Axially Symmetric Dark Energy Cosmological Model in Scale Covariant Theory of Gravitation

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Abstract: ***In this paper, we have investigated axially symmetric dark energy cosmological model with variable equation of state (EoS) in scale covariant theory of gravitation proposed by Canuto et al. (Phys. Rev. Lett. 39: 429, 1977). The solution obtained, with the help of special law of variation for Hubble's parameter formulated by Bermann (Nuovo Cimento 74 B: 182, 1983), represents a dark energy cosmological model. Some physical and kinematical properties are also discussed.***

Keywords: Axially symmetric, Dark energy, Scale covariant theory

1. Introduction

In recent years there has been an immense interest in constructing the cosmological models which are of vital important for better understanding of large scale structure of the universe in several alternative theories of gravitation. The most important among them are scalar-tensor theories of gravitation formulated by Brans-Dicke [1], Nordt-Vedt [2], Sen and Dunn [3] and Saez-Ballester [4]. All versions of the scalar tensor theories are based on the introduction of a scalar field ϕ into the formulation of general relativity. This scalar field together with the metric tensor field then forms a scalar-tensor field representing the gravitational field.

Canuto et al. [5] formulated a scale-covariant theory of gravitation which also admits a variable G as well as it is a viable alternative to general relativity [6, 7]. In this theory, Einstein's field equations are valid in gravitational units where as physical quantities are measured in atomic units. The components of metric tensor in the two systems of units are related by a conformal transformation

$$\bar{g}_{ij} = \phi^2(x^k) g_{ij} \tag{1}$$

wherein Latin indices takes values 1, 2, 3 and 4. The barred quantities denote gravitational units and unbarred quantities denote atomic units. The gauge function $\phi(0<\phi<\infty)$ in its most general formulation is a function of all space time coordinates. Thus, using the conformal transformation of the type given by (1), Canuto et al. [5] transformed the usual Einstein equations into

$$R_{ij} - \frac{1}{2} R g_{ij} + f_{ij}(\phi) = -8\pi G(\phi) T_{ij} + \Lambda(\phi) g_{ij} \tag{2}$$

where $\phi^2 f_{ij} = 2\phi\phi_{i;j} - 4\phi_i\phi_j - g_{ij}(\phi\phi^k_{;k} - \phi^k\phi_k)$. (3)

Here R_{ij} is the Ricci tensor, R is the Ricci scalar, Λ is the cosmological 'constant', G is the gravitational 'constant' and T_{ij} is the energy momentum tensor. A semi colon denotes covariant derivative and ϕ_i denotes ordinary derivative with respect to x^i. A particular feature of this theory is that no independent equation for ϕ exists. The possibilities that have been considered for gauge functions ϕ (Canuto et al. [5]) are

$$\phi(t) = \left(\frac{t_0}{t}\right)^{\varepsilon} \quad \varepsilon = \pm 1, \ \pm\frac{1}{2} \tag{4}$$

where t_0 is a constant.

Reddy and Venkateshwarlu [8], Reddy et al. [9], Beesham [10-12], Singh and Devi [13] Adhav et al. [14] Ram et al. [15] and Venkateshwarlu [16] are some of the authors who have investigated several aspects of the scale covariant theory of gravitation.

Moreover, the discovery of the accelerated expansion of the universe supposedly driven by exotic dark energy (Perlumutter et al. [17], Reiss et al. [18], Spergel et al. [19], Copeland et al. [20]) has lead, in recent years, to the investigations of dark energy models both in general relativity and in alternative theories of gravitation. The nature and composition of dark energy is still unknown and therefore many radically different models have been proposed (Caldwell [21], Frieman et al. [22], Liddle and Scherrer [23] and Peebles and Ratra [24]). Dark energy is usually characterized by the (EoS) parameter $\omega(t) = \frac{p}{\rho}$ where p is the fluid pressure and ρ is energy density (Carroll and Hoffman [25]). Recently, dark energy models with variable EoS parameter have been investigated by Pradhan et al. [26], Akarsu and Kilinc [27], Yadav and Yadav [28], Yadav et al. [29], Ray et al. [30], Pradhan and Amirhashchi [31] and Tade et al. [32]. Reddy et al. [33] have discussed dark energy model in anisotropic Bianchi type-I space time with variable EoS parameter in scale covariant theory of gravitation. Recently Singh and Sharma [34] have studied anisotropic dark energy model with EoS parameter in Bianchi type-II space time in scale covariant theory of gravitation. With these motivations, in this paper we have investigated axially symmetric dark energy cosmological model with variable equation of state EoS parameter and constant deceleration parameter in scale covariant theory of gravitation.

