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A New Class of Contra Continuous Functions via b-Open Sets in SETS

B. Kanchana[1], F. Nirmala Irudayam[2]

[1]Research Scholar, Department of Mathematics, Nirmala College for Women, Red Fields, Coimbatore, India

[2]Assistant Professor, Department of Mathematics, Nirmala College for Women, Red Fields, Coimbatore, India

Abstract: ***In this paper, we define and study a new class of functions named contra g^{*+}b-continuous and almost contra g^{*+}b-continuous functions in simple extended topological spaces (SETS) and investigate some of its basic properties and relations concerning the above newly introduced functions.***

Keywords: contra g^{*+}b-continuous, almost contra g^{*+}b-continuous, g^{*+}b-locally indiscrete

1. Introduction

In 1996, Dontchev [3] initiated the notion of contra continuous functions and an year later Dontchev, Ganster and Reilly [5] studied a new class of functions called regular set connected functions. Dontchev and Noiri [4], Jafari and Noiri [6,7] investigated the concepts of contra semi-continuous functions, contra pre-continuous functions and contra α-continuous functions between topological spaces respectively. Nasef [15] defined the so called contra b-continuous functions in topological spaces. A.A.Omari and M.S.M.Noorani [18] discussed the further properties of contra b-continuous functions and established the idea of almost contra b-continuous functions. Caldas, Jafari, Noiri and Simoes [2] proposed a new class of functions called generalized contra continuous (contra g-continuous) functions. New types of contra generalized continuity such as contra αg-continuity [6] and contra gc-continuity [4] have been introduced and investigated. Metin Akdag and Alkan Ozkan [14] introduced some of the fundamental properties of contra generalized b-continuous (contra gb-continuous) via the concept of gb-open sets. Thirumalaiswamy and Saranya [22], Vidhya and Parimelazhagan [25] devised and presented a new class of functions called contra g^*b-continuous and almost contra g^*b-continuous functions in topological spaces. The concept of extending a topology by a non-open set was proposed by Levine [11] in 1963. A simple extension of a topology τ is defined as $\tau(B) = \{(B \cap O) \cup O' / O, O' \in \tau\}$ by Levine. B.Kanchana and F.Nirmala Irudayam [9, 10] formulated the concept of g^{*+}b-closed sets and g^{*+}b-continuity in extended topological spaces. The purpose of this paper is to introduce the notion of contra g^{*+}b-continuous, almost contra g^{*+}b-continuous and study some of their properties.

Throughout this paper X, Y and Z (or (X,τ^+), (Y,σ^+) and (Z,η^+)) are simple extension topological space in which no separation axioms are assumed unless and otherwise stated. For any subset A of X, the interior of A is same as the interior in usual topology and the closure of A is newly defined in simple extension topological spaces.

2. Preliminaries

The following definitions are useful in the sequel.

Definition 2.1: A subset A of a topological space (X,τ) is called a,

(i) regular open set [21], if A = int(cl(A)) and a b-open set [1], if A ⊆ cl(int(A)) ∪ int(cl(A)).
(ii) generalized closed set (briefly g-closed)[12], if cl(A) ⊆ U, whenever A ⊆ U and U is open in X.
(iii) generalized b-closed set (briefly gb-closed) [17], if bcl(A) ⊆ U, whenever A ⊆ U and U is open in X.
(iv) g^*b-closed set [23], if bcl(A) ⊆ U, whenever A ⊆ U and U is g-open in X.

Definition 2.2: A subset A of a topological space (X,τ^+) is called a,

(i) regular^+ open set [8], if A = $\text{int}(\text{cl}^+(A))$.
(ii) b^+-open set [16], if A ⊆ $\text{cl}^+(\text{int}(A))$ ∪ $\text{int}(\text{cl}^+(A))$.
(iii) generalized^+ closed set (briefly g^+-closed) [9], if $\text{cl}^+(A)$ ⊆ U, whenever A ⊆ U and U is open in X.
(iv) generalized b^+-closed set (briefly gb^+-closed) [9], if $\text{bcl}^+(A)$ ⊆ U, whenever A ⊆ U and U is open in X.
(v) g^{*+}b-closed set [9], if $\text{bcl}^+(A)$ ⊆ U, whenever A ⊆ U and U is g^+-open in X.

