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On Fuzzy Soft Ring and Its Properties

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Abstract: ***In this paper, the study of fuzzy soft ring by combining soft set theory. The notions of fuzzy soft ring as defined and several related properties and structural characteristics are investigated .***

Keywords: Soft set, Fuzzy soft set, soft ring, Fuzzy soft ring, soft homomorphism, Fuzzy Soft isomorphism, Fuzzy soft ring

1. Introduction

In many complicated problems arising in the fields of engineering, social science, economics, medical science etc involving uncertainties, classical methods are found to be inadequate in recent times.

Molodtsov initiated soft set theory [8] as a new mathematical tool for dealing with uncertainties. In recent times, researchers have contributed a lot towards fuzzification of soft set theory. It is well known that the concept of fuzzy sets, introduced by Zadeh [10], has been extensively applied to many scientific fields.

In 1971, Rosenfeld [9] applied the concept to the theory of groupoids and groups. Inan et al. have already introduced the definition of fuzzy soft rings and studied some of their basic properties [11]. (Dr.N.sarala and B.Suganya, 2014) presented some properties of fuzzy soft groups. Further (Dr.N.sarala and B.Suganya, 2014) introduced on normal fuzzy soft groups.

In this paper, we first present the basic definitions of soft sets, fuzzy sets , fuzzy soft sets , soft ring . We study fuzzy soft ring theory by using fuzzy soft sets and studied some of algebraic properties.

Throughout this work, *U* refers to the initial universe, *P*(*U*) is the power set of *U*, *E* is a set of parameters and $A \subseteq E$. *S*(*U*) denotes the set of all soft sets over *U*.

2. Preliminaries

In this section, we first recall the basic definitions related to fuzzy soft sets which would be used in the sequel.

Definition2.1
Suppose that U is an initial universe set and E is a set of parameters, Let P(U) denotes the power set of U .A pair (F,E) is called a ***soft set*** over U where F is a mapping given by F: E⟶ P(U) .

Clearly, a soft set is a mapping from parameters to P(U), and it is not a set, but a parameterized family of subsets of the Universe.

Definition 2.2.
Let U be an initial Universe set and E be the set of parameters. Let $A \subset E$. A pair (F, A) is called ***fuzzy soft set*** over U where F is a mapping given by F: A⟶ P(U), where P(U) denotes the collection of all fuzzy subsets of U.

Definition 2.3
Let X be a group and (F,A) be a soft set over X. Then (F,A) is said to be a ***soft group*** over X iff $F(a) < X$, for each $a \in A$.

Definition 2.4
Let X be a group and (f, A) be a fuzzy soft set over X. Then (f, A) is said to be a ***fuzzy soft group*** over X iff for each $a \in A$ and $x, y \in X$,
(i) $f_a(x \,.\, y) \geq T\,(f_a(x), f_a(y)$
(ii) $f_a(x^{-1}) \geq f_a(x)$
That is, for each $a \in A$, f_a is a fuzzy subgroup in Rosenfeld's sense [9]

Definition 2.5
Let (f,A) be a soft set over a ring R. Then (f, A) is said to be a ***soft ring*** over R if and only if f(a) is sub ring of R for each $a \in A$.

Definition 2.6
Let (φ,ψ): X → Y is a fuzzy soft function, if φ is a homomorphism from x → y then (φ,ψ) is said to be **fuzzy soft homomorphism**. If φ is an isomorphism from X → Y and ψ is 1-1 mapping from A on to B then (φ,ψ) is said to be **fuzzy soft isomorphism**.

3. Properties of Fuzzy soft ring

Throughout this paper R denotes a commutative ring and all fuzzy soft sets are considered over R. In this section, we presented the basic concepts of fuzzy soft ring theory and then we discuss basic algebraic structures of fuzzy soft ring and some related properties are investigated.

Definition: 3.1
Let R be a soft ring. A fuzzy set 'μ' in R is called ***fuzzy soft ring*** in R. i.e. $(x,y) \in R$
(FSR1) (i) $\mu\,(x+y) \geq T\{\mu(x), \mu(y)\}$
(FSR2) (ii) $\mu\,(-x) \geq \mu\,(x)$ and
(FSR3) (iii) $\mu\,(xy) \geq T\{\mu(x), \mu(y)\}$, for all $x,y \in R$.

