

k-Super Mean Labeling of Some Graphs

Dr. M. Tamilselvi¹, K. Akilandeswari², V. Suguna³

¹Associate Professor, Department of Mathematics, SeethalakshmiRamaswami College, Tirchirappalli – 620002

^{2,3}Research Scholar, Department of Mathematics, SeethalakshmiRamaswami College, Tirchirappalli – 620002

Abstract: Let G be a (p, q) graph and $f: V(G) \rightarrow \{1, 2, 3, \dots, p + q\}$ be an injection. For each edge $e = uv$, let $f^*(e) = \frac{f(u)+f(v)}{2}$ if $f(u) + f(v)$ is even and $f^*(e) = \frac{f(u)+f(v)+1}{2}$ if $f(u) + f(v)$ is odd, then f is called super mean labeling if $f(V) \cup \{f^*(e): e \in E(G)\} = \{1, 2, 3, \dots, p + q\}$. A graph that admits a super mean labeling is called Super mean graph. Let G be a (p, q) graph and $f: V(G) \rightarrow \{k, k + 1, k + 2, \dots, p + q + k - 1\}$ be an injection. For each edge $e = uv$, let $f^*(e) = \frac{f(u)+f(v)}{2}$ if $f(u) + f(v)$ is even and $f^*(e) = \frac{f(u)+f(v)+1}{2}$ if $f(u) + f(v)$ is odd, then f is called k -super mean labeling if $f(V) \cup \{f^*(e): e \in E(G)\} = \{k, k + 1, \dots, p + q + k - 1\}$. A graph that admits a k -super mean labeling is called k -Super mean graph. In this paper we investigate k -super mean labeling of $\langle B_{n,n}, w \rangle$, $S(B_{n,n})$, and $T_n \odot K_1$.

Keywords: Super mean labeling, Super mean graph, k -Super mean labeling, k -Super mean graph, Bistar graphs, $T_n \odot K_1$.

1. Introduction

All graphs in this paper are finite, simple and undirected. Terms not defined here are used in the sense of Harary [7]. The symbols $V(G)$ and $E(G)$ will denote the vertex set and edge set of a graph G .

Graph labeling was first introduced in the late 1960's. Many studies in graph labeling refer to Rosa's research in 1967 [11]. Labeled graphs serve as useful models for a broad range of applications such as X-ray, crystallography, radar, coding theory, astronomy, circuit design and communication network addressing. Particularly interesting applications of graph labeling can be found in [1-4].

The concept of mean labeling was introduced and studied by S. Somasundaram and R. Ponraj [12].

The concept of super mean labeling was introduced and studied by D. Ramya et al. [11]. Further some results on super mean graphs are discussed in [8,9,10,13,15].

B. Gayathri and M. Tamilselvi [5-6, 14] extended super mean labeling to k -super mean labeling.

In this paper we investigate k -Super mean labeling of Bistar graphs and $T_n \odot K_1$. For brevity, we use k -SML for k -super mean labeling.

Definition 1.1

Let G be a (p, q) graph and $f: V(G) \rightarrow \{1, 2, 3, \dots, p + q\}$ be an injection. For each edge $e = uv$, let $f^*(e) = \frac{f(u)+f(v)}{2}$ if $f(u) + f(v)$ is even and $f^*(e) = \frac{f(u)+f(v)+1}{2}$ if $f(u) + f(v)$ is odd, then f is called **Super mean labeling** if $f(V) \cup \{f^*(e): e \in E(G)\} = \{1, 2, 3, \dots, p + q\}$. A graph that admits a super mean labeling is called **Super mean graph**.

Definition 1.2

Let G be a (p, q) graph and

$f: V(G) \rightarrow \{k, k + 1, k + 2, \dots, p + q + k - 1\}$ be an injection. For each edge $e = uv$, let $f^*(e) = \frac{f(u)+f(v)}{2}$ if $f(u) + f(v)$ is even and $f^*(e) = \frac{f(u)+f(v)+1}{2}$ if $f(u) + f(v)$ is odd, then f is called **k -Super mean labeling** if $f(V) \cup \{f^*(e): e \in E(G)\} = \{k, k + 1, \dots, p + q + k - 1\}$. A graph that admits a k -super mean labeling is called **k -Super mean graph**.

Definition 1.3

A bistar $(B_{n,n})$ is the graph obtained by joining the central vertices of two copies of star $K_{1,n}$ by an edge.

Definition 1.4

If G is a graph, then $S(G)$ is a graph obtained by subdividing each edge of G by a vertex.