Definition 2.3: A function $f: X \to Y$ is called,

(i) gb-continuous [17], if $f^{-1}(V)$ is gb-closed in X for every closed set V of Y.
(ii) g^*b-continuous [24], if $f^{-1}(V)$ is g^*b-closed in X for every closed set V of Y.
(iii) contra continuous [3], if $f^{-1}(V)$ is closed in X for each open set V of Y.
(iv) contra pre-continuous [7], if $f^{-1}(V)$ is pre-closed in X for each open set V of Y.
(v) contra semi-continuous [4], if $f^{-1}(V)$ is semi-closed in X for each open set V of Y.
(vi) contra α-continuous [6], if $f^{-1}(V)$ is α-closed in X for every open set V of Y.

(vii) contra b-continuous [18], if $f^{-1}(V)$ is b-closed in X for every open set V of Y.
(viii) contra gb-continuous [14], if $f^{-1}(V)$ is gb-closed in X for every open set V of Y.
(ix) contra g*b-continuous [22,25], if $f^{-1}(V)$ is g*b-closed in X for every open set V of Y.
(x) almost continuous (almost contra-continuous) [20], if $f^{-1}(V)$ is open (closed) in X for each regular open set V of Y.
(xi) almost contra g*b-continuous [22,25], if $f^{-1}(V)$ is g*b-closed in X for every regular open set V of Y.
(xii) regular set connected [22], if $f^{-1}(V)$ is clopen in X for every regular open set V of Y.

Definition 2.4: A function $f:(X,\tau^+)\to(Y,\sigma^+)$ is called,
(i) gb$^+$-continuous [19], if $f^{-1}(V)$ is gb$^+$-closed in X for every closed set V of Y.
(ii) g^{*+}b-continuous [10], if $f^{-1}(V)$ is g^{*+}b-closed in X for every closed set V of Y.

Definition 2.5: A space (X,τ) is called a g*b-locally indiscrete [22], if every g*b-open set in it is closed.

3. Contra g^{*+}b-Continuous and Almost Contra g^{*+}b-Continuous Functions

In this section we promote the new idea of contra g^{*+}b-continuous functions and almost contra g^{*+}b-continuous functions in simple extended topological spaces.

Definition 3.1: A function $f:(X,\tau^+)\to(Y,\sigma^+)$ is called,
(i) contra$^+$-continuous if $f^{-1}(V)$ is closed in X for each open set V of Y.
(ii) contra pre$^+$-continuous if $f^{-1}(V)$ is pre$^+$-closed in X for each open set V of Y.
(iii) contra semi$^+$-continuous if $f^{-1}(V)$ is semi$^+$-closed in X for each open set V of Y.
(iv) contra α$^+$-continuous if $f^{-1}(V)$ is α$^+$-closed in X for every open set V of Y.
(v) contra b$^+$-continuous if $f^{-1}(V)$ is b$^+$-closed in X for every open set V of Y.
(vi) contra gb$^+$-continuous if $f^{-1}(V)$ is gb$^+$-closed in X for every open set V of Y.
(vii) almost$^+$ continuous (almost contra$^+$-continuous) if $f^{-1}(V)$ is open (closed) in X for each regular$^+$ open set V of Y.
(viii) almost contra pre$^+$-continuous if $f^{-1}(V)$ is pre$^+$-closed in X for each regular$^+$ open set V of Y.
(ix) almost contra semi$^+$-continuous if $f^{-1}(V)$ is semi$^+$-closed in X for each regular$^+$ open set V of Y.
(x) almost contra α$^+$-continuous if $f^{-1}(V)$ is α$^+$-closed in X for each regular$^+$ open set V of Y.
(xi) almost contra b$^+$-continuous if $f^{-1}(V)$ is b$^+$-closed in X for each regular$^+$ open set V of Y.
(xii) almost contra gb$^+$-continuous if $f^{-1}(V)$ is gb$^+$-closed in X for each regular$^+$ open set V of Y.

Definition 3.2: A function $f:(X,\tau^+)\to(Y,\sigma^+)$ is called contra g^{*+}b-continuous if $f^{-1}(V)$ is g^{*+}b-closed in X for every open set V of Y.

