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k–Super Mean Labeling of Some Graphs

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Abstract: *Let G be a (p, q) graph and $f: V(G) \to \{1,2,3,\dots,p+q\}$ be an injection. For each edge e = uv, let $f^*(e) = \frac{f(u)+f(v)}{2}$ if $f(u)+f(v)$ is even and $f^*(e) = \frac{f(u)+f(v)+1}{2}$ if $f(u)+f(v)$ is odd, then f is called super mean labeling if $f(V) \cup \{f^*(e): e \in E(G)\} = \{1,2,3,\dots,p+q\}$. A graph that admits a super mean labeling is called Super mean graph. Let G be a (p, q) graph and $f: V(G) \to \{k, k+1, k+2, \dots, p+q+k-1\}$ be an injection. For each edge e = uv, let $f^*(e) = \frac{f(u)+f(v)}{2}$ if $f(u)+f(v)$ is even and $f^*(e) = \frac{f(u)+f(v)+1}{2}$ if $f(u)+f(v)$ is odd, then f is called k- super mean labeling if $f(V) \cup \{f^*(e): e \in E(G)\} = \{k, k+1, \dots, p+q+k-1\}$. A graph that admits a k-super mean labeling is called k-Super mean graph. In this paper we investigate k – super mean labeling of <$B_{n,n}$, w>, S($B_{n,n}$), and $T_n \odot K_1$.*

Keywords: Super mean labeling, Super mean graph,k–Super mean labeling, k–Super mean graph, Bistar graphs, $T_n \odot K_1$.

1. Introduction

All graphs in this paper are finite, simple and undirected. Terms not defined here are used in the sense of Harary [7]. The symbols V(G) and E(G) will denote the vertex set and edge set of a graph G.

Graph labeling was first introduced in the late 1960's. Many studies in graph labeling refer to Rosa's research in 1967 [11]. Labeled graphs serve as useful models for a broad range of applications such as X-ray, crystallography, radar, coding theory, astronomy, circuit design and communication network addressing. Particularly interesting applications of graph labeling can be found in [1-4].

The concept of mean labeling was introduced and studied by S. Somasundaram and R.Ponraj [12].

The concept of super mean labeling was introduced and studied by D. Ramya et al. [11]. Futher some results on super mean graphs are discussed in [8,9,10,13,15].

B. Gayathri and M. Tamilselvi [5-6, 14] extended super mean labeling to k-super mean labeling.

In this paper we investigate k – Super mean labeling of Bistar graphs and $T_n \odot K_1$. For brevity, we use k-SML for k-super mean labeling.

Definition1.1
Let G be a (p,q) graph and $f: V(G) \to \{1,2,3,\dots,p+q\}$ be an injection. For each edge e = uv, let $f^*(e) = \frac{f(u)+f(v)}{2}$ if $f(u)+f(v)$ is even and $f^*(e) = \frac{f(u)+f(v)+1}{2}$ if $f(u)+f(v)$is odd, then f is called **Super mean labeling** if $f(V) \cup \{f^*(e): e \in E(G)\} = \{1,2,3,\dots,p+q\}$. A graph that admits a super mean labeling is called **Super mean graph.**

Definition1.2
Let G be a (p,q) graph and $f: V(G) \to \{k, k+1, k+2, \dots, p+q+k-1\}$be an injection. For each edge e = uv, let $f^*(e) = \frac{f(u)+f(v)}{2}$ if $f(u)+f(v)$ is even and $f^*(e) = \frac{f(u)+f(v)+1}{2}$ if $f(u)+f(v)$is odd, then f is called **k-Super mean labeling** if $f(V) \cup \{f^*(e): e \in E(G)\} = \{k, k+1, \dots, p+q+k-1\}$. A graph that admits a k-super mean labeling is called **k-Super mean graph**.

Definition1.3
A bistar ($B_{n,n}$) is the graph obtained by joining the central vertices of two copies of star $K_{1,n}$ by an edge.

Definition1.4
If G is a graph, then S(G) is a graph obtained by subdividing each edge of G by a vertex.

Definition1.5
A triangular snake (T_n) is obtained from a path by identifying each edge of the path with an edge of the cycle C_3.

2. Main Results

Definition 2.1
The graph $< B_{n,n}, w >$is the graph obtained by subdividing the central edge of the bistar$B_{n,n}$ with the vertex w.

Theorem 2.2
The graph $< B_{n,n}, w >$ is a k-super mean graph for all $n \geq 2$.

Proof
Let $V(< B_{n,n}, w >) = \{u, v, w, u_i, v_i; 1 \leq i \leq n\}$ and

$$E(< B_{n,n}, w >) = \{e = (u, w)\} \cup \{e'' = (w, v)\} \cup \{e_i = (u, u_i): 1 \leq i \leq n\} \cup \{e_i' = (v, v_i): 1 \leq i \leq n\}$$

be the vertices and edges of $< B_{n,n}, w >$ respectively.
Case (i): n is even
Let n = 2*l*, for some *l*.

