International Journal of Science and Research (IJSR)
ISSN (Online): 2319-7064
Index Copernicus Value (2013): 6.14 | Impact Factor (2015): 6.391

Some More Results on k – Super Mean Graphs

Dr. M. Tamilselvi[1], K. Akilandeswari[2], N. Revathi[3]

[1]Associate Professor, Department of Mathematics, Seethalakshmi Ramaswami College, Tirchirappalli – 620002

[2]Research Scholar, Department of Mathematics, Seethalakshmi Ramaswami College, Tirchirappalli – 620002

[3]Lecturer, Department of Mathematics, Seethalakshmi Ramaswami College, Tirchirappalli – 620002

Abstract: *Let G be a (p, q) graph and $f: V(G) \to \{1,2,3,\ldots,p+q\}$ be an injection. For each edge e = uv, let $f^*(e) = \frac{f(u)+f(v)}{2}$ if $f(u)+f(v)$ is even and $f^*(e) = \frac{f(u)+f(v)+1}{2}$ if $f(u)+f(v)$ is odd, then f is called super mean labeling if $f(V) \cup \{f^*(e): e \in E(G)\} = \{1,2,3,\ldots,p+q\}$. A graph that admits a super mean labeling is called Super mean graph. Let G be a (p, q) graph and $f: V(G) \to \{k, k+1, k+2, \ldots, p+q+k-1\}$ be an injection. For each edge e = uv, let $f^*(e) = \frac{f(u)+f(v)}{2}$ if $f(u)+f(v)$ is even and $f^*(e) = \frac{f(u)+f(v)+1}{2}$ if $f(u)+f(v)$ is odd, then f is called k-super mean labeling if $f(V) \cup \{f^*(e): e \in E(G)\} = \{k, k+1, \ldots, p+q+k-1\}$. A graph that admits a k-super mean labeling is called k-Super mean graph. In this paper we investigate k – super mean labeling of H_n, $H_n \odot mK_1$, $A(DT_n)$, $n(P_1 \odot \overline{K_2})$ and KP(r, s, l).*

Keywords: k-Super mean labeling, k-Super mean graph, H_n, $H_n \odot mK_1$, $A(DT_n)$, $n(P_1 \odot \overline{K_2})$ and *KP(r, s, l)*.

1. Introduction

All graphs in this paper are finite, simple and undirected. Terms not defined here are used in the sense of Harary [7]. The symbols V(G) and E(G) will denote the vertex set and edge set of a graph G.

Graph labeling was first introduced in the late 1960's. Many studies in graph labeling refer to Rosa's research in 1967 [11]. Labeled graphs serve as useful models for a broad range of applications such as X-ray, crystallography, radar, coding theory, astronomy, circuit design and communication network addressing. Particularly interesting applications of graph labeling can be found in [1-4].

The concept of mean labeling was introduced and studied by S. Somasundaram and R.Ponraj [12].

The concept of super mean labeling was introduced and studied by D. Ramya et al. [11]. Futher some results on super mean graphs are discussed in [8,9,10,13,15].

B. Gayathri and M.Tamilselvi [5-6,14] extended super mean labeling to k-super mean labeling.

In this paper we investigate k – Super mean labeling of $H_n, H_n \odot mK_1$, $A(DT_n)$, $n(P_1 \odot \overline{K_2})$and *KP(r,s,l)*. For brevity, we use k-SML for k-super mean labeling.

Definition1.1
Let G be a (p, q) graph and $f: V(G) \to \{1,2,3,\ldots,p+q\}$ be an injection. For each edge e = uv, let $f^*(e) = \frac{f(u)+f(v)}{2}$ if $f(u)+f(v)$ is even and $f^*(e) = \frac{f(u)+f(v)+1}{2}$ if $f(u)+f(v)$is odd, then f is called **Super mean labeling** if $f(V) \cup \{f^*(e): e \in E(G)\} = \{1,2,3,\ldots,p+q\}$. A graph that admits a super mean labeling is called **Super mean graph.**

Definition1.2
Let G be a (p, q) graph and $f: V(G) \to \{k, k+1, k+2, \ldots, p+q+k-1\}$be an injection. For each edge e = uv, let $f^*(e) = \frac{f(u)+f(v)}{2}$ if $f(u)+f(v)$ is even and $f^*(e) = \frac{f(u)+f(v)+1}{2}$ if $f(u)+f(v)$is odd, then f is called **k-Super mean labeling** if $f(V) \cup \{f^*(e): e \in E(G)\} = \{k, k+1, \ldots, p+q+k-1\}$.
A graph that admits a k-super mean labeling is called
k-Super mean graph.