Proposition 3.2. [6]

Let μ and λ be two fuzzy soft ring of R, then
$\mu \cap \lambda$ is a fuzzy soft ring of R.

Proof :
It is obvious

Proposition 3.3. [6]
Let μ and λ be two fuzzy soft ring of R, then
$\mu \cup \lambda$ is a fuzzy soft ring of R.

Proof:
It is obvious

Theorem:3.4
Let μ and λ be two fuzzy soft rings over R, then $\mu \wedge \lambda$ is fuzzy soft ring of R.

Proof:
Let x, y $\in$G.

FSR(iii)

$$(\mu \wedge \lambda)(x+y) = (\mu(x+y) \wedge \lambda(x+y))$$
$$\geq T\{\mu(x), \mu(y)\} \wedge T\{\lambda(x), \lambda(y)\}$$
$$\geq T\{\mu(x) \wedge \lambda(x)\}, T\{\mu(y) \wedge \lambda(y)\}$$
$$\geq T\{\mu \wedge \lambda)(x), (\mu\wedge\lambda)(y)\}$$

(i.e) $(\mu \wedge \lambda)(x+y) \geq T\{(\mu\wedge \lambda)(x), (\mu \wedge \lambda)(y)\}$

FSR(ii)

$$(\mu \wedge \lambda)(-x) \geq \mu(-x) \wedge \lambda(-x)$$
$$\geq \mu(x) \wedge \lambda(x)$$
$$\geq (\mu \wedge\lambda)(x)$$

(i.e) $(\mu \wedge \lambda)(-x) \geq (\mu\wedge\lambda)(x)$

FSR(iii)

$$(\mu \wedge \lambda)(xy) = \mu(xy) \wedge \lambda(xy)$$
$$\geq T\{(\mu(x), \mu(y)\} \wedge T\{\lambda(x), \lambda(y)\}$$
$$\geq T\{\mu(x) \wedge \lambda(x)\}, T\{\mu(y) \wedge \lambda(y)\}$$
$$\geq T\{(\mu \wedge \lambda)(x), (\mu \wedge \lambda)(y)\}$$

(i.e)$(\mu \wedge \lambda)(xy) \geq T\{(\mu\wedge \lambda)(x), (\mu \wedge \lambda)(y)\}$
Hence $\mu \wedge \lambda$ is fuzzy soft ring of R.

Theorem:3.5
Let μ and λ be two fuzzy soft rings over R, then $\mu \vee \lambda$ is fuzzy soft ring of R.

Proof:
Let x, y $\in$G.

FSR(iii)

$$(\mu \vee \lambda)(x+y) = (\mu(x+y) \vee \lambda(x+y))$$
$$\geq T(\mu(x), \mu(y)) \vee T(\lambda(x), \lambda(y))$$
$$\geq T(\mu(x) \vee \lambda(x)), T(\mu(y) \vee \lambda(y))$$
$$\geq T(\mu \vee \lambda)(x), (\mu\vee \lambda)(y))$$

(i.e) $(\mu\vee \lambda)(x+y) \geq T(\mu\vee \lambda)(x), (\mu \vee \lambda)(y))$

FSR(ii)

$$(\mu\vee\lambda)(-x) \geq \mu(-x) \vee \lambda(-x)$$
$$\geq \mu(x) \vee \lambda(x)$$
$$\geq (\mu \vee\lambda)(x)$$

(i.e) $(\mu\vee(-x) \geq (\mu\vee\lambda)(x)$

FSR(iii)

$$(\mu \vee \lambda)(xy) = \mu(xy) \vee \lambda(xy)$$
$$\geq T\{\mu(x), \mu(y)\} \vee T\{\lambda(x), \lambda(y)\}$$
$$\geq T\{\mu(x) \vee \lambda(x)\}, T\{\mu(y) \vee \lambda(y)\}$$
$$\geq T\{(\mu \vee \lambda)(x), (\mu\vee \lambda)(y)\}$$

(i.e)$(\mu\vee\lambda)(xy) \geq T(\mu\vee\lambda)(x), (\mu \vee \lambda)(y))$
Hence $\mu \vee \lambda$ is fuzzy soft ring of R.

