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3-Equitable Labeling in Context of the Barycentric Subdivision of Cycle Related Graph

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Abstract: ***A function from the vertex set of a graph Gto the set {0, 1, 2} is called 3-equitable labeling if the induced edge labels are produced by the absolute difference of labels of end vertices such that the absolute difference of number of vertices of G labeled with 0,1and 2differbyat most 1and similarly the absolute difference of number of edges of G labeled with 0,1and 2differbyat most 1[1]. In this paper, wediscuss3-equitable labeling in the context of barycentric subdivision of different cycle related graphs.***

Keywords: 3-equitable labeling, barycentric subdivision

AMS Subject classification number (2010): 05C78

1. Introduction

We consider simple, finite, undirected graph. If the vertices of the graph are assigned values subject to certain conditions, then it is known as a graph labeling [5]. A survey on graph labeling is given by Gallian[3].

We follow Gross and Yellen [5] for the graph theoretical terminology and notations. The vertex set and the edge set of graph G are denoted by $V(G)$ and $E(G)$ respectively. A mapping f from $V(G)$ to $\{0,1,2\}$ is called ternary vertex labeling of G[1]. Aternary vertex labeling of a graph G iscalled*3-equitable labeling* if the induced edge labeling function f^* from $E(G)$ to the set $\{0,1,2\}$ is defined as $f^*(e=uv)=|f(u)-f(v)|$ such that the absolute difference of number of vertices of G with label0,1and 2differbyat most 1and similarly absolute difference of number of edges of G with label 0,1and 2 differ by at most 1. A graph which admits 3-equitable labeling is called a *3-equitable graph* [1].

$C_n \odot K_1$	Crown obtained from the corona of C_n with K_1.
$C_n \odot 2K_1$	Double crown obtained from the corona of C_n with $2K_1$.
AC_n	Armed crown obtained from $C_m \oplus P_n$.
C_n'	Obtained by duplicating a vertex by an edge in a cycle C_n ,where $\{v_1', v_2', ..., v_{2n}'\}$ are vertices added to obtain C_n' corresponding to the vertices $v_1, v_2, ..., v_n$ in C_n.

Notation 1.1: Standard graphs used in this paper [2]:
Let $e = uv$ bean edge of a graph G and w is not a vertex of G. Then edge is said to be subdivided when it is replaced by edges $e' = uw$ and $e'' = vw$[5]. If every edge of graph G is subdivided, then the resulting graph is called barycentric subdivision of the graph G. It is denoted by $S(G)$.Vaidya et al.[6] proved that cycle with twin chords is cordial as well as 3-equitable. In [7] Vaidyaetal. proved that the barycentric subdivision of cycle with one chord, cycle with twin chords and cycle with triangle are cordial. G. V. Ghodasra and I.I. Jadav[4] proved that barycentric subdivision of cycle with one chord, cycle with twin chords, cycle with triangle, shell graph and wheel graph are3-equitable. In this paper, we prove that the barycentric subdivision of crown, double crown, armed crown and some standard graph are3-equitable graphs.

2. Main Results

Theorem 1: The barycentric subdivision of crown $C_n \odot K_1$ *is 3-equitable.*

Proof: Let $S(C_n \odot K_1)$ denote the barycentric subdivision of crown $C_n \odot K_1$. Let $u_1, u_2, ..., u_n$ be pendent vertices of $C_n \odot K_1$ and $v_1, v_2, ..., v_{2n}$ are the vertices corresponding to C_n, where v_j is the vertex added due to the barycentric subdivision of edge $v_{j-1}v_{j+1}$, $j = 2,4,6,...,2n-2$ and v_{2n} is the vertex added due to the barycentric subdivision of edge $v_{2n-1}v_1$. Here, w_i is the vertex added due to the barycentric subdivision of edge u_iv_{2i-1}, $(i = 1,2,3,..., n)$. Note that $|V(S(C_n \odot K_1))| = 4n$ and $|E(S(C_n \odot K_1))| = 4n$. We define labeling function $f: V(S(C_n \odot K_1)) \to \{0,1,2\}$ as follows

$$f(v_i) = \begin{cases} 0; i \equiv 0,3 (\text{mod } 6) \\ 1; i \equiv 4,5 (\text{mod } 6) \\ 2; i \equiv 1,2 (\text{mod } 6), 1 \leq i \leq 2n. \end{cases}$$

$$f(u_i) = \begin{cases} 1; i \equiv 1,4 (\text{mod } 6) \\ 2; i \equiv 0,2,3,5 (\text{mod } 6), 1 \leq i \leq n. \end{cases}$$

$$f(w_i) = \begin{cases} 0; i \equiv 1,2,4,5 (\text{mod } 6) \\ 2; i \equiv 0,3 (\text{mod } 6), 1 \leq i \leq n. \end{cases}$$

Hence $S(C_n \odot K_1)$ is 3-equitable, for all $n \in \mathbb{N}, n \geq 3$.