Definition 1.3
The H – graph of a path P_n, denoted by H_n is the graph obtained from two copies of P_n with vertices $v_1, v_2, \ldots, v_n$ and $u_1, u_2, \ldots, u_n$ by joining the vertices $v_{\frac{n+1}{2}}$ and $u_{\frac{n+1}{2}}$; if n is odd and the vertices $v_{\frac{n}{2}+1}$and $u_{\frac{n}{2}}$; if n is even.

Definition 1.4
A triangular snake (T_n) is obtained from a path $v_1, v_2, \ldots, v_n$ by joining v_i and v_{i+1} to a new vertices w_i for i = 1,2,…,n-1.

Definition 1.5
A double triangular snake (DT_n) consists of two triangular snake that have a common path. That is, a double triangular snake is obtained from a path $v_1, v_2, \ldots, v_n$ by joining v_i and v_{i+1} to a new vertices w_i for i = 1,2,…,n-1and to a new vertices u_i for i = 1,2,…,n-1.

Definition 1.6
A double alternate triangular snake $AD(T_n)$ consists of alternate triangular snake that have a common path. That is, double triangular snake is obtained from the path $u_1, u_2, \ldots, u_n$ by joining u_i and u_{i+1} (alternatively) to two new vertices v_i and w_i.

Definition 1.7
If G has order n, the corona of G with H, $G \odot H$ is the graph obtained by taking one copy of G and n copies of H and joining the i th vertex of G with an edge to every vertex in the i th copy of H.

Definition 1.8
For any graph G, the graph mG denotes the disjoint union of m copies of G.

Definition 1.9
A Kayak paddle KP(r, s, l) is obtained from two cycles C_r and C_s that are joined together by a path of length l.

2. Main Results

Theorem 2.1
The graph H_n is a k - Super mean graph for $n \geq 1$.

Proof
Let $V(H_n) = \{u_i; 1 \leq i \leq n\} \cup \{v_i; 1 \leq i \leq n\}$
be the vertices of H_n and

$$E(H_n) = \left\{ e = \begin{Bmatrix} (v_{\frac{n+1}{2}}, u_{\frac{n+1}{2}});\ if\ n\ is\ odd \\ (v_{\frac{n}{2}+1}, u_{\frac{n}{2}});\ if\ n\ is\ even \end{Bmatrix} \right\} \cup$$

$$\{e_i = (u_i, u_{i+1}); 1 \leq i \leq n-1\} \cup$$
$$\{e_i' = (v_i, v_{i+1}); 1 \leq i \leq n-1\}$$

be the edges of H_n.
Define $f: V(H_n) \rightarrow \{k, k+1, k+2, \ldots, 4n+k-2\}$ by
$f(u_i) = 2i + k - 2; 1 \leq i \leq n,$
$f(v_i) = 2n + 2i + k - 2; 1 \leq i \leq n.$
Now the induced edge labels as follows:
$f^*(e_i) = 2i + k - 1; 1 \leq i \leq n-1,$
$f^*(e_i') = 2n + 2i + k - 1; 1 \leq i \leq n-1,$

$$f^*(e) = \begin{cases} 2n + k - 1, if\ n\ is\ odd \\ 2n + k - 1, if\ n\ is\ even \end{cases}$$

Here p = 2n and q = 2n-1.
Clearly, $f(V) \cup \{f^*(e): e \in E(H_n)\} = \{k, k+1, k+2, \ldots, 4n+k-2\}.$
So f is a k – super mean labeling.
Hence H_n is a k – super mean graph.

