

Fuzzy Inventory Model without Shortages Using Signed Distance Method

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Abstract: In this paper an inventory model without shortage is considered under fuzzy environment. Our objective is to determine the optimal total cost and optimal order quantity for proposed inventory model. The optimum order quantity is calculated using Signed Distance Method for defuzzification. Cost involved; ordering cost and holding cost considered as fuzzy parameters. The Trapezoidal fuzzy numbers are used to achieve the goal. The proposed model is illustrated numerically.

Keywords: Defuzzification, inventory model, signed distance method, trapezoidal fuzzy number.

1. Introduction

Inventory is one of the most expensive and important assets of companies. Managers have long recognized that good inventory control is crucial. There are many reasons of maintaining inventories, on one hand, a firm can try to reduce costs by reducing on – hand inventory levels. On the other hand, customers become dissatisfied when frequent inventory outages. Therefore, the proper inventory control help in growth of an organization.

Inventory is a stored resource that is used to satisfy a current or future need. Raw materials, work-in-process, and finished goods are examples of inventory. There are only two fundamental decisions that you have to make when controlling inventory: how much to order and when to order. A major objective in controlling inventory is to minimize total inventory costs.

The economic order quantity (EOQ) model is one of the oldest and most commonly known inventory control techniques. Research on its use dates back to a 1915 by Ford W. Harris. This model is still used by a large number of organizations today. This technique is relatively easy to use, but it makes a number of assumptions. Some of the more important assumptions follow:

- 1) Demand is known and constant.
- 2) The lead time – that is, the time between the placement of the order and the receipt of the order – is known and constant.
- 3) The receipt of inventory is instantaneous. In other words, the inventory from an order arrives in one batch, at one point in time.
- 4) Quantity discounts are not possible.
- 5) No safety stock.

with these assumptions, inventory usage has a saw tooth shape, as in Figure 1.1. Here, Q represents the amount that is ordered. In general, the inventory level increases from 0 to Q units when an order arrives. Because demand is constant over time, inventory drops at a uniform rate over time. Another order is placed such that when the inventory level reaches to 0, the new order is received and inventory level again jumps to Q units, represented by the vertical lines. This process continues indefinitely over time.

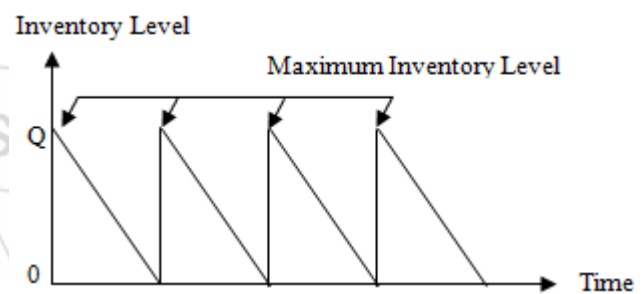


Figure 1.1: Inventory usage over time

2. Literature Review

The basic well known square root formula for EOQ model for constant demand was first given by Harris (1915). Some of the important works done by many researchers like Buchanan (1980), Goyal (1986), Goyal et al (1986), Hariga (1996), Teng and Thompson (1996). Sarkar and Sana (2010) have developed an inventory model with increasing demand under inflation. In all the above mentioned works, the parameters were taken in crisp environment.

In literature, there are many papers on fuzzified problems of EOQ model. Urgeletti treated EOQ model in fuzzy sense, and used triangular fuzzy number. Chen and Wang used trapezoidal fuzzy number to fuzzify the order cost, inventory cost and back-order cost in the total cost of inventory model without backorder. Park and Vujosevic et al developed the inventory models in fuzzy sense where ordering cost and holding cost are represented by Fuzzy numbers. Vujosevic has represented ordering cost by triangular fuzzy number and holding cost by trapezoidal fuzzy number.

For defuzzification, the study shows that signed distance method is better than centroid Yao and Lee. De and Rawant, proposed an EOQ model without shortage cost by using triangular fuzzy number. The total cost has been computed by using signed distance method. For different fuzzy numbers and methods of defuzzification, Sen et al (2014) and Dutta and Kumar (2012) were referred.