Example 3.3: Let X = Y = {a, b, c} with topologies $\tau=\{X,\emptyset,\{a,b\}\},\tau^+=\{X,\emptyset,\{a\},\{a,b\}\}$ and $\sigma=\{Y,\emptyset,\{b,c\}\},\sigma^+=\{Y,\emptyset,\{b\},\{b,c\}\}$. Let $f:X\to Y$ be the identity function. Then the function *f* is contra g^{*+}b-continuous.

Theorem 3.4: Every contra$^+$-continuous function is contra g^{*+}b-continuous but not conversely.
Proof: Let $f:X\to Y$ be contra$^+$-continuous. Let V be any open set in Y. Then the inverse image $f^{-1}(V)$ is closed in X. Since every closed set is g^{*+}b-closed, $f^{-1}(V)$ is g^{*+}b-closed in X. Therefore *f* is contra g^{*+}b-continuous.

Example 3.5: Let X = Y = {a, b, c} with topologies $\tau=\{X,\emptyset,\{a,b\}\},\tau^+=\{X,\emptyset,\{a\},\{a,b\}\}$ and $\sigma=\{Y,\emptyset,\{b\}\},\sigma^+=\{Y,\emptyset,\{b\},\{a,c\}\}$. Let $f:X\to Y$ be the identity function. Then the function *f* is contra g^{*+}b-continuous but not contra$^+$-continuous.

Theorem 3.6: Every contra pre$^+$-continuous function is contra g^{*+}b-continuous but not conversely.
Proof: Let $f:X\to Y$ be contra pre$^+$-continuous. Let V be any open set in Y. Then the inverse image $f^{-1}(V)$ is pre$^+$-closed in X. Since every pre$^+$-closed set is g^{*+}b-closed, $f^{-1}(V)$ is g^{*+}b-closed in X. Therefore *f* is contra g^{*+}b-continuous.

Example 3.7: Let X = Y = {a, b, c} with topologies $\tau=\{X,\emptyset,\{a\},\{a,b\}\}$, $\tau^+=\{X,\emptyset,\{a\},\{b\},\{a,b\}\}$ and $\sigma=\{Y,\emptyset,\{a\}\},\sigma^+=\{Y,\emptyset,\{a\},\{b,c\}\}$. Let $f:X\to Y$ be the identity function. Then the function *f* is contra g^{*+}b-continuous but not contra pre$^+$-continuous.

Theorem 3.8: Every contra semi$^+$-continuous function is contra g^{*+}b-continuous but not conversely.
Proof: Let $f:X\to Y$ be contra semi$^+$-continuous. Let V be any open set in Y. By the property of contra semi$^+$-continuity we have the inverse image $f^{-1}(V)$ to be semi$^+$-closed in X. But we know that every semi$^+$-closed set is g^{*+}b-closed. Hence $f^{-1}(V)$ is g^{*+}b-closed in X. Therefore *f* is contra g^{*+}b-continuous.

Example 3.9: Let X = Y = {a, b, c} with topologies $\tau=\{X,\emptyset,\{a,b\}\},\tau^+=\{X,\emptyset,\{a\},\{a,b\}\}$ and

$\sigma = \{Y, \emptyset, \{a\}\}, \sigma^+ = \{Y, \emptyset, \{a\}, \{a, c\}\}$. Let $f: X \to Y$ be the identity function. Then the function f is contra g^{*+}b-continuous but not contra semi$^+$-continuous.

Theorem 3.10: Every contra α^+-continuous function is contra g^{*+}b-continuous but not conversely.
Proof: Let $f: X \to Y$ be contra α^+-continuous. Let V be any open set in Y. Then the inverse image $f^{-1}(V)$ is α^+-closed in X. Since every α^+-closed set is g^{*+}b-closed, $f^{-1}(V)$ is g^{*+}b-closed in X. Therefore f is contra g^{*+}b-continuous.

Example 3.11: Let X = Y = {a, b, c} with topologies $\tau = \{X, \emptyset, \{a, b\}\}, \tau^+ = \{X, \emptyset, \{a\}, \{a, b\}\}$ and $\sigma = \{Y, \emptyset, \{b, c\}\}, \sigma^+ = \{Y, \emptyset, \{b\}, \{b, c\}\}$. Define a function $f: X \to Y$ by f(a) = b, f(b) = a, f(c) = c. Then the function f is contra g^{*+}b-continuous but not contra α^+-continuous.