Proposition 3.6
If $\{\mu_i\}_{i \in\mu}$ is a family of fuzzy soft ring of R, then $\cap\mu_i$ is fuzzy soft ring R whose $\cap\mu_i = \{(x, \wedge\mu_i(x) / x\in R\}$, where $i \in \mu$.

Proof:
Let x, y $\in$G, then for $i \in f_a$, it follows that

(FSR(i))

$$\cap\mu_i(x+y) = \wedge \mu_i(x+y)$$
$$\geq \wedge \{T(\mu_i(x), \mu_i(y))\}$$
$$\geq T\{(\wedge \mu_i(x)), (\wedge \mu_i(y)\}$$
$$\geq T\{(\cap\mu_i(x), (\cap\mu_i(y)\}$$

(i.e) $\cap\mu_i(x+y) \geq T\{(\cap\mu_i(x), (\cap\mu_i(y)\}$

(FSR(ii))

$$\cap \mu_i(-x) = \wedge \mu_i(-x)$$
$$\geq \wedge \mu_i(x)$$
$$\geq (\cap\mu_i)(x)$$

(i.e) $(\cap \mu_i)(-x) \geq (\cap \mu_i)(x)$

(FSR(iii))

$$\cap \mu_i(xy) = \wedge \mu_i(xy)$$
$$\geq \wedge T\{\mu_i(x), \mu_i(y)\}$$
$$\geq T\{(\wedge \mu_i(x)), (\wedge \mu_i(y)\}$$
$$\geq T\{\cap\mu_i(x), \cap\mu_i(y)\}$$

(i.e) $\cap \mu_i(xy) \geq T((\cap\mu_i(x), (\cap\mu_i(y))$
Hence $(\cap f_{ai})$ is fuzzy soft ring of R.

Proposition 3.7
If $\{\mu_i\}_{i \in\mu}$ is a family of fuzzy soft ring of R, then $\cup \mu_i$ is fuzzy soft ring R whose $\cup \mu_i = \{(x, \vee \mu_i(x) / x\in R\}$, where $i \in \mu$.

Proof:
Let x, y $\in$G, then for $i \in \mu$, it follows that

(FSR(i))

$$\cup\mu_i(x+y) = \vee \mu_i(x+y)$$
$$\geq \vee \{T(\mu_i(x), \mu_i(y))\}$$
$$\geq T\{(\vee\mu_i(x)), (\vee\mu_i(y)\}$$
$$\geq T\{\cup\mu_i(x), \cup\mu_i(y)\}$$

(i.e) $\cup\mu_i(x+y) \geq T\{(\cup\mu_i(x), (\cup\mu_i(y)\}$

(FSR(ii))

$$\cup \mu_i(-x) = \vee \mu_i(-x)$$
$$\geq \vee \mu_i(x)$$
$$\geq (\cup\mu_i)(x)$$

(i.e) $(\cup \mu_i)(-x) \geq (\cup \mu_i)(x)$

(FSR(iii))

$$\cup\mu_i(xy) = \vee\mu_i(xy)$$
$$\geq \vee T\{\mu_i(x), \mu_i(y)\}$$
$$\geq T\{(\vee\mu_i(x)), (\vee\mu_i(y)\}$$
$$\geq T\{\cup\mu_i(x), \cup\mu_i(y)\}$$

(i.e) $\cup\, \mu_i\,(xy) \geq T\,((\cup\mu_i\,(x), (\cup\mu_i(y))$

Hence $(\cup\, \mu_i)$ is fuzzy soft ring of R.

Proposition 3.8

Let R and R' be two rings and θ: R $\rightarrow$R' be a soft homomorphism. If μ is fuzzy soft ring of R then the pre-image $\theta^{-1}(\mu)$ is fuzzy soft ring of R.

Proposition 3.9 [6]

Let θ: R $\rightarrow$ R' be an epimorphism and μ be fuzzy soft set in R'. If $\theta^{-1}(\mu)$ is fuzzy soft ring of R, μ is fuzzy soft ring of R'.

Theorem:3.10

If 'μ' is a fuzzy soft ring of R, then μ^C is also fuzzy soft ring of R.