Example 2.2

25-super mean graph of H_5 is given in figure 2.1:

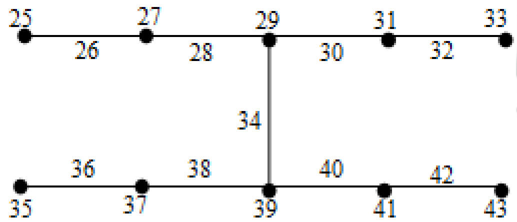

Figure 2.1: 25-SML of H_5

Theorem 2.3
The $H_n \odot mK_1$ is a k – super mean graph for all positive integers $m \geq 1$ and $n \geq 1$.

Proof
Let $V(H_n \odot mK_1) = \{u_i; 1 \leq i \leq n\} \cup$
$\{v_i; 1 \leq i \leq n\} \cup$
$\{x_{i,l};\ 1 \leq l \leq m, 1 \leq i \leq n\} \cup$
$\{y_{i,l};\ 1 \leq l \leq m, 1 \leq i \leq n\}$
be the vertices of $H_n \odot mK_1$.

$$E(H_n \odot mK_1) = \left\{ e = \begin{Bmatrix} (v_{\frac{n+1}{2}}, u_{\frac{n+1}{2}});\ if\ n\ is\ odd \\ (v_{\frac{n}{2}+1}, u_{\frac{n}{2}});\ if\ n\ is\ even \end{Bmatrix} \right\} \cup$$

$$\{e_i = (u_i, u_{i+1}); 1 \leq i \leq n-1\} \cup$$
$$\{e_i' = (v_i, v_{i+1}); 1 \leq i \leq n-1\} \cup$$
$$\{e_{i,l} = (u_i, x_{i,l});\ 1 \leq l \leq m, 1 \leq i \leq n\} \cup$$
$$\{e_{i,l}' = (v_i, y_{i,l});\ 1 \leq l \leq m, 1 \leq i \leq n\}$$

be the edges of $H_n \odot mK_1$.
Define
$f: V(H_n \odot mK_1) \rightarrow \{k, k+1, \ldots, 4n(m+1)+k-2\}$ by
For $1 \leq i \leq n,$

$$f(u_i) = \begin{cases} 2(m+1)(i-1) + k; i\ is\ odd \\ 2(m+1)i + k - 2; i\ is\ even \end{cases}$$

$$f(v_i) = \begin{cases} f(u_i) + 2n(m+1) + 2m; i\ is\ odd\ and\ n\ is\ odd \\ f(u_i) + 2n(m+1) - 2m; i\ is\ even\ and\ n\ is\ odd \\ f(u_i) + 2n(m+1); n\ is\ even \end{cases}$$

For $1 \leq i \leq n\ and\ 1 \leq l \leq m,$

$$f(x_{i,l}) = \begin{cases} 2(m+1)(i-1) + 4l + k - 2; i\ is\ odd \\ 2(m+1)(i-2) + 4l + k; i\ is\ even \end{cases}$$

$$f(y_{i,l}) = \begin{cases} f(x_{i,l}) + 2n(m+1) - 2m; i\ is\ odd\ and\ n\ is\ odd \\ f(x_{i,l}) + 2n(m+1) + 2m; i\ is\ even\ and\ n\ is\ odd \\ f(x_{i,l}) + 2n(m+1); n\ is\ even \end{cases}$$

Now the induced edge labels are as follows:
For $1 \leq i \leq n-1,$
$f^*(e_i) = 2i(m+1) + k - 1,$
$f^*(e_i') = f^*(e_i) + 2n(m+1).$
For $1 \leq i \leq n\ and\ 1 \leq l \leq m,$
$f^*(e_{i,l}) = 2(m+1)(i-1) + k - 1 + 2l,$
$f^*(e_{i,l}') = f^*(e_{i,l}) + 2n(m+1),$
$f^*(e) = 2n(m+1) + k - 1; if\ n\ is\ odd,$
$f^*(e) = 2n(m+1) + k - 1; if\ n\ is\ even.$
Here p = 2n(m+1) and q = 2n(m+1)-1.
Clearly,
$f(V) \cup \{f^*(e): e \in E(H_n \odot mK_1)\} = \{k, k+1, \ldots, 4n(m+1) + k - 2\}.$
So f is a k – super mean labeling.
Hence $H_n \odot mK_1$ is a k – super mean graph.

