematica pannonica, 10 (2), pp.159-168, 1999.
[5] J.Dontchev, M.Ganster and I.Reilly, "More on almost s-continuity," Indian J. Math, 41, pp.139-146, 1999.
[6] S.Jafari and T.Noiri, "On contra α-continuous functions between topological spaces," Iran. Int. J. Sci, 2 (2), pp.153-167, 2001.
[7] S.Jafari and T.Noiri, "On contra pre-continuous functions," Bull. Malays. Math. Sci. Soc (2), 25 (2), pp.115-128, 2002.
[8] B.Kanchana and F.Nirmala Irudayam, "On a weaker form of b-open sets in extended topological spaces," International Journal of Innovative Research in Science, Engineering and Technology, 5 (5), pp. 6919-6926, May 2016.
[9] B.Kanchana and F.Nirmala Irudayam, "On the generalization of $^{*+}$b sets in simple extended topological spaces," International Journal of Innovative Research in Science, Engineering and Technology, 5 (5), pp.7107-7114, May 2016.
[10] B.Kanchana and F.Nirmala Irudayam, "On generalized $^{*+}$b-continuous and irresolute maps," International Journal of Engineering Science and Computing, 6 (7), pp.1667-1671, July 2016.
[11] N.Levine, "Simple extension of topology," Amer. Math. Monthly, 71, pp.22-105, 1964.
[12] N.Levine, "Generalized closed sets in topology," Rend. Circ. Mat. Palermo, (2) 19, pp.89-96, 1970.
[13] N.Levine, "Semi-open sets and semi-continuity in topological spaces," Amer. Math. Monthly, 70, pp.36-41, 1963.
[14] Metin Akdag and Alkan Ozkan, "Some properties of contra gb-continuous functions," Journal of New Results in Science, 1, pp.40-49, 2012.
[15] A.A.Nasef, "Some properties of contra γ-continuous functions," Chaos, Solitons and Fractals, 24, pp.471-477, 2005.
[16] F.Nirmala Irudayam and Sr.I.Arockiarani, "A note on the weaker form of bI set and its generalization in SEITS," International Journal of Computer Application, 4 (2), pp.42-54, Aug 2012.
[17] A.A.Omari and M.S.M.Noorani, "On generalized b-closed sets," Bull. Malays. Math. Sci. Soc(2), 32 (1), pp.19-30, 2009.

[18] A.A.Omari and M.S.M.Noorani, "Some properties of contra b-continuous and almost contra b-continuous functions," European Journal of Pure and Applied Mathematics, 2 (2), pp.213-230, 2009.

[19] S.Reena and F.Nirmala Irudayam, "A new weaker form of πgb-continuity," International Journal of Innovative Research in Science, Engineering and Technology, 5 (5), pp.8676-8682, May 2016.

[20] M.K.Singal and A.R.Singal, "Almost continuous mappings," Yokohama Math, 3, pp.63-73, 1968.

[21] M.H.Stone, "Application of the theory of Boolean Rings to General Topology," Trans. Amer. Math. Soc, 41, pp.374-481, 1937.

[22] M.Thirumalaiswamy and S.Saranya, "On some new functions of g^*b-continuity in topological spaces," Int. J. of Math. Archive, 3 (11), pp.4081-4084, 2012.

[23] D.Vidhya and R.Parimelazhagan, "g^*b-closed sets in topological spaces," Int. J. Contemp. Math. Sciences, 7, pp.1305-1312, 2012.

[24] D.Vidhya and R.Parimelazhagan, "g^*b-continuous maps and pasting lemma in topological spaces," Int. Journal of Math. Analysis, 6 (47), pp.2307-2315, 2012.

[25] D.Vidhya and R.Parimelazhagan, "g^*b-Homomorphisms and Contra g^*b-continuous maps in topological spaces," International Journal of Computer Applications, 58 (14), pp.1-7, 2012.