Above defined labeling pattern satisfies the conditions of3-equitable labeling as shown in Table1.

Let $n = 6a + b$, where $b \in \mathbb{N}$ and $a \in \mathbb{N} \cup \{0\}$.

Table 1: Table for Theorem 1

b	Vertex Conditions	Edge Conditions
0,3	$v_f(0) = v_f(1) = v_f(2)$	$e_f(0) = e_f(1) = e_f(2)$
1,4	$v_f(0) + 1 = v_f(1) + 1 = v_f(2)$	$e_f(0) = e_f(1) + 1 = e_f(2) + 1$
2,5	$v_f(0) = v_f(1) + 1 = v_f(2)$	$e_f(0) + 1 = e_f(1) = e_f(2)$

Example 1.*3-equitable labeling of the graph obtained by barycentric subdivision of crown* $C_5 \odot K_1$ *is shown in Figure 1. It is the case related to* $n \equiv 5(mod 6)$.

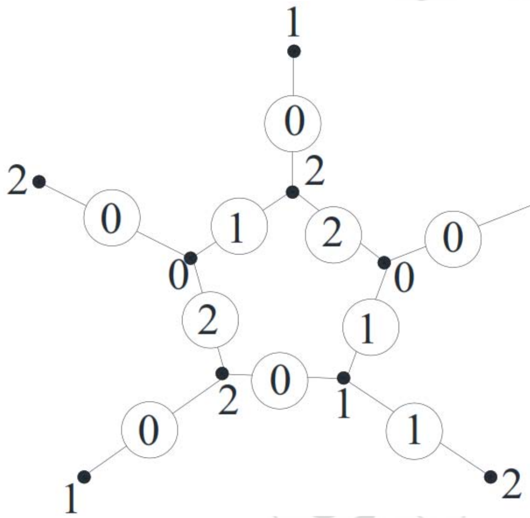

Figure 1

$f(u_i) = 1; if\ 1 \leq i \leq 2n.$

$f(v_i) = 0; if\ 1 \leq i \leq 2n.$

$f(w_i) = 2; if\ 1 \leq i \leq 2n.$

From the above labeling pattern, we get $v_f(0) = v_f(1) = v_f(2)$ and $e_f(0) = e_f(1) = e_f(2)$.

Hence $S(C_n \odot K_1)$ is 3-equitable graph.

Example 2.*3-equitable labeling of the graph obtained by barycentric subdivision of double crown* $C_4 \odot 2K_1$ *is shown in Figure 2. It is the case related to* $n \equiv 4(mod 6)$.

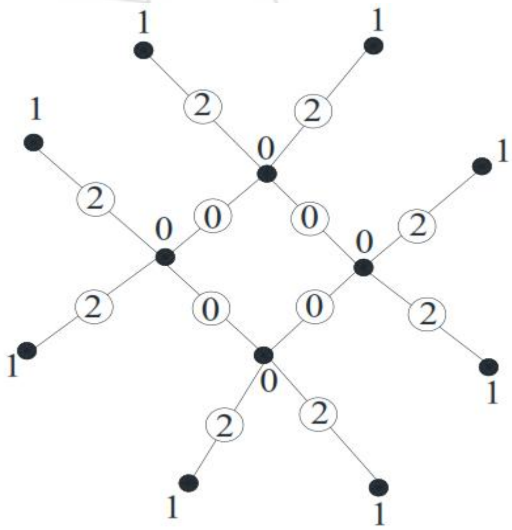

Figure 2

Theorem 2. *The barycentric subdivision of double crown* $C_n \odot 2K_1$ *is 3-equitable.*