Example 2.4

50-super mean graph of $H_4 \odot 2K_1$ is given in figure 2.2:

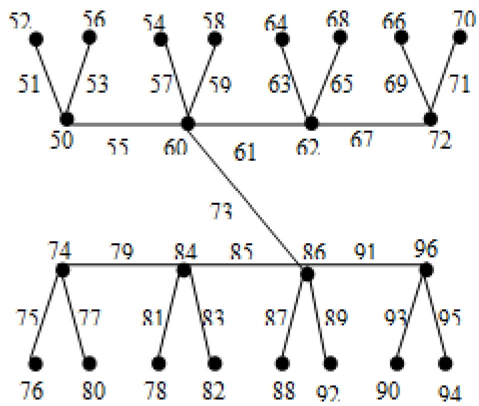

Figure 2.2: 50-SML of $H_4 \odot 2K_1$

Paper ID: NOV164767 http://dx.doi.org/10.21275/v5i6.NOV164767

Theorem 2.5
The graph AD(T_n) is a k-Super mean graph, for n ≥ 2.

Proof
Let $V(AD(T_n)) = \left\{u_i, u_i', v_i, w_i; 1 \le i \le \frac{n}{2}\right\}$, and

$$E(AD(T_n)) = \{e_i = (u_i, u_i'); 1 \le i \le n-1\} \cup \{e_i' = (v_i, u_i); 1 \le i \le \tfrac{n}{2}\} \cup \{e_i'' = (v_i, u_i'); 1 \le i \le \tfrac{n}{2}\} \cup \{e_i''' = (u_i, w_i); 1 \le i \le \tfrac{n}{2}\} \cup \{e_i^{iv} = (u_i', w_i); 1 \le i \le \tfrac{n}{2}\}$$

be the vertices and edges of AD(T_n) respectively.
Define
$f: V(AD(T_n)) \to \{k, k+1, k+2, \dots, 5n+k-2\}$ by
$f(u_i) = 10i + k - 10; 1 \le i \le \frac{n}{2}$,
$f(u_i') = 10i + k - 2; 1 \le i \le \frac{n}{2}$,
$f(u_i) = 10i + k - 8; 1 \le i \le \frac{n}{2}$,
$f(w_i) = 10i + k - 4; 1 \le i \le \frac{n}{2}$.
Now the induced edge labels are as follows:
$f^*(e_i) = 5i + k - 1; 1 \le i \le n-1$,
$f^*(e_i') = 10i + k - 9; 1 \le i \le \frac{n}{2}$,
$f^*(e_i'') = 10i + k - 5; 1 \le i \le \frac{n}{2}$,
$f^*(e_i''') = 10i + k - 7; 1 \le i \le \frac{n}{2}$,
$f^*(e_i^{iv}) = 10i + k - 3; 1 \le i \le \frac{n}{2}$.
Here p =2n and q = 3n-1.
Clearly,
$f(V) \cup \{f^*(e): e \in E(AD(T_n))\} = \{k, k+1, \dots, 5n+k-2\}$.
So f is a k – super mean labeling.
Hence $AD(T_n)$ is a k – super mean graph, for n ≥ 2.

Example 2.6
40-super mean graph of $AD(T_6)$ is given in figure 2.3:

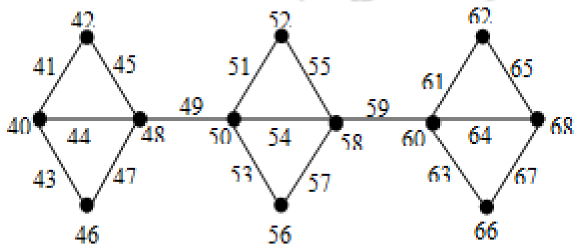

Figure 2.3: 40-SML of AD(T_6)

Theorem 2.7
The graph $n(P_2 \odot \overline{K_2})$ is a k-Super mean graph, for $n \ge 1$.