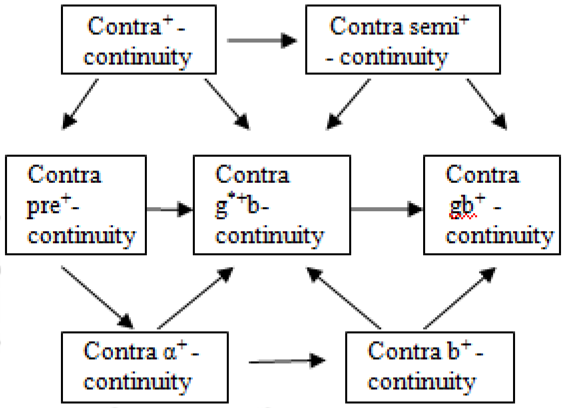
Theorem 3.12: Every contra b^+-continuous function is contra g^{*+}b-continuous but not conversely.
Proof: Let $f: X \to Y$ be contra b^+-continuous. Let V be any open set in Y. By the property of contra b^+-continuity we have the inverse image $f^{-1}(V)$ to be b^+-closed in X. But we know that every b^+-closed set is g^{*+}b-closed. Hence $f^{-1}(V)$ is g^{*+}b-closed in X. Therefore f is contra g^{*+}b-continuous.

Example 3.13: Let X = Y = {a, b, c, d} with topologies $\tau = \{X, \emptyset, \{a\}, \{a, d\}, \{c, d\}, \{a, c, d\}\}$, $\tau^+ = \{X, \emptyset, \{a\}, \{d\}, \{a, d\}, \{c, d\}, \{a, c, d\}\}$ and $\sigma = \{Y, \emptyset, \{a\}, \{b, c\}, \{a, b, c\}, \{a, b, d\}\}$, $\sigma^+ = \{Y, \emptyset, \{a\}, \{b\}, \{a, b\}, \{b, c\}, \{a, b, c\}, \{a, b, d\}\}$
Let $f: X \to Y$ be the identity function. Then the function f is contra g^{*+}b-continuous but not contra b^+-continuous.

Theorem 3.14: Every contra g^{*+}b-continuous function is contra gb^+-continuous but not conversely.
Proof: Let $f: X \to Y$ be contra g^{*+}b-continuous. Let V be any open set in Y. Then the inverse image $f^{-1}(V)$ is g^{*+}b-closed in X. Since every g^{*+}b-closed set is gb^+-closed, $f^{-1}(V)$ is gb^+-closed in X. Therefore f is contra gb^+-continuous.
Example 3.15: Let X = Y = {a, b, c} with topologies $\tau = \{X, \emptyset, \{a\}\}, \tau^+ = \{X, \emptyset, \{a\}, \{a, c\}\}$ and $\sigma = \{Y, \emptyset, \{a, b\}\}, \sigma^+ = \{Y, \emptyset, \{b\}, \{a, b\}\}$. Let $f: X \to Y$ be the identity function. Then the function f is contra gb^+-continuous but not contra g^{*+}b-continuous.

Theorem 3.16:
(i) Every contra$^+$-continuous function is contra semi$^+$-continuous function.
(ii) Every contra$^+$-continuous function is contra pre$^+$-continuous function.
(iii) Every contra semi$^+$-continuous function is contra gb^+-continuous function.
(iv) Every contra pre$^+$-continuous function is contra α^+-continuous function.
(v) Every contra α^+-continuous function is contra b^+-continuous function.
(vi) Every contra b^+-continuous function is contra gb^+-continuous function.
Proof: The proof is obvious.

Remark 3.17: As a summation of the concepts the above theorems we propose are in the following diagrammatic representation.



Remark 3.18: The following two examples will show that the concept of g^{*+}b-continuous and contra g^{*+}b-continuous are independent from each other.

Example 3.19: Let X = Y = {a, b, c} with topologies $\tau = \{X, \emptyset, \{a, b\}\}, \tau^+ = \{X, \emptyset, \{a\}, \{a, b\}\}$ and $\sigma = \{Y, \emptyset, \{b, c\}\}, \sigma^+ = \{Y, \emptyset, \{c\}, \{b, c\}\}$. Let $f: X \to Y$ be the identity function. Then the function f is contra g^{*+}b-continuous but not g^{*+}b-continuous.