Proof. Let $S(C_n \odot 2K_1)$ denotes the barycentric subdivision of double crown $C_n \odot 2K_1$. Let $u_1, u_2, ..., u_{2n}$ be the pendant vertices of $C_n \odot 2K_1$ and $v_1, v_2, ..., v_{2n}$ are vertices corresponding to C_n, where v_j is the vertex added due to barycentric subdivision of edge $v_{j-1}v_{j+1}$ $j = 2,4,6,...,2n-2$ and v_{2n} is the vertex added due to the barycentric subdivision of edge $v_{2n-1}v_1$. Here, w_i is the vertex added due to the barycentric subdivision of edge u_iv_i for odd i and u_iv_{i-1} for even $i\ (i = 2,4,6,...,2n)$. Note that $|V(S(C_n \odot 2K_1))| = 6n$ and $|E(S(C_n \odot 2K_1))| = 6n$. We define labeling function $f: V(S(C_n \odot 2K_1)) \to \{0,1,2\}$ as follows.

Theorem 3: The barycentric subdivision of armed crown AC_n *is 3-equitable.*

Proof: Let $S(AC_n)$ denote the barycentric subdivision of armed crown AC_n. Let u_i denote the pendant vertices of AC_n and $v_1, v_2, ..., v_{2n}$ are the vertices corresponding to C_n, where v_j is the vertex added due to the barycentric subdivision of edge $v_{j-1}v_{j+1}$, $j = 2,4,6,...,2n-2$ and v_{2n} is the vertex added due to the barycentric subdivision of edge $v_{2n-1}v_1$. Here w_i is the vertices of degree wo and w_i' is the vertex added due to barycentric subdivision of edge

$w_i v_{2i-1}$, u_i' is the vertex added due to barycentric subdivision of edge $w_i u_i, 1 \leq i \leq n$.

Note that $|V(S(AC_n))| = 6n$ and $|E(S(AC_n))| = 6n$.

We define labeling function $f: V(S(AC_n)) = 6n \rightarrow \{0,1,2\}$ as follows.

Case 1: n is even

$$f(v_i) = \begin{cases} 2; i \equiv 0,1 (\text{mod } 4) \\ 0; i \equiv 2,3, (\text{mod } 4), 1 \leq i \leq 2n. \end{cases}$$

$f(w_i) = 1; if\ 1 \leq i \leq n.$

$f(w_i') = 1; if\ 1 \leq i \leq n.$

$f(u_i) = 0; if\ 1 \leq i \leq n.$

$f(u_i') = 2; if\ 1 \leq i \leq n.$

Case 2: n is odd.

$$f(v_i) = \begin{cases} 2; i \equiv 0,1 (\text{mod } 4) \\ 0; i \equiv 2,3, (\text{mod } 4), 1 \leq i \leq 2n. \end{cases}$$

$f(w_n) = 1, f(w_n') = 2, f(u_n) = 0, f(u_n') = 1$

$f(w_i) = 1; if\ 1 \leq i \leq n-1.$

$f(w_i') = 1; if\ 1 \leq i \leq n-1.$

$f(u_i) = 0; if\ 1 \leq i \leq n-1.$

$f(u_i') = 2; if\ 1 \leq i \leq n-1.$

From the above labeling pattern, we get $v_f(0) = v_f(1) = v_f(2)$ and $e_f(0) = e_f(1) = e_f(2)$. Hence $S(AC_n)$ is 3-equitable graph.

Example 3. *3-equitable labeling of the graph obtained by barycentric subdivision of armed crown AC_3 is shown in Figure 3. It is the case related to $n \equiv 3 (mod 6)$.*