3. Model Formulation

3.1 Definitions and Preliminaries

Definition 3.1: Let \tilde{A} be a fuzzy set on $\mathbb{R} = (-\infty, \infty)$. It is called a fuzzy point if its membership function is

$$\mu_{\tilde{A}}(x) = \begin{cases} 1, & \text{if } x = a \\ 0, & \text{if } x \neq a \end{cases}$$

Definition 3.2: Let $[a, b; \alpha]$ be a fuzzy set on \mathbb{R} . It is called a level α fuzzy interval, $0 \leq \alpha \leq 1, a < b$, if its membership function is

$$\mu_{[a,b;\alpha]}(x) = \begin{cases} \alpha, & \text{if } a \leq x \leq b \\ 0, & \text{if } x \neq a \end{cases}$$

If $a = b$, we call $[a, b; \alpha]$ a level α fuzzy point at a .

Definition 3.3: A trapezoidal fuzzy number $\bar{A} = (a, b, c, d)$ is represented with membership function $\mu_{\bar{A}}$ as:

$$\mu_{\bar{A}}(x) = \begin{cases} L(x) = \frac{x-a}{b-a}; & \text{when } a \leq x \leq b \\ 1 & \text{when } b \leq x \leq c \\ R(x) = \frac{d-x}{d-c}; & \text{when } c \leq x \leq d \\ 0 & \text{otherwise} \end{cases}$$

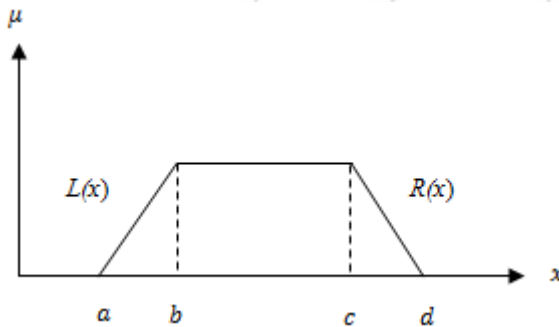


Figure 3.1: Trapezoidal fuzzy number

Definition 3.4: A fuzzy set is called in LR – form, if there exist reference function L (for left), R (for right), and scalars $m > 0$ and $n > 0$ with membership function.

$$\mu_{\bar{A}}(x) = \begin{cases} L\left(\frac{\sigma - x}{m}\right), & \text{for } x \leq \sigma; \\ 1, & \text{for } \sigma \leq x \leq \gamma; \\ R\left(\frac{x - \gamma}{n}\right), & \text{for } x \geq \gamma. \end{cases}$$

Where σ a real number is called the mean value of \bar{A} , m and n are called the left and right spreads, respectively. The function L and R map $\mathbb{R}^+ \rightarrow [0,1]$, and are decreasing. A LR-Type fuzzy number can be represented as $\bar{A} = (\sigma, \gamma, m, n)$.

Definition 3.5: Suppose $\bar{A} = (a_1, a_2, a_3, a_4)$ and $\bar{B} = (b_1, b_2, b_3, b_4)$ are two trapezoidal fuzzy numbers, then arithmetical operations are defined as:

1. The addition of \bar{A} and \bar{B} is
 $\bar{A} \oplus \bar{B} = (a_1 + b_1, a_2 + b_2, a_3 + b_3, a_4 + b_4)$

2. The multiplication of \bar{A} and \bar{B} is
 $\bar{A} \otimes \bar{B} = (a_1 b_1, a_2 b_2, a_3 b_3, a_4 b_4)$

3. $\bar{A} - \bar{B} = (a_1 - b_4, a_2 - b_3, a_3 - b_2, a_4 - b_1)$

4. For any real number K ,
 $K\bar{A} = (Ka_1, Ka_2, Ka_3, Ka_4)$ if $K > 0$
 $K\bar{A} = (Ka_4, Ka_3, Ka_2, Ka_1)$ if $K < 0$

Definition 3.6: Defuzzification of \bar{A} can be found by signed distance method. If A is a trapezoidal fuzzy number then signed distance from A to 0 is defined as;

$$d(\bar{A}, 0) = \frac{1}{2} \int_0^1 ([A_L(\alpha), A_R(\alpha)], 0) d\alpha$$

where

$$A_\alpha = [A_L(\alpha), A_R(\alpha)]$$

$$A_\alpha = [a + (b-a)\alpha, d - (d-c)\alpha], \alpha \in [0,1]$$

is α -cut of fuzzy set \bar{A} , which is a close interval.