Example 3.20: Let X = Y = {a, b, c} with topologies $\tau = \{X, \emptyset, \{a, b\}\}, \tau^+ = \{X, \emptyset, \{a\}, \{a, b\}\}$ and $\sigma = \{Y, \emptyset, \{a, b\}\}, \sigma^+ = \{Y, \emptyset, \{b\}, \{a, b\}\}$. Let $f: X \to Y$ be the identity function. Then the function f is g^{*+}b-continuous but not contra g^{*+}b-continuous.

Theorem 3.21: If $f: X \to Y$ is contra g^{*+}b-continuous map and $g: Y \to Z$ is continuous map, then their composition $g \circ f: X \to Z$ is contra g^{*+}b-continuous.
Proof: Let U be any open set in Z. Since $g: Y \to Z$ is continuous, $g^{-1}(U)$ is open in Y. Since $f: X \to Y$ is contra g^{*+}b-continuous, $f^{-1}(g^{-1}(U))$ is g^{*+}b-closed in X. Hence $(g \circ f)^{-1}(U)$ is g^{*+}b-closed in X. Thus $g \circ f$ is contra g^{*+}b-continuous.

Theorem 3.22: If $f: X \to Y$ is g^{*+}b-irresolute map and $g: Y \to Z$ is g^{*+}b-continuous map, then their composition $g \circ f: X \to Z$ is contra g^{*+}b-continuous.
Proof: Let U be any open set in Z. Then $g^{-1}(U)$ is g^{*+}b-closed in Y, because $g: Y \to Z$ is contra g^{*+}b-continuous. Since $f: X \to Y$ is g^{*+}b-irresolute,

$f^{-1}(g^{-1}(U)) = (g \circ f)^{-1}(U)$ is g^{*+}b-closed in X. Thus $g \circ f$ is contra g^{*+}b-continuous.

Remark 3.23: The composition of two contra g^{*+}b-continuous maps need not be a contra g^{*+}b-continuous map as seen from the following example.

Example 3.24: Let X = Y = Z = {a, b, c} with topologies $\tau = \{X, \emptyset, \{a, b\}\}$, $\tau^+ = \{X, \emptyset, \{a\}, \{a, b\}\}$; $\sigma = \{Y, \emptyset, \{b\}\}$, $\sigma^+ = \{Y, \emptyset, \{b\}, \{b, c\}\}$ and $\eta = \{Z, \emptyset, \{a, c\}\}$, $\eta^+ = \{Z, \emptyset, \{a\}, \{a, c\}\}$. Let $f: (X, \tau^+) \to (Y, \sigma^+)$ and $g: (Y, \sigma^+) \to (Z, \eta^+)$ be the identity map. Both f and g are contra g^{*+}b-continuous but their composition $g \circ f: (X, \tau^+) \to (Z, \eta^+)$ is not a contra g^{*+}b-continuous. Since for the open set {a,c} in Z, $(g \circ f)^{-1}(\{a, c\})$ = {a,b} is not g^{*+}b-closed in X and hence $g \circ f$ is not contra g^{*+}b-continuous.

Definition 3.25: A space (X, τ^+) is called a g^{*+}b-locally indiscrete if every g^{*+}b-open set in it is closed.

Example 3.26: Let X = {a, b} with the topology $\tau = \{X, \emptyset, \{a\}\}$, $\tau^+ = \{X, \emptyset, \{a\}, \{b\}\}$. Then (X, τ^+) g^{*+}b-locally indiscrete space.

Theorem 3.27: If a function $f: X \to Y$ is g^{*+}b-continuous and (X, τ^+) is g^{*+}b-locally indiscrete, then f is contra$^+$-continuous.

Proof: Let V be any open set in Y. Then the inverse image $f^{-1}(V)$ is g^{*+}b-open in X as f is g^{*+}b-continuous. Since (X, τ^+) is g^{*+}b-locally indiscrete $f^{-1}(V)$ is closed in X. Hence f is contra$^+$-continuous.

Theorem 3.28: If a function $f: X \to Y$ is contra g^{*+}b-continuous and X is the space where "every g^{*+}b-closed set is closed", then f is contra$^+$-continuous.