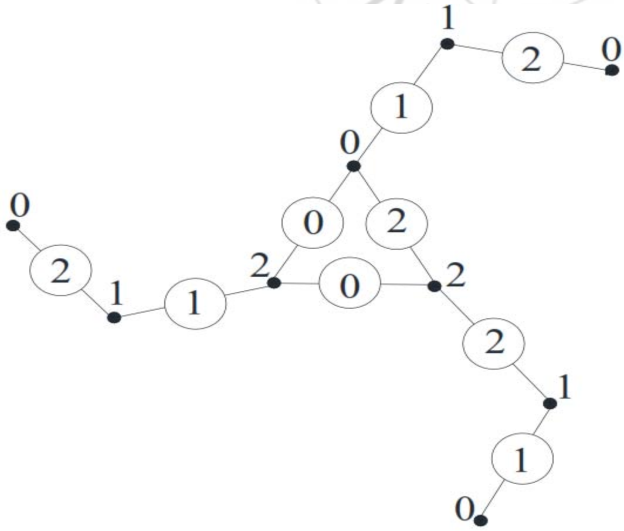

Figure 3

Theorem4. *The barycentric subdivision of C_n' is 3-equitable.*

Proof. Let denotes the barycentric subdivision of C_n'. Let $u_1, u_2, ..., u_{2n}$ be the outer vertices of C_n' and $v_1, v_2, ..., v_{2n}$ are the vertices corresponding to C_n, where v_j is the vertex added due to the barycentric subdivision of edge $v_{j-1}v_{j+1}$, $j = 2,4,6,...,2n-2$ and v_{2n} is the vertex added due to the barycentric subdivision of edge $v_{2n-1}v_1$ and w_i is the vertex added due to barycentric subdivision of edge $u_{2n-1}u_{2i}, i = 1,2,3,\dots,n$. Here u_i' is the vertex added due to barycentric subdivision of edge $u_i v_i$ for odd i and edge $u_i v_{i-1}$ for even i, $(i = 1,2,3,\dots,2n)$.

Note that $|V(S(C_n'))| = 8n$ and $|E(S(C_n'))| = 8n$

We define labeling function $f: V(S(C_n')) = 6n \rightarrow \{0,1,2\}$ as follows

Case 1: $n \equiv 0,3\ (mod6)$

$f(v_i) = 1; if\ 1 \leq i \leq 2n.$

$f(u_i) = 2; if\ 1 \leq i \leq 2n.$

$f(u_i') = 0; if\ 1 \leq i \leq 2n.$

$$f(w_i) = \begin{cases} 0; i \equiv 1,4 (\text{mod } 6) \\ 1; i \equiv 2,5 (\text{mod } 6) \\ 2; i \equiv 0,3 (\text{mod } 6), 1 \leq i \leq n \end{cases}$$

Case 2: $n \equiv 1,2,4,5(\ mod6)$

$f(u_{2n}') = 2.$

$f(v_i) = 1; if\ 1 \leq i \leq 2n$

$f(u_i) = 2; if\ 1 \leq i \leq 2n.$

$f(u_i') = 0; if\ 1 \leq i \leq 2n-1$

$$f(w_i) = \begin{cases} 0; i \equiv 1,4 (\text{mod } 6) \\ 1; i \equiv 2,5 (\text{mod } 6) \\ 2; i \equiv 0,3 (\text{mod } 6), 1 \leq i \leq n \end{cases}$$

Above defined labeling pattern satisfies the conditions of 3-equitable labeling as shown in Table4.

Let $n = 6a + b$, where $b \in \mathbb{N}$ and $a \in \mathbb{N} \cup \{0\}$.

Table 2: Table for Theorem 4

b	Vertex Conditions	Edge Conditions
0,3	$v_f(0) = v_f(1) = v_f(2)$	$e_f(0) = e_f(1) = e_f(2)$
2,5	$v_f(0) + 1 = v_f(1) = v_f(2)$	$e_f(0) + 1 = e_f(1) = e_f(2) + 1$
1,4	$v_f(0) + 1 = v_f(1) + 1 = v_f(2)$	$e_f(0) = e_f(1) + 1 = e_f(2)$

Hence the barycentric subdivision of C'_n is 3-equitable.

Example 3.3: equitable labeling of the graph obtained by barycentric subdivision of C'_n is shown in Figure 4.

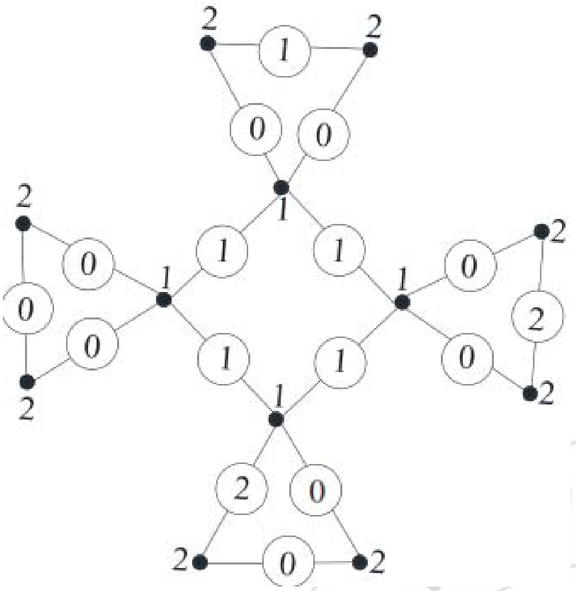

Figure 4

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