Definition 1.5

A triangular snake (T_n) is obtained from a path by identifying each edge of the path with an edge of the cycle C_3 .

2. Main Results

Definition 2.1

The graph $\langle B_{n,n}, w \rangle$ is the graph obtained by subdividing the central edge of the bistar $B_{n,n}$ with the vertex w .

Theorem 2.2

The graph $\langle B_{n,n}, w \rangle$ is a k -super mean graph for all $n \geq 2$.

Proof

Let $V(\langle B_{n,n}, w \rangle) = \{u, v, w, u_i, v_i; 1 \leq i \leq n\}$ and $E(\langle B_{n,n}, w \rangle) = \{e = (u, w)\} \cup \{e'' = (w, v)\} \cup \{e_i = (u, u_i): 1 \leq i \leq n\} \cup \{e_i = (v, v_i): 1 \leq i \leq n\}$

be the vertices and edges of $\langle B_{n,n}, w \rangle$ respectively.

Case (i): n is even

Let $n = 2l$, for some l .

Define

$$f: V(\langle B_{n,n}, w \rangle) \rightarrow \{k, k+1, k+2, \dots, 4n+k+5\}$$

$$f(u) = k,$$

$$f(w) = 4n+k+4,$$

$$f(v) = 4n+k+2$$

$$f(u_i) = 4i+k-2; \text{ if } 1 \leq i \leq 2l \text{ and if } i \neq l+1,$$

$$f(u_{l+1}) = 4l+k+1,$$

$$f(v_i) = 4i+k; \text{ if } 1 \leq i \leq 2l.$$

Now the induced edge labels are

$$f^*(e) = 2n+k+2,$$

$$f^*(e'') = 4n+k+3,$$

$$f^*(e_i) = 2i+k-1; 1 \leq i \leq n,$$

$$f^*(e_1) = f^*(e_n) + 4,$$

$$f^*(e_i) = f^*(e_1) + 2(i-1); 2 \leq i \leq n.$$

Here $p = 2n+3, q = 2n+2$.

$$\text{Clearly, } f(V) \cup \{f^*(e): e \in E(\langle B_{n,n}, w \rangle)\} = \{k, k+1, \dots, 4n+k+4\}.$$

So f is a k -super mean labeling, when n is even.

Hence $\langle B_{n,n}, w \rangle$ is a k -super mean graph, when n is even.

Case (ii): n is odd

Then $n = 2l+1$, for some l .

We label the vertices of $\langle B_{n,n}, w \rangle$ as follows:

$$f(u) = k,$$

$$f(w) = 4n+k+4$$

$$f(v) = 4n+k+2,$$

$$f(u_i) = 4i+k-2; \text{ if } 1 \leq i \leq 2l+1,$$

$$f(v_i) = 4i+k; \text{ if } 1 \leq i \leq 2l+1 \text{ and if } i \neq l+1,$$

$$f(v_{l+1}) = 4l+k+3.$$

Now the induced edge labels are

$$f^*(e) = 2n+k+2,$$

$$f^*(e'') = 4n+k+3,$$

$$f^*(e_i) = 2i+k-1; \text{ if } 1 \leq i \leq n$$

$$f^*(e_1) = f^*(e_n) + 4,$$

$$f^*(e_i) = f^*(e_1) + 2(i-1); 2 \leq i \leq n.$$

Here $p = 2n+3, q = 2n+2$.

$$\text{Clearly, } f(V) \cup \{f^*(e): e \in E(\langle B_{n,n}, w \rangle)\} = \{k, k+1, \dots, 4n+k+4\}.$$

So f is a k -super mean labeling, when n is odd.

Hence $\langle B_{n,n}, w \rangle$ is a k -super mean graph, when n is odd.

Example 2.3

50 - Super mean graph of $\langle B_{2,2}, w \rangle$ is given in figure 2.1:

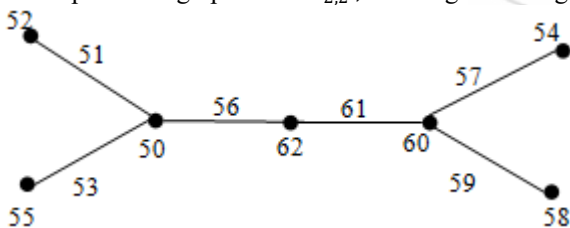


Figure 2.1: 50-SML of $\langle B_{2,2}, w \rangle$

Example 2.4

50 - Super mean graph of $\langle B_{3,3}, w \rangle$ is given in figure 2.2:

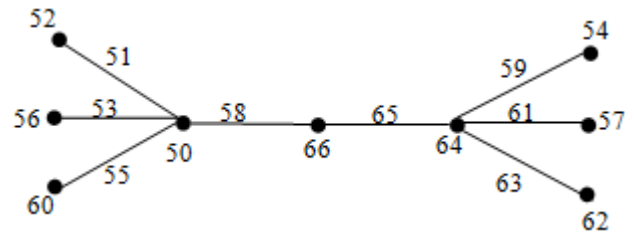


Figure 2.2: 50-SML of $\langle B_{3,3}, w \rangle$

Theorem 2.5

The graph $S(B_{n,n})$ is a k -Super mean graph for all $n \geq 2$.

Proof

Let $V(S(B_{n,n})) = \{u, v, w, u_i, v_i, u'_i, v'_i; 1 \leq i \leq n\}$ and

$$E(S(B_{n,n})) = \{e = (w, v)\} \cup \{e_i = (u, u_i); 1 \leq i \leq n\} \cup \{e_{n+1} = (u, w)\} \cup \{e_j = (v, v_i); 1 \leq i \leq n, n+2 \leq j \leq 2n+1\} \cup \{e'_i = (u_i, u'_i); 1 \leq i \leq n\} \cup \{e_i = (v_i, v'_i); 1 \leq i \leq n\}$$

be the vertices and edges of $S(B_{n,n})$ respectively.

Define

$$f: V(S(B_{n,n})) \rightarrow \{k, k+1, k+2, \dots, 8n+k+4\}$$

$$f(u) = k,$$

$$f(w) = 8n+k+4,$$

$$f(v) = 8n+k+2$$

$$f(u_i) = 8i+k-4; 1 \leq i \leq n,$$

$$f(u'_i) = 8i+k-6; 1 \leq i \leq n,$$

$$f(v_i) = 8i+k-2; 1 \leq i \leq n,$$

$$f(v'_i) = 8i+k; 1 \leq i \leq n.$$

Now the induced edge labels are

$$f^*(e) = 8n+k+3,$$

$$f^*(e_i) = 4i+k-3; 1 \leq i \leq 2n+1,$$

$$f^*(e'_i) = 8i+k-5; 1 \leq i \leq n,$$

$$f^*(e_i) = 8i+k-1; 1 \leq i \leq n.$$

Here $p = 4n+3, q = 4n+2$.

$$\text{Clearly, } f(V) \cup \{f^*(e): e \in E(S(B_{n,n}))\} = \{k, k+1, \dots, 8n+k+4\}.$$

So f is a k -super mean labeling.

Hence $S(B_{n,n})$ is a k -super mean graph for $n \geq 2$.

Example 2.6

25-Super mean labeling of $S(B_{5,5})$ is given in figure 2.3:

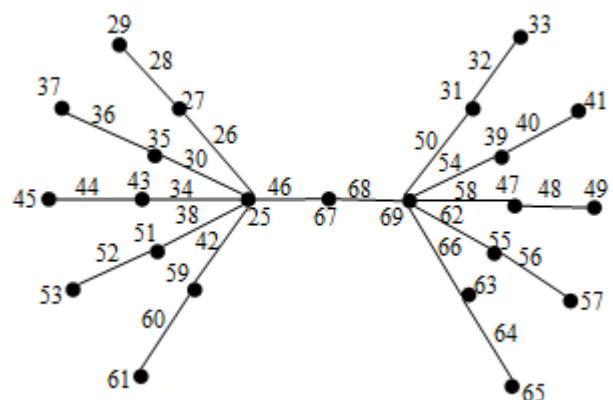


Figure 2.3: 25-SML of $S(B_{5,5})$

Theorem 2.7

The graph $T_n \odot K_1$ is a k-Super mean graph, for $n \geq 2$.

Proof:

$$\begin{aligned} \text{Let } V(T_n \odot K_1) &= \{u_i, u'_i; 1 \leq i \leq n-1\} \cup \\ &\quad \{v_i, v'_i; 1 \leq i \leq n\} \text{ and} \\ E(T_n \odot K_1) &= \{e_i = (u_i, u'_i); 1 \leq i \leq n-1\} \cup \\ &\quad \{e'_i = (v_i, u'_i); 1 \leq i \leq n-1\} \cup \\ &\quad \{e''_i = (v_{i+1}, u'_i); 1 \leq i \leq n-1\} \cup \\ &\quad \{e'''_i = (v_i, v_{i+1}); 1 \leq i \leq n-1\} \cup \\ &\quad \{e^{iv}_i = (v_i, v'_i); 1 \leq i \leq n\} \end{aligned}$$

be the vertices and edges of $T_n \odot K_1$ respectively.