Proof: Let V be any open set in Y. Then the inverse image $f^{-1}(V)$ is g^{*+}b-closed in X as f is contra g^{*+}b-continuous. By hypothesis, $f^{-1}(V)$ is closed in X. Hence f is contra$^+$-continuous.

Theorem 3.29: If a function $f: X \to Y$ is g^{*+}b-irresolute map with Y as g^{*+}b-locally indiscrete space and $g: Y \to Z$ is contra g^{*+}b-continuous map, then their composition $g \circ f: X \to Z$ is contra g^{*+}b-continuous.

Proof: Let U be any closed set in Z. Since $g: Y \to Z$ is contra g^{*+}b-continuous, $g^{-1}(U)$ is g^{*+}b- open in Y. Since Y is g^{*+}b-locally indiscrete, $g^{-1}(U)$ is closed in Y. Hence $g^{-1}(U)$ is g^{*+}b-closed set in Y. Since $f: X \to Y$ is g^{*+}b-irresolute, $f^{-1}(g^{-1}(U)) = (g \circ f)^{-1}(U)$ is g^{*+}b-closed in X. Thus $g \circ f$ is contra g^{*+}b-continuous.

Definition 3.30: A function $f: (X, \tau^+) \to (Y, \sigma^+)$ is called almost contra g^{*+}b-continuous if $f^{-1}(V)$ is g^{*+}b-closed in X for every regular$^+$ open set V of Y.

Example 3.31: Let X = Y = {a, b, c} with topologies $\tau = \{X, \emptyset, \{b\}\}, \tau^+ = \{X, \emptyset, \{b\}, \{b, c\}\}$ and $\sigma = \{Y, \emptyset, \{a\}, \{a, b\}\}, \sigma^+ = \{Y, \emptyset, \{a\}, \{b\}, \{a, b\}\}$. Let $f: X \to Y$ be the identity function. Then the function f is almost contra g^{*+}b-continuous.

Theorem 3.32: Every contra g^{*+}b-continuous function is almost contra g^{*+}b-continuous but not conversely.

Proof: Since every regular$^+$ open set is open set, the proof follows.

Example 3.33: Let X = Y = {a, b, c} with topologies $\tau = \{X, \emptyset, \{a, b\}\}, \tau^+ = \{X, \emptyset, \{a\}, \{a, b\}\}$ and $\sigma = \{Y, \emptyset, \{a\}, \{a, b\}\}, \sigma^+ = \{Y, \emptyset, \{a\}, \{b\}, \{a, b\}\}$. Let $f: X \to Y$ be the identity function. Then the function f is almost contra g^{*+}b-continuous but not contra g^{*+}b-continuous.

Theorem 3.34:
(i) Every almost contra$^+$-continuous function is almost contra g^{*+}b-continuous function.
(ii) Every almost contra pre$^+$-continuous function is almost contra g^{*+}b-continuous function.
(iii) Every almost contra semi$^+$-continuous function is almost contra g^{*+}b-continuous function.
(iv) Every almost contra α^+-continuous function is almost contra g^{*+}b-continuous function.
(v) Every almost contra b$^+$-continuous function is almost contra g^{*+}b-continuous function.
(vi) Every almost contra g^{*+}b-continuous function is almost contra gb$^+$-continuous function.

Proof: The proof is obvious.

Remark 3.35: Converse of the above statements is not true as shown in the following example.

Example 3.36:

(i) Let X = Y = {a, b, c} with topologies $\tau = \{X, \emptyset, \{a, b\}\}, \tau^+ = \{X, \emptyset, \{a\}, \{a, b\}\}$ and $\sigma = \{Y, \emptyset, \{a\}, \{a, c\}\}$, $\sigma^+ = \{Y, \emptyset, \{a\}, \{c\}, \{a, c\}\}$. Let $f: X \to Y$ be the identity function. Then the function f is almost contra g^{*+}b-continuous but not almost contra$^+$-continuous.

(ii) Let X = Y = {a, b, c} with topologies $\tau = \{X, \emptyset, \{a\}, \{a, b\}\}$, $\tau^+ = \{X, \emptyset, \{a\}, \{b\}, \{a, b\}\}$ and $\sigma = \{Y, \emptyset, \{b\}, \{b, c\}\}$, $\sigma^+ = \{Y, \emptyset, \{b\}, \{c\}, \{b, c\}\}$. Define a function $f: X \to Y$ by $f(a) = b, f(b) = a, f(c) = c$. Then the function f is almost contra g^{*+}b-continuous but not almost contra pre$^+$-continuous.