3.2 Notations and Assumptions

Notations

- Let us now define the following parameters:
 Q^* = optimal order quantity (i.e., the EOQ)
 Q = order quantity per cycle
 D = annual demand, in units, for the inventory item
 C_0 = ordering cost per order
 C_h = carrying or holding cost per unit per year
 TC = total cost for the period $[0, T]$
 \widetilde{TC} = fuzzy total cost for the period $[0, T]$

Assumptions

- In this model, the following assumptions are considered:
 1) Total demand is considered as constant.
 2) Time of the plan is constant
 3) Shortage is not allowed

Finding the Economic order Quantity in Crisp Sense

$$\text{Total ordering cost} = \left(\frac{D}{Q}\right) \times C_0$$

$$\text{Total carrying cost} = \left(\frac{Q}{2}\right) \times C_h$$

Total cost = Total ordering cost + Total carrying cost

$$TC = \left(\frac{D}{Q}\right) \times C_0 + \left(\frac{Q}{2}\right) \times C_h \quad (3.1)$$

The presence of Q in the denominator of the first term makes equation (3.1) a nonlinear equation with respect to Q . The optimum Q^* can be obtained by equating the first order derivative at TC to zero and solving the resulting equations.

$$\text{Optimal order quantity} = Q^* = \sqrt{\frac{2DC_0}{C_h}}$$

$$F(Q) \text{ is minimum when } \frac{dF(Q)}{dQ} = 0,$$

$$\text{and where } \frac{d^2 F(Q)}{dQ^2} > 0.$$

Now,

$$Q^* = \sqrt{2D \frac{(a_1 + b_1 + c_1 + d_1)}{(a_2 + b_2 + c_2 + d_2)}}$$

$$\text{Also, at } Q = Q^* \text{ we have } \frac{d^2 F(Q)}{dQ^2} > 0$$

This shows that $F(Q)$ is minimum at $Q = Q^*$,

$$\text{An } F(Q^*) = (a_1 + b_1 + c_1 + d_1) \frac{D}{4Q^*} + (a_2 + b_2 + c_2 + d_2) \frac{Q^*}{8}$$

Finding the Economic Order Quantity in Fuzzy Sense

We consider the model in fuzzy environment, since the inventory cost and holding cost are in fuzzy nature, we represent them by trapezoidal fuzzy numbers.

Let

\tilde{C}_h : Fuzzy carrying cost per unit quantity per year

\tilde{C}_o : Fuzzy ordering cost per order

The total demand and time of plan are considered as constants. The fuzzy total cost is given by

$$\tilde{TC} = \left(\frac{D}{Q}\right) \times \tilde{C}_o + \left(\frac{Q}{2}\right) \times \tilde{C}_h$$

Now, we defuzzify the fuzzy total cost by using signed distance method.

Suppose $\tilde{C}_o = (a_1, b_1, c_1, d_1)$ and $\tilde{C}_h = (a_2, b_2, c_2, d_2)$ are trapezoidal numbers in LR form and $a_1, b_1, c_1, d_1, a_2, b_2, c_2, d_2$ are known positive numbers.

$$\begin{aligned} &= \left(\frac{D}{Q}\right) \times (a_1, b_1, c_1, d_1) \oplus \left(\frac{Q}{2}\right) \times (a_2, b_2, c_2, d_2) \\ \tilde{TC} &= \left[a_1 \frac{D}{Q}, b_1 \frac{D}{Q}, c_1 \frac{D}{Q}, d_1 \frac{D}{Q} \right] \oplus \left[a_2 \frac{Q}{2}, b_2 \frac{Q}{2}, c_2 \frac{Q}{2}, d_2 \frac{Q}{2} \right] \\ &= \left[a_1 \frac{D}{Q} + a_2 \frac{Q}{2}, b_1 \frac{D}{Q} + b_2 \frac{Q}{2}, c_1 \frac{D}{Q} + c_2 \frac{Q}{2}, d_1 \frac{D}{Q} + d_2 \frac{Q}{2} \right] \\ &= (a, b, c, d) \text{ (say)} \end{aligned}$$