Define

$f: V(< B_{n,n}, w >) \rightarrow \{k, k+1, k+2, \ldots, 4n+k+5\}$by

$f(u) = k,$

$f(w) = 4n + k + 4,$

$f(v) = 4n + k + 2$

$f(u_i) = 4i + k - 2;\ if\ 1 \leq i \leq 2l\ and\ if\ i \neq l+1,$

$f(u_{l+1}) = 4l + k + 1,$

$f(v_i) = 4i + k; if\ 1 \leq i \leq 2l.$

Now the induced edge labels are

$f^*(e) = 2n + k + 2,$

$f^*(e'') = 4n + k + 3,$

$f^*(e_i) = 2i + k - 1; 1 \leq i \leq n,$

$f^*(e_1') = f^*(e_n) + 4,$

$f^*(e_i') = f^*(e_1') + 2(i-1);\ 2 \leq i \leq n.$

Here p = 2n+3, q = 2n+2.

Clearly, $f(V) \cup \{f^*(e): e \in E(< B_{n,n}, w >)\} = \{k, k+1, \ldots, 4n+k+4\}.$

So f is a k – super mean labeling, when n is even.

Hence $< B_{n,n}, w >$ is a k–super mean graph, when n is even.

Case (ii): n is odd

Then n = 2*l* + 1, for some *l*.

We label the vertices of $< B_{n,n}, w >$ as follows:

$f(u) = k,$

$f(w) = 4n + k + 4$

$f(v) = 4n + k + 2,$

$f(u_i) = 4i + k - 2;\ if\ 1 \leq i \leq 2l+1,$

$f(v_i) = 4i + k; if\ 1 \leq i \leq 2l+1\ and\ if\ i \neq l+1,$

$f(v_{l+1}) = 4l + k + 3.$

Now the induced edge labels are

$f^*(e) = 2n + k + 2,$

$f^*(e'') = 4n + k + 3,$

$f^*(e_i) = 2i + k - 1; if\ 1 \leq i \leq n$

$f^*(e_1') = f^*(e_n) + 4,$

$f^*(e_i') = f^*(e_1') + 2(i-1);\ 2 \leq i \leq n.$

Here p = 2n+3, q = 2n+2.

Clearly, $f(V) \cup \{f^*(e): e \in E(< B_{n,n}, w >)\} = \{k, k+1, \ldots, 4n+k+4\}.$

So f is a k – super mean labeling, when n is odd.

Hence $< B_{n,n}, w >$ is a k – super mean graph, when n is odd.

Example 2.3

50 - Super mean graph of $< B_{2,2}, w >$ is given in figure 2.1:

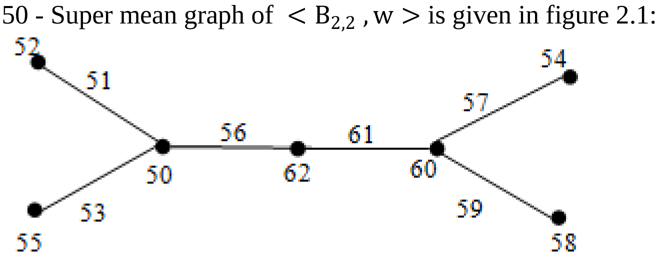

Figure 2.1: 50-SML of $< B_{2,2}, w >$

Example 2.4

50 - Super mean graph of $< B_{3,3}, w >$ is given in figure 2.2:

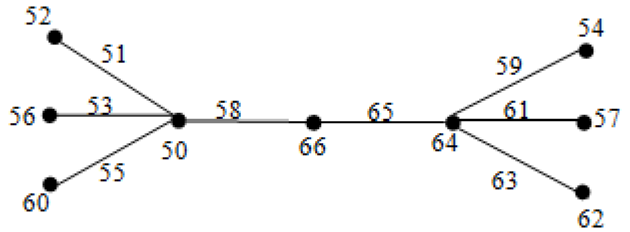

Figure 2.2: 50-SML of $< B_{3,3}, w >$

Theorem 2.5

The graph $S(B_{n,n})$ is a k-Super mean graph for all $n \geq 2$.

Proof

Let$V(S(B_{n,n})) = \{u, v, w, u_i, v_i, u_i', v_i'; 1 \leq i \leq n\}$and

$E(S(B_{n,n})) = \{e = (w, v)\} \cup$
$\{e_i = (u, u_i); 1 \leq i \leq n\} \cup$
$\{e_{n+1} = (u, w)\} \cup$
$\{e_j = (v, v_i); 1 \leq i \leq n, n+2 \leq j \leq 2n+1\} \cup$
$\{e_i' = (u_i, u_i'): 1 \leq i \leq n\} \cup$
$\{e_i'' = (v_i, v_i'): 1 \leq i \leq n\}$

be the vertices and edges of $S(B_{n,n})$respectively.