(iii) Let X = Y = {a, b, c} with topologies $\tau = \{X, \emptyset, \{a,b\}\}, \tau^+ = \{X, \emptyset, \{a\}, \{a,b\}\}$ and $\sigma = \{Y, \emptyset, \{a\}, \{a,c\}\}$, $\sigma^+ = \{Y, \emptyset, \{a\}, \{c\}, \{a,c\}\}$. Let $f: X \to Y$ be the identity function. Then the function f is almost contra g^{*+}b-continuous but not almost contra semi$^+$-continuous.

(iv) Let X = Y = {a, b, c} with topologies $\tau = \{X, \emptyset, \{a,b\}\}, \tau^+ = \{X, \emptyset, \{a\}, \{a,b\}\}$ and $\sigma = \{Y, \emptyset, \{a\}, \{a,c\}\}$, $\sigma^+ = \{Y, \emptyset, \{a\}, \{c\}, \{a,c\}\}$. Let $f: X \to Y$ be the identity function. Then the function f is almost contra g^{*+}b-continuous but not almost contra α^+-continuous.

(v) Let X = Y = {a, b, c, d} with topologies $\tau = \{X, \emptyset, \{a\}, \{a,d\}, \{c,d\}, \{a,c,d\}\}$, $\tau^+ = \{X, \emptyset, \{a\}, \{d\}, \{a,d\}, \{c,d\}, \{a,c,d\}\}$ and $\sigma = \{Y, \emptyset, \{c\}, \{a,c\}, \{a,b,d\}\}$, $\sigma^+ = \{Y, \emptyset, \{c\}, \{d\}, \{a,c\}, \{c,d\}, \{a,c,d\}, \{a,b,d\}\}$. Let $f: X \to Y$ be the identity function. Then the function f is almost contra g^{*+}b-continuous but not almost contra b^+-continuous.

(vi) Let X = Y = {a, b, c} with topologies $\tau = \{X, \emptyset, \{a\}\}, \tau^+ = \{X, \emptyset, \{a\}, \{a,c\}\}$ and $\sigma = \{Y, \emptyset, \{a,b\}\}$, $\sigma^+ = \{Y, \emptyset, \{c\}, \{a,b\}\}$. Let $f: X \to Y$ be the identity function. Then the function f is almost contra gb^+-continuous but not almost contra g^{*+}b-continuous.

Theorem 3.37: If a map $f: X \to Y$ from a topological space X into a topological space Y, then the following statements are equivalent:
(a) f is almost contra g^{*+}b-continuous.
(b) for every regular$^+$ closed set F of Y $f^{-1}(F)$ is g^{*+}b- open in X.

Proof:

(a) ⇒ (b) : Let F be a regular$^+$ closed set in Y, then Y – F is a regular$^+$ open set in Y. By (a), $f^{-1}(Y - F) = X - f^{-1}(F)$ is g^{*+}b-closed set in X. This implies $f^{-1}(F)$ is g^{*+}b-open set in X. Therefore (b) holds.

(b) ⇒ (a) : Let G be a regular$^+$ open set of Y. Then Y – G is a regular$^+$ closed set in Y. By (b), $f^{-1}(Y - G)$ is g^{*+}b-open set in X. This implies $X - f^{-1}(G)$ is g^{*+}b-open set in X, which implies $f^{-1}(G)$ is g^{*+}b-closed set in X. Therefore (a) holds.

Definition 3.38: A function $f: (X, \tau^+) \to (Y, \sigma^+)$ is said to be regular$^+$ set connected if $f^{-1}(V)$ is clopen in X for every regular$^+$ open set V of Y.

Theorem 3.39: If a function $f: X \to Y$ is almost contra g^{*+}b-continuous and almost$^+$ continuous together with X is the space where "every g^{*+}b-closed set is closed", then f is regular$^+$ set connected.

Proof: Let V be a regular$^+$ open set in Y. Since f is almost contra g^{*+}b-continuous and almost$^+$ continuous, $f^{-1}(V)$ is g^{*+}b-closed and open in X. By hypothesis, $f^{-1}(V)$ is clopen in X. Therefore f is regular$^+$ set connected.

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