Proof
Let $V(n(P_2 \odot \overline{K_2})) = \{v_i, v_i', v_i''; 1 \le i \le n\} \cup \{u_i, u_i', u_i''; 1 \le i \le n\}$ and

$$E(n(P_2 \odot \overline{K_2})) = \{e_i = (v_i, v_i'); 1 \le i \le n\} \cup \{e_i' = (v_i, v_i''); 1 \le i \le n\} \cup \{e_i'' = (v_i'', u_i''); 1 \le i \le n\} \cup \{e_i''' = (u_i', u_i); 1 \le i \le n\} \cup \{e_i^{iv} = (u_i'', u_i); 1 \le i \le n\}$$

be the vertices and edges of $n(P_2 \odot \overline{K_2})$ respectively.
Define
$f: V(n(P_2 \odot \overline{K_2})) \to \{k, k+1, k+2, \dots, 11n+k-1\}$ by
$f(v_i) = 11i + k - 11;\ 1 \le i \le n$,
$f(v_i') = 11i + k - 7;\ 1 \le i \le n$,
$f(v_i'') = 11i + k - 9;\ 1 \le i \le n$,
$f(u_i) = 11i + k - 5;\ 1 \le i \le n$,
$f(u_i') = 11i + k - 1;\ 1 \le i \le n$,
$f(u_i'') = 11i + k - 3;\ 1 \le i \le n$.
Now the induced edge labels are as follows:
$f^*(e_i) = 11i + k - 10; 1 \le i \le n$,
$f^*(e_i') = 11i + k - 8; 1 \le i \le n$,
$f^*(e_i'') = 11i + k - 6; 1 \le i \le n$,
$f^*(e_i''') = 11i + k - 4; 1 \le i \le n$,
$f^*(e_i^{iv}) = 11i + k - 2; 1 \le i \le n$.
Here p = 6n and q = 5n.
Clearly, $f(V) \cup \{f^*(e): e \in E(n(P_2 \odot \overline{K_2}))\} = \{k, k+1, \dots, 11n+k-1\}$.
So f is a k – super mean labeling.
Hence $n(P_2 \odot \overline{K_2})$ is a k – super mean graph, for $n \ge 1$.

Example 2.8
35-super mean graph of 4($P_2 \odot \overline{K_2}$) is given in figure 2.4:

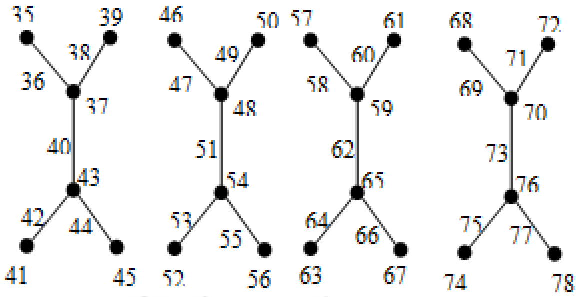

Figure 2.4: 35-SML of 4($P_2 \odot \overline{K_2}$)

Theorem 2.9
The graph *KP(r, s, l)* is a k–Super mean graph, for all $r, s \ge 3, r, s \ne 4$ and $l \ge 1$.

Proof
Case 1: r,s is odd
Let r, s = 2n+1 for all $n \ge 1$.
Let $V(KP(r,s,l)) = \{v_i, 1 \le i \le r\} \cup \{u_i, 1 \le i \le l+1\} \cup \{v_i', 1 \le i \le s\}$
be the vertices of *KP(r, s, l)* and
$E(KP(r,s,l)) = \{e_i = (v_i, v_{i+1}), 1 \le i \le r\} \cup \{e_i' = (v_i', v_{i+1}'), 1 \le i \le s\} \cup \{e_i'' = (u_i, u_{i+1}), 1 \le i \le l\}$
be the edges of *KP(r, s, l)*.
Define
$f: V(KP(r,s,l)) \to \{k, k+1, \dots., 2(r+s+l)+k-2\}$ by
$f(v_i) = 2i + k - 2; 1 \le i \le n+1$,
$f(v_{n+1+i}) = 2(n+1+i) + k - 1;\ 1 \le i \le n$,
$f(u_1) = f(v_r)$,
$f(u_i) = f(v_r) + 2(i-1);\ 2 \le i \le l+1$,
$f(v_1') = f(u_{l+1})$,
$f(v_i') = f(v_1') + 2(i-1);\ 2 \le i \le n+1$,
$f(v_{n+1+i}') = f(v_1') + 2(n+i+1);\ 1 \le i \le n$.
Now the induced edge labels are as follows:
$f^*(e_i) = 2i + k - 1;\ 1 \le i \le n$,
$f^*(e_{n+i}) = 2(n+i) + k;\ 1 \le i \le n$,
$f^*(e_{2n+1}) = k + 2n + 1$,
$f^*(e_i'') = f(v_r) + 2i - 1;\ 1 \le i \le l$,