Proof:

For any x,y ε R

(FSR1)

$$\mu^C(x+y) = 1- \mu\,(x+y)$$
$$\leq 1\text{-}T\,\{\mu\,(x), \mu\,(y)\}$$
$$= \max\,\{1\text{-}\,\mu\,(x), 1\text{-}\,\mu\,(y)\}$$
$$= \max\,\{\mu^C(x), \mu^C(y)\}$$

(i.e) $\mu^C(x+y) = \max\,\{\mu^C(x), \mu^C(y)\}$

(FSR2)

$$\mu^C(-x) = 1- \mu(-x)$$
$$= 1- \mu(x)$$
$$= \mu^C(x).$$

(i.e) $\mu^C(-x) = \mu^C(x)$.

(FSR3)

$$\mu^C(xy) = 1- \mu\,(xy)$$
$$\leq 1\text{-}T\,\{\mu\,(x), \mu\,(y)\}$$
$$= \max\,\{1\text{-}\,\mu\,(x), 1\text{-}\,\mu\,(y)\}$$
$$= \max\,\{\mu^C(x), \mu^C(y)\}$$

(i.e) $\mu^C(xy) = \max\,\{\mu^C(x), \mu^C(y)\}$

Hence μ^C is fuzzy soft ring of R.

Theorem :3.11

If μ and λ are fuzzy soft rings of R and R' respectively, then μ x λ is a fuzzy soft rings of Rx R'.

Proof:

Let μ and λ are fuzzy soft rings of the rings R and R' respectively

Let x_1 and x_2 be in R, y_1 and y_2 be in R'. Then (x_1, y_1) and (x_2, y_2) are in Rx R' .

Now,

(FSR1)

$$\mu \times \lambda\ [(x_1+ y_1)(x_2+ y_2)] = \mu \times \lambda\ (x_1+x_2, y_1+y_2)$$
$$= T\,\{\mu\,(x_1+x_2), \lambda(y_1+y_2)\}$$
$$\geq T\{T\{\mu\,(x_1), \mu\,(x_2)\}, T\{\lambda\,(y_1), \lambda(y_2)\}\}$$
$$= T\{T\{\mu\,(x_1), \lambda(y_1)\}, T\{\mu\,(x_2), \lambda(y_2)\}\}$$
$$= T\{\mu x\lambda\,(x_1, y_1), \mu\, x\lambda\,(x_2, y_2)\}.$$

Therefore **μ x $\lambda[(x_1+ y_1)(x_2+ y_2)]=T\{\mu x\lambda(x_1,y_1), \mu x\lambda(x_2,y_2)\}$.**

(FSR2)

$$\mu \times \lambda\ [-(x_1, y_1)] = \mu \times \lambda\ (-x_1, -y_1)$$
$$= T\,\{\mu\,(-x_1), \lambda\,(-y_1)\}$$
$$\geq T\,\{\mu\,(x_1), \lambda\,(y_1)\}$$
$$= \mu \times \lambda\ (x_1, y_1).$$

Therefore **$\mu \times \lambda\ [-(x_1, y_1)] = \mu \times \lambda\ (x_1, y_1)$.**

(FSR2)

$$\mu \times \lambda\ [(x_1, y_1)(x_2, y_2)] = \mu \times \lambda\ (x_1x_2, y_1y_2)$$
$$= T\,\{\mu\,(x_1x_2), \lambda(y_1y_2)\}$$
$$\geq T\{T\{\mu\,(x_1), \mu\,(x_2)\}, T\{\lambda\,(y_1), \lambda(y_2)\}\}$$
$$= T\{T\{\mu\,(x_1), \lambda(y_1)\}, T\{\mu\,(x_2), \lambda(y_2)\}\}$$
$$= T\{\mu x\lambda\,(x_1, y_1), \mu\, x\lambda\,(x_2, y_2)\}.$$

Therefore **μ x $\lambda[(x_1,y_1)(x_2,y_2)]=T\{\mu x\lambda\,(x_1, y_1), \mu\, x\lambda\,(x_2, y_2)\}$.**

Hence μ x λ is fuzzy soft rings of Rx R'.

4. Conclusion

In this paper, we have studied the concept of fuzzy soft rings and their properties. This concept can further be extended for new results.

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