Define

$f: V(S(B_{n,n})) \rightarrow \{k, k+1, k+2, \ldots, 8n+k+4\}$by

$f(u) = k,$

$f(w) = 8n + k + 4,$

$f(v) = 8n + k + 2$

$f(u_i') = 8i + k - 4;\ 1 \leq i \leq n,$

$f(u_i) = 8i + k - 6;\ 1 \leq i \leq n,$

$f(v_i) = 8i + k - 2;\ 1 \leq i \leq n,$

$f(v_i') = 8i + k;\ 1 \leq i \leq n.$

Now the induced edge labels are

$f^*(e) = 8n + k + 3,$

$f^*(e_i) = 4i + k - 3; 1 \leq i \leq 2n+1,$

$f^*(e_i') = 8i + k - 5;\ 1 \leq i \leq n,$

$f^*(e_i'') = 8i + k - 1;\ 1 \leq i \leq n.$

Here p = 4n+3, q = 4n+2.

Clearly, $f(V) \cup \{f^*(e): e \in E(S(B_{n,n}))\} = \{k, k+1, \ldots, 8n+k+4\}.$

So f is a k – super mean labeling.

Hence$S(B_{n,n})$is a k – super mean graphfor$n \geq 2$.

Example 2.6

25-Super mean labeling of $S(B_{5,5})$ is given in figure2.3:

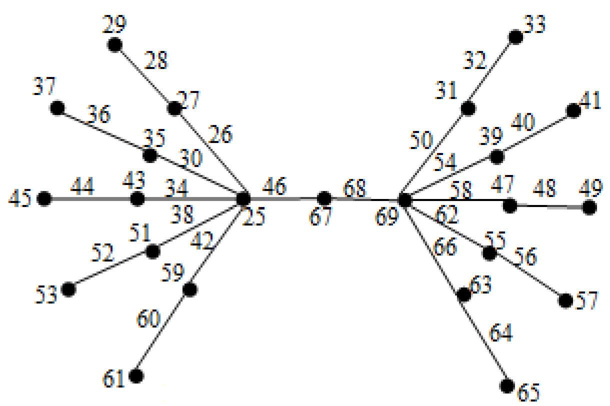

Figure 2.3: 25- SML of $S(B_{5,5})$

Theorem 2.7

The graph $T_n \odot K_1$ is a k-Super mean graph, for $n \geq 2$.

Proof:

Let $V(T_n \odot K_1) = \{u_i, u_i', ; 1 \leq i \leq n-1\} \cup \{v_i, v_i'; 1 \leq i \leq n\}$ and

$$E(T_n \odot K_1) = \{e_i = (u_i, u_i'); 1 \leq i \leq n-1\} \cup \{e_i' = (v_i, u_i): 1 \leq i \leq n-1\} \cup \{e_i'' = (v_{i+1}, u_i'): 1 \leq i \leq n-1\} \cup \{e_i''' = (v_i, v_{i+1}): 1 \leq i \leq n-1\} \cup \{e_i^{iv} = (v_i, v_i'): 1 \leq i \leq n\}$$

be the vertices and edges of $T_n \odot K_1$ respectively.

Define

$f: V(T_n \odot K_1) \rightarrow \{k, k+1, k+2, \dots, 9n+k-7\}$ by

$f(u_i') = 9i + k - 3;\ 1 \leq i \leq n-1,$

$f(u_i) = 9i + k - 5;\ 1 \leq i \leq n-1,$

$f(v_i) = 9i + k - 7;\ 1 \leq i \leq n,$

$f(v_i') = 9i + +9k;\ 1 \leq i \leq n.$

Now the induced edge labels are

$f^*(e_i) = 9i + k - 4; 1 \leq i \leq n-1,$

$f(e_i') = 9i + k - 6;\ 1 \leq i \leq n-1,$

$f(e_i'') = 9i + k - 1;\ 1 \leq i \leq n-1,$

$f(e_i''') = 9i + k - 2;\ 1 \leq i \leq n-1,$

$f(e_i^{iv}) = 9i + k - 8;\ 1 \leq i \leq n.$

Here p = 4n-2, q = 5n-4.

Clearly, $f(V) \cup \{f^*(e): e \in E(T_n \odot K_1)\} = \{k, k+1, \dots, 9n+k-7\}.$

So f is a k – super mean labeling.

Hence $T_n \odot K_1$ is a k – super mean graph.

Example 2.8

50-Super mean labeling of $T_4 \odot K_1$ is given in figure 2.4:

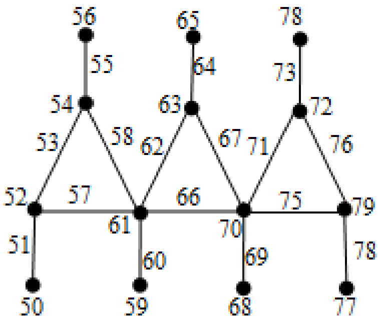

Figure 2.6: 50-SMLof $T_4 \odot K_1$

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