Now,

$$\begin{aligned} A_L(\alpha) &= a + (b - a)\alpha \\ &= a_1 \frac{D}{Q} + a_2 \frac{Q}{2} + \left[(b_1 - a_1) \frac{D}{Q} + (b_2 - a_2) \frac{Q}{2} \right] \alpha \\ A_R(\alpha) &= d - (d - c)\alpha \\ &= d_1 \frac{D}{Q} + d_2 \frac{Q}{2} - \left[(d_1 - c_1) \frac{D}{Q} + (d_2 - c_2) \frac{Q}{2} \right] \alpha \end{aligned}$$

Defuzzifying \tilde{TC} by using signed distance method, we have

$$\begin{aligned} d(\tilde{TC}, 0) &= \\ &= \frac{1}{2} \int [A_L(\alpha) + A_R(\alpha)] d\alpha \\ &= \frac{1}{2} \left[(a_1 + d_1) \frac{D}{Q} + (a_2 + d_2) \frac{Q}{2} \right] + \\ &\quad \frac{1}{4} \left[(b_1 + c_1 - a_1 - d_1) \frac{D}{Q} + (b_2 - a_2 + d_2 + c_2) \frac{Q}{2} \right] \\ &= (a_1 + b_1 + c_1 + d_1) \frac{D}{4Q} + (a_2 + b_2 + c_2 + d_2) \frac{Q}{8} \\ &= F(Q) \text{ (say)} \end{aligned}$$

Algorithm for Finding Fuzzy Total Cost and Fuzzy Optimal Order Quantity:

- 1) Calculate total cost (TC) for the crisp model.
- 2) Determine fuzzy total cost (\tilde{TC}) using fuzzy arithmetic operations on fuzzy carrying and holding cost, taken as fuzzy trapezoidal numbers.
- 3) Apply signed distance method for defuzzification of (\tilde{TC}). Then, find fuzzy optimal order quantity Q^* , using first and second derivative test.

4. Numerical Example

To illustrate the develop model (both in crisp & fuzzy sense), we have taken an example.

In crisp sense

Let,
 $D = 600$ units per units
 $C_h = \text{Rs. } 10$ per month
 $C_o = \text{Rs. } 100$ per order

Then, optimal order quantity $Q^* = 109.54$ units and the minimum cost per year $TC_{min} = TC^* = \text{Rs. } 1095.50$.

In fuzzy sense

Let,
 $D = 600$ units per units
 $C_h = (7, 9, 11, 12)$
 $C_o = (96, 98, 101, 103)$

Then, we get $Q^* = 110.66$ units and the minimum cost per year $TC_{min} = TC^* = \text{Rs. } 1078.96$.

Table 3.1: Sensitivity Analysis

No	Demand	$C_o = (96, 98, 101, 103)$		$C_o = (96, 97, 102, 103)$	
		$C_h = (7, 9, 11, 12)$		$C_h = (7, 8, 10, 12)$	
		Q^*	TC^* (Rs.)	Q^*	TC^* (Rs.)
1	600	110.66	1078.96	113.61	1470.39
2	625	112.94	1101.21	115.96	1481.22
3	650	115.18	1123.01	118.25	1491.84
4	675	117.38	1144.41	120.51	1502.26
5	700	119.53	1165.41	122.72	2937.00

5. Conclusion

In this study, we have used signed distanced method for defuzzifying the holding cost and ordering cost. These costs are considered as trapezoidal fuzzy numbers. Finally, numerical examples are given to illustrate this model and we observed that the EOQ obtained by signed distance method is closer to crisp EOQ and total cost obtained by signed distance method is less than crisp total cost. Also, EOQ is more sensitive towards demand and total cost increase as demand increases.

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Author Profile



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