$f^*(e_i^{'}) = f(v_1^{'}) + 2i - 1;\ 1 \le i \le n,$
$f^*(e_i^{'}) = f(v_1^{'}) + 2(n + i); 1 \le i \le n,$
$f^*(e_{2n+1}^{'}) = \ f(v_1^{'}) + 2n + 1.$
Here $p = r + s + l - 1, q = r + s + l$.
Clearly, $f(V) \cup \{f^*(e): e \in E(KP(r, s, l))\} =$
$\{k, k + 1, \ldots, 2(r + s + l) + k - 2\}.$
So f is a k – super mean labeling.
Hence *KP(r, s, l)* is a k – super mean graph, when r, s is odd.

Case 2: r, s is even
Let r, s = 2nfor all$n \ge 3$.
Let$V(KP(r, s, l)) = \{v_i, 1 \le i \le r\} \cup$
$\{u_i, 1 \le i \le l + 1\} \cup$
$\{v_i^{'}, 1 \le i \le s\}$
be the vertices of *KP(r, s, l)* and
$E(KP(r, s, l)) = \{e_i = (v_i, v_{i+1}), 1 \le i \le r\} \cup$
$\{e_i^{'} = (v_i^{'}, v_{i+1}^{'}), 1 \le i \le s\} \cup$
$\{e_i^{''} = (u_i, u_{i+1}), 1 \le i \le l\}$
be the edges of *KP(r, s, l)*.
Define
$f: V(KP(r, s, l)) \to \{k, k + 1, \ldots, 2(r + s + l) + k - 2\}$ by
$f(v_1) = k$
$f(v_i) = 4i + k - 6; 2 \le i \le n,$
$f(v_{n+j}) = 4n - 3j + k + 2;\ 1 \le j \le 2,$
$f(v_{n+j+2}) = 4(n - j) + k - 3;\ 1 \le j \le n - 2,$
$f(u_1) = f(v_{n+1}),$
$f(u_i) = f(v_{n+1}) + 2(i - 1);\ 2 \le i \le l + 1,$
$f(v_1^{'}) = f(u_{l+1}),$
$f(v_i^{'}) = f(v_1^{'}) + 4i - 6;\ 2 \le i \le n,$
$f(v_{n+j}^{'}) = f(v_1^{'}) + 4n - 3j + 2;\ 1 \le j \le 2,$
$f(v_{n+j+2}^{'}) = f(v_1^{'}) + 4(n - j) - 3;\ 1 \le j \le n - 2.$
Now the induced edge labels are as follows:
$f^*(e_1) = k + 1,$
$f^*(e_i) = k + 4(i - 1);\ 2 \le i \le n - 1,$
$f^*(e_n) = 4n + k - 3,$
$f^*(e_{n+1}) = 4n + k - 2,$
$f^*(e_{n+1+j}) = 4(n - j) + k - 1;\ 1 \le j \le n - 1,$
$f^*(e_i^{''}) = f(v_{n+1}) + 2i - 1;\ 1 \le i \le l,$
$f^*(e_1^{'}) = f(v_1^{'}) + 1,$
$f^*(e_i^{'}) = f(v_1^{'}) + 4(i - 1);\ 2 \le i \le n - 1,$
$f^*(e_n^{'}) = f(v_1^{'}) + 4n - 3,$
$f^*(e_{n+1}^{'}) = f(v_1^{'}) + 4n - 2,$
$f^*(e_{n+1+j}^{'}) = f(v_1^{'}) + 4(n - j) - 1;\ 1 \le j \le n - 1.$
Here $p = r + s + l - 1, q = r + s + l$.
Clearly,
$f(V) \cup \{f^*(e): e \in E(KP(r, s, l))\} =$
$\{k, k + 1, \ldots, 2(r + s + l) + k - 2\}.$
So f is a k – super mean labeling.
Hence *KP(r, s, l)* is a k – super mean graph, when r, s is even.