Define

$$f: V(T_n \odot K_1) \rightarrow \{k, k+1, k+2, \dots, 9n+k-7\} \text{ by}$$

$$f(u_i) = 9i+k-3; 1 \leq i \leq n-1,$$

$$f(u'_i) = 9i+k-5; 1 \leq i \leq n-1,$$

$$f(v_i) = 9i+k-7; 1 \leq i \leq n,$$

$$f(v'_i) = 9i+k+9; 1 \leq i \leq n.$$

Now the induced edge labels are

$$f^*(e_i) = 9i+k-4; 1 \leq i \leq n-1,$$

$$f(e'_i) = 9i+k-6; 1 \leq i \leq n-1,$$

$$f(e''_i) = 9i+k-1; 1 \leq i \leq n-1,$$

$$f(e'''_i) = 9i+k-2; 1 \leq i \leq n-1,$$

$$f(e^{iv}_i) = 9i+k-8; 1 \leq i \leq n.$$

Here $p = 4n-2, q = 5n-4$.

$$\text{Clearly, } f(V) \cup \{f^*(e): e \in E(T_n \odot K_1)\} = \{k, k+1, \dots, 9n+k-7\}.$$

So f is a k -super mean labeling.

Hence $T_n \odot K_1$ is a k -super mean graph.

Example 2.8

50-Super mean labeling of $T_4 \odot K_1$ is given in figure 2.4:

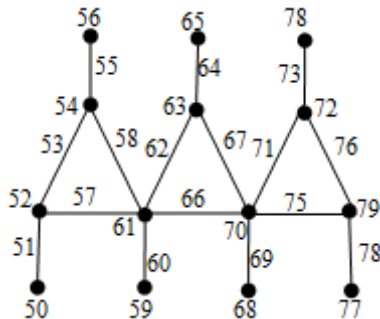


Figure 2.6: 50-SML of $T_4 \odot K_1$

[6] B. Gayathri and M. Tamilselvi, k-super mean labeling of some trees and cycle related graphs, Bulletin of Pure and Applied Sciences, Volume 26E(2) (2007) 303-311.

[7] F. Harary, Graph Theory, Addison Wesley, Massachusetts (1972).

[8] P. Jeyanthi and D. Ramya, Super mean labeling of some classes of graphs, International J. Math. Combin., 1 (2012) 83-91.

[9] P. Jeyanthi, D. Ramya and P. Thangavelu, On super mean graphs, AKCE J. Graphs Combin., 6 No. 1 (2009) 103-112.

[10] D. Ramya, R. Ponraj and P. Jeyanthi, Super mean labeling of graphs, ArsCombin., 112 (2013) 65-72.

[11] Rosa, On certain valuations of the vertices of a graph Theory of Graphs (Internet Symposium, Rome, July (1966), Gordon and Breach, N.Y. and Duhod, Paris (1967) 349-355.

[12] S. Somasundaram and R. Ponraj, Mean labeling of graphs, National Academy Science Letter, 26 (2003), 210-213.

[13] P.Sugirtha, R. Vasuki and J. Venkateswari, Some new super mean graphs, International Journal of Mathematics Trends and Technology, Vol. 19 No. 1 March 2015.

[14] M. Tamilselvi, A study in Graph Theory-Generalization of super mean labeling, Ph.D. Thesis, Vinayaka Mission University, Salem, August (2011).

[15] R. Vasuki and A. Nagarajan, Some results on super mean graphs, International J. Math. Combin., 3 (2009) 82-96.

References

[1] G.S. Bloom, S.W. Golomb, Applications of numbered undirected graphs, Proc. IEEE, 65 (1977), 562-570.

[2] G.S. Bloom, S.W. Golomb, Numbered complete graphs unusual rulers and assorted applications, Theory and Applications of Graphs-Lecture notes in Math., Springer Verlag, New York, 642 (1978), 53-65.

[3] G.S. Bloom, D.F. Hsu, On graceful digraphs and a problem in network addressing, CongressusNumerantium, 35 (1982) 91-103.

[4] J.A. Gallian, A dynamic survey of graph labeling, Electronic Journal of Combinatorics, 18 (2015) # DS6.

[5] B. Gayathri, M. Tamilselvi, M. Duraisamy, k-super mean labeling of graphs, In: Proceedings of the International Conference on Mathematics and Computer Sciences, Loyola College, Chennai (2008), 107-111.