2. Metric and Field Equations

We consider the axially symmetric metric (Bhattacharya and Karade [35]) in the form

$$ds^2 = dt^2 - A^2[d\chi^2 + f^2(\chi)d\Phi^2] - B^2 dz^2, \tag{5}$$

where A, B are functions of t only and f is a function of the co-ordinate χ only.

The simplest generalization of equation of state (EoS) parameter of perfect fluid may be used to determine the EoS parameter separately on each spatial axis by preserving the diagonal form of the energy momentum tensor in a way consistent with the considered metric. Therefore, the energy momentum tensor of the fluid is taken as

$$T_j^i = diag\,[T_0^0, T_1^1, T_2^2, T_3^3] \tag{6}$$

One can parameterize this as follows:

$$\begin{aligned} T_j^i &= diag[\rho, -p_\chi, -p_\Phi, -p_z] \\ &= diag\,[1, -\omega_\chi, -\omega_\Phi, -\omega_z]\rho \\ &= diag\,[1, -\omega_\chi, -(\omega+\delta)_\Phi, -(\omega+\delta)_z]\rho, \end{aligned} \tag{7}$$

where ρ is the energy density of the fluid, p_χ, p_Φ, p_z are the pressures and $\omega_\chi, \omega_\Phi, \omega_z$ are the directional EoS parameters along the χ, Φ, z axes respectively. Here ω is the deviation free EoS parameter of the fluid. We have parameterized the deviation from isotropy by setting $\omega_\chi = \omega$ and then introducing skewness parameters δ which is the deviation from ω along Φ axis and z axis respectively. Since in axially symmetric space time $T_1^1 = T_2^2$, so we obtain

$$\delta = 0. \tag{8}$$

In a commoving co-ordinate system, the field equations (2) and (3) for metric (5) with the help of (7) and (8) lead to the following system of equations

$$\frac{\ddot{A}}{A} + \frac{\ddot{B}}{B} + \frac{\dot{A}\dot{B}}{AB} + \frac{\dot{B}\dot{\phi}}{B\phi} + \frac{\ddot{\phi}}{\phi} - \left(\frac{\dot{\phi}}{\phi}\right)^2 = -8\pi G\omega\rho \tag{9}$$

$$\frac{2\ddot{A}}{A} + \left(\frac{\dot{A}}{A}\right)^2 - \frac{1}{A^2}\left(\frac{f''}{f}\right) + \frac{2\dot{A}\dot{\phi}}{A\phi} - \frac{\dot{B}\dot{\phi}}{B\phi} + \frac{\ddot{\phi}}{\phi} - \left(\frac{\dot{\phi}}{\phi}\right)^2 = -8\pi G\omega\rho \tag{10}$$

$$\left(\frac{\dot{A}}{A}\right)^2 + 2\frac{\dot{A}\dot{B}}{AB} - \frac{1}{A^2}\left(\frac{f''}{f}\right) + 2\frac{\dot{A}\dot{\phi}}{A\phi} + \frac{\dot{B}\dot{\phi}}{B\phi} - \frac{\ddot{\phi}}{\phi} + 3\left(\frac{\dot{\phi}}{\phi}\right)^2 = 8\pi G\rho. \tag{11}$$

Here the over head dot and dash denote differentiation with respect to t and χ respectively. The functional dependence of the metric together with (10) and (11) imply

$$\frac{f''}{f} = k^2,\ k^2 \text{ is constant.} \tag{12}$$

If $k = 0$, then $f(\chi) = \text{(constant)},\ 0 < \chi < \infty$. This constant can be made equal to 1. Thus we shall have $f(\chi) = \chi$ resulting in flat model of the universe (Hawking and Ellise [36]). Now the field equations (9)-(11) reduces to

$$\frac{\ddot{A}}{A} + \frac{\ddot{B}}{B} + \frac{\dot{A}\dot{B}}{AB} + \frac{\dot{B}\dot{\phi}}{B\phi} + \frac{\ddot{\phi}}{\phi} - \left(\frac{\dot{\phi}}{\phi}\right)^2 = -8\pi G\omega\rho \tag{13}$$

$$\frac{2\ddot{A}}{A} + \left(\frac{\dot{A}}{A}\right)^2 + \frac{2\dot{A}\dot{\phi}}{A\phi} - \frac{\dot{B}\dot{\phi}}{B\phi} + \frac{\ddot{\phi}}{\phi} - \left(\frac{\dot{\phi}}{\phi}\right)^2 = -8\pi G\omega\rho \tag{14}$$

$$\left(\frac{\dot{A}}{A}\right)^2 + 2\frac{\dot{A}\dot{B}}{AB} + 2\frac{\dot{A}\dot{\phi}}{A\phi} + \frac{\dot{B}\dot{\phi}}{B\phi} - \frac{\ddot{\phi}}{\phi} + 3\left(\frac{\dot{\phi}}{\phi}\right)^2 = 8\pi G\rho \tag{15}$$