Example 2.10
10-super mean graph of KP(3,5,2) is given in figure 2.5:

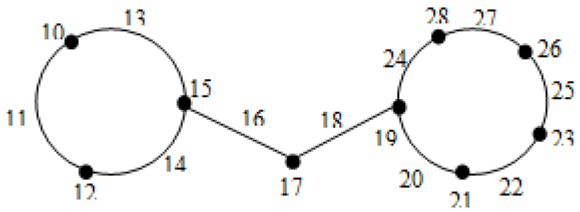

Figure 2.5: 10-SMLof KP(3,5,2)

Remark 2.11
KP(4,4,l) and *KP(r, 4 ,l)* are not k-super mean graph because C_4 is not a k-super mean graph.
Remark 2.12
KP(4, s ,l) is a k-super mean graph, for $s \ge 3, s \ne 4$ and $l \ge 2$.

Example 2.13
KP(4, 5, 2) is a 5-super mean graph is given in figure 2.6:

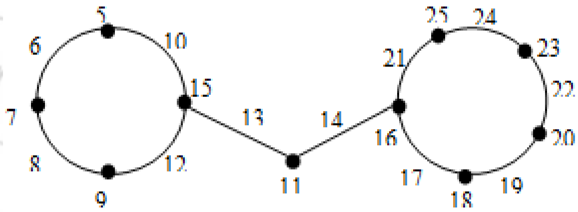

Figure 2.6: 5-SML of KP(4,5,2)

References

[1] G.S. Bloom, S.W. Golomb, Applications of numberedundirected graphs, Proc. IEEE, 65 (1977), 562-570.
[2] G.S. Bloom, S.W. Golomb, Numbered complete graphsunusual rulers and assorted applications, Theory andApplications of Graphs-Lecture notes in Math., Springer Verlag, New York, 642 (1978), 53-65.
[3] G.S. Bloom, D.F. Hsu, On graceful digraphs and a problem in network addressing, CongressusNumerantium, 35 (1982) 91-103.
[4] J.A. Gallian, A dynamic survey of graph labeling, Electronic Journal of Combinatorics, 18 (2015) # DS6.
[5] B. Gayathri, M. Tamilselvi, M. Duraisamy, k-super mean labeling of graphs, In: Proceedings of the International Conference on Mathematicsand Computer Sciences, Loyola College, Chennai (2008), 107-111.
[6] B. Gayathri and M. Tamilselvi, k-super mean labeling of some trees and cycle related graphs, Bulletin of Pure and Applied Sciences, Volume 26E(2) (2007) 303-311.
[7] F. Harary, Graph Theory, Addison Wesley, Massachusetts (1972).
[8] P. Jeyanthi and D. Ramya, Super mean labeling of some classes of graphs, International J. Math. Combin., 1 (2012) 83-91.
[9] P. Jeyanthi, D. Ramya and P. Thangavelu, On super mean graphs, AKCE J. Graphs Combin., 6 No. 1 (2009) 103-112.
[10] D. Ramya, R. Ponraj and P. Jeyanthi, Super mean labeling of graphs, ArsCombin., 112 (2013) 65-72.
[11] Rosa, On certain valuations of the vertices of a graph Theory of Graphs (Internet Symposium, Rome, July (1966), Gordon and Breach, N.Y. and Duhod, Paris (1967) 349-355.

[12] S. Somasundaram and R. Ponraj, Mean labeling of graphs, National Academy Science Letter, 26 (2003), 210-213.
[13] P.Sugirtha, R. Vasuki and J. Venkateswari, Some new super mean graphs, International Journal of Mathematics Trends and Technology, Vol. 19 No. 1 March 2015.
[14] M. Tamilselvi, A study in Graph Theory-Generalization of super mean labeling, Ph.D. Thesis, Vinayaka Mission University, Salem, August (2011).
[15] R. Vasuki and A. Nagarajan, Some results on super mean graphs, International J. Math. Combin., 3 (2009) 82-96.