3. Solution of the Field Equations

Equations (13)-(15) are three independent equations in five unknowns A, B, ϕ, ω and ρ. We solve the above set of highly non-linear equations with the help of special law of variation of Hubble's parameter proposed by Berman [37] which yields constant deceleration parameter of the models of the universe. We consider the constant deceleration parameter model defined by

$$q = \frac{-R\ddot{R}}{\dot{R}^2} = \text{Constant}, \tag{16}$$

where $R = (A^2B)^{\frac{1}{3}}$ is the average scale factor. Here the constant is taken as negative so that it is an accelerating model of the universe. Therefore, the average scale factor R obtained from the equation (16) is

$$R = (at+b)^{\frac{1}{1+q}}, \tag{17}$$

where $a \neq 0$ and b are constants of integration. This equation implies that the condition of expansion is $1+q > 0$.

Also, the set of equations being highly non-linear, we assume a relation between metric coefficients given by

$$A = B^n, \quad \text{where } n \text{ is constant.} \tag{18}$$

The field equation (13)-(15) admit solutions given by

$$B = (at+b)^{\frac{3}{(2n+1)(1+q)}} \tag{19}$$

$$\text{and } A = (at+b)^{\frac{3n}{(2n+1)(1+q)}}. \tag{20}$$

Hence, the metric (5) through a proper choice of integration constant (i.e. $a = 1, b = 0$) can be written as

$$ds^2 = dt^2 - t^{\frac{6n}{(2n+1)(1+q)}}\left(d\chi^2 + f^2(\chi)d\phi^2\right) - t^{\frac{6}{(2n+1)(1+q)}}dz^2. \tag{21}$$

Equation (21) represents axially symmetric radiating cosmological model in presence of dark energy with negative constant declaration parameter in scale covariant theory.

4. Physical Properties

The metric (21) represents an axially symmetric dark energy cosmological model with the following physical and kinematical parameters in scale covariant theory of gravitation.

The energy density

$$\rho = \frac{1}{8\pi G}\left[\frac{9n(n+2)}{(2n+1)^2(1+q)^2 t^2} - \frac{3\varepsilon}{(1+q)t^2} + \frac{\varepsilon(2\varepsilon-1)}{t^2}\right] \tag{22}$$

EoS parameter

$$\omega = \frac{1}{8\pi G\rho}\left[\begin{array}{l}\dfrac{6n}{(2n+1)(1+q)t^2} - \dfrac{27n^2}{(2n+1)^2(1+q)^2 t^2} \\ + \dfrac{3(2n-1)\varepsilon}{(2n+1)(1+q)t^2} - \dfrac{\varepsilon}{t^2}\end{array}\right] \tag{23}$$

Skewness parameter

$$\delta = 0 \tag{24}$$

Special volume

$$V = t^{\frac{3}{1+q}} \tag{25}$$

Scalar expansion

$$\theta = \frac{3}{(1+q)t} \tag{26}$$

Shear scalar

$$\sigma^2 = \frac{3}{2}\frac{1}{(1+q)^2 t^2} \tag{27}$$

Hubble parameter

$$H = \frac{1}{(1+q)t} \tag{28}$$

It is observed that from the model (21) has no initial singularity, i.e. at $t=0$. Physical quantities ρ, ω diverge at $t=0$ while they vanish for large values of t. The spatial volume increases as t increases (since $1+q>0$) which shows that the universe is expanding. The scalar of expansion θ, shear scalar σ^2 and the Hubble's parameter H diverge at $t=0$ and vanish for large t. Since $\frac{\sigma^2}{\theta^2} = \frac{1}{6}$, the model does not approach isotropy for large values of t.

However, since $1+q>0$, the universe is expanding with the increase of cosmic time but the rate of expansion and shear scalar decrease to zero and tend to isotropic. Thus this case implies an accelerating model of universe. Hence it follows that our model represents physical dark energy model.

5. Conclusions

Dark energy cosmological models are, recently, playing a vital role in the discussion of accelerated expansion of the universe in general relativity. With the advent of alternative theories of gravitation study of these models is gaining importance. Here we have investigated axially symmetric, dark energy model with variable EoS parameter in a scale-covariant theory of gravitation formulated by Canuto et al. [5]. It is observed that the model has no initial singularity and all the physical parameters are infinite at the initial epoch, $t=0$ and tend to zero for large t. It is also observed that the model does not approach isotropy through the whole evolution of the universe. This model, definitely, throws some light on the understanding of dark energy model in scale-covariant theory of gravitation.

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