Some More Results on $k$ – Super Mean Graphs

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Abstract: Let $G$ be a $(p, q)$ graph and $f: V(G) \rightarrow \{1, 2, 3, ..., p + q\}$ be an injection. For each edge $e = uv$, let $f^*(e) = \frac{f(u)+f(v)}{2}$ if $f(u) + f(v)$ is even and $f^*(e) = \frac{f(u)+f(v)+1}{2}$ if $f(u) + f(v)$ is odd, then $f$ is called super mean labeling if $f(V) \cup \{f^*(e): e \in E(G)\} = \{1, 2, 3, ..., p + q\}$. A graph that admits a super mean labeling is called Super mean graph. Let $G$ be a $(p, q)$ graph and $f: V(G) \rightarrow \{k, k+1, k+2, ..., p + q + k - 1\}$ be an injection. For each edge $e = uv$, let $f^*(e) = \frac{f(u)+f(v)}{2}$ if $f(u) + f(v)$ is even and $f^*(e) = \frac{f(u)+f(v)+1}{2}$ if $f(u) + f(v)$ is odd, then $f$ is called $k$-super mean labeling if $f(V) \cup \{f^*(e): e \in E(G)\} = \{k, k+1, ..., p + q + k - 1\}$. A graph that admits a $k$-super mean labeling is called $k$-Super mean graph. In this paper we investigate $k$ – super mean labeling of $H_n \circ KP(r, s, l)$ and $A(DT_4)$, $n(P_1 \circ K_2)$ and $KP(r, s, l)$.

Keywords: k-Super mean labeling, k-Super mean graph, $H_n \circ KP(r, s, l)$, $A(DT_4)$, $n(P_1 \circ K_2)$ and $KP(r, s, l)$.

1. Introduction

All graphs in this paper are finite, simple and undirected. Terms not defined here are used in the sense of Harary [7]. The symbols V(G) and E(G) will denote the vertex set and edge set of a graph G.

Graph labeling was first introduced in the late 1960’s. Many studies in graph labeling refer to Rosa’s research in 1967 [11]. Labeled graphs serve as useful models for a broad range of applications such as X-ray, crystallography, radar, coding theory, astronomy, circuit design and communication network addressing. Particularly interesting applications of graph labeling can be found in [1-4].

The concept of mean labeling was introduced and studied by S. Somasundaram and R.Ponraj [12].

The concept of super mean labeling was introduced and studied by D. Ramya et al. [11]. Further results on super mean graphs are discussed in [8,9,10,13,15].

B. Gayathri and M.Tamiilselvi [5-6,14] extended super mean labeling to k-super mean labeling.

In this paper we investigate k – Super mean labeling of $H_n \circ mK_1$, $A(DT_4)$, $n(P_1 \circ K_2)$ and $KP(r, s, l)$. For brevity, we use k-SML for k-super mean labeling.

Definition 1.1
Let $G$ be a $(p, q)$ graph and $f: V(G) \rightarrow \{1, 2, 3, ..., p + q\}$ be an injection. For each edge $e = uv$, let $f^*(e) = \frac{f(u)+f(v)}{2}$ if $f(u) + f(v)$ is even and $f^*(e) = \frac{f(u)+f(v)+1}{2}$ if $f(u) + f(v) is odd, then f is called Super mean labeling if $f(V) \cup \{f^*(e): e \in E(G)\} = \{1, 2, 3, ..., p + q\}$. A graph that admits a super mean labeling is called Super mean graph.

Definition 1.2
Let $G$ be a $(p, q)$ graph and $f: V(G) \rightarrow \{k, k+1, k+2, ..., p + q + k - 1\}$ be an injection. For each edge $e = uv$, let $f^*(e) = \frac{f(u)+f(v)}{2}$ if $f(u) + f(v)$ is even and $f^*(e) = \frac{f(u)+f(v)+1}{2}$ if $f(u) + f(v)$ is odd, then $f$ is called $k$-super mean labeling if $f(V) \cup \{f^*(e): e \in E(G)\} = \{k, k+1, ..., p + q + k - 1\}$. A graph that admits a $k$-super mean labeling is called $k$-Super mean graph.

Definition 1.3
The $H$ – graph of a path $P_n$, denoted by $H_n$, is the graph obtained from two copies of $P_n$ with vertices $v_1, v_2, ..., v_n$ and $u_1, u_2, ..., u_n$ by joining the vertices $v_{n+1}$ and $u_{n+1}$; if $n$ is odd and the vertices $v_{n+1}$ and $u_{n+1}$; if $n$ is even.

Definition 1.4
A triangular snake $(T_n)$ is obtained from a path $v_1, v_2, ..., v_n$ by joining $v_1$ and $v_{n+1}$ to a new vertices $w_i$ for $i = 1, 2, ..., n-1$.

Definition 1.5
A double triangular snake $(DT_n)$ consists of two triangular snake that have a common path. That is, a double triangular snake is obtained from a path $v_1, v_2, ..., v_n$ by joining $v_i$ and $v_{i+1}$ to a new vertices $w_i$ for $i = 1, 2, ..., n-1$ and to a new vertices $u_i$ for $i = 1, 2, ..., n-1$.

Definition 1.6
A double alternate triangular snake $AD(T_n)$ consists of alternate triangular snake that have a common path. That is, double triangular snake is obtained from the path $u_1, u_2, ..., u_n$ by joining $u_i$ and $u_{i+1}$ (alternatively) to two new vertices $v_i$ and $w_i$.

Definition 1.7
If $G$ has order $n$, the corona of $G$ with $H, G \circ H$ is the graph obtained by taking one copy of $G$ and $n$ copies of $H$ and joining the $i$ th vertex of $G$ with an edge to every vertex in the $i$ th copy of $H$. 

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Definition 1.8
For any graph G, the graph mG denotes the disjoint union of m copies of G.

Definition 1.9
A Kayak paddle KP(r, s, l) is obtained from two cycles C_r and C_s that are joined together by a path of length l.

2. Main Results

Theorem 2.1
The graph H_n is a k - Super mean graph for n ≥ 1.

Proof
Let V(H_n) = {u_i; 1 ≤ i ≤ n} ∪ {v_i; 1 ≤ i ≤ n} be the vertices of H_n and

\[ E(H_n) = \begin{cases} e = \left( \frac{u_{i+1}, v_{i+1}}{2}, \text{ if } n \text{ is odd} \right) \cup \left( \frac{u_{i+1}, u_i}{2}, \text{ if } n \text{ is even} \right) \cup e_i = (u_i, u_i+1); 1 ≤ i ≤ n-1 \} \cup e_i = (v_i, v_i+1); 1 ≤ i ≤ n-1 \} \]

be the edges of H_n ∩ mK_1.

Define f: V(H_n) → {k, k + 1, k + 2, ..., 4n + k - 2} by

f(u_i) = 2i + 2k - 2; 1 ≤ i ≤ n,
f(v_i) = 2n + 2i + 2k - 2; 1 ≤ i ≤ n.

Now the induced edge labels are as follows:

f^*(e_i) = 2i + 1 - k; 1 ≤ i ≤ n - 1, 
f^*(e_i) = 2n + 2i + 2k - 3; 1 ≤ i ≤ n - 1.

f^*(e) = \begin{cases} 2n + k - 1, \text{ if } n \text{ is odd} \right) \cup \left( 2n + k - 1, \text{ if } n \text{ is even} \right) 

Here p = 2n and q = 2n - 1.

Clearly, f(V) ∪ f^*(E): e ∈ E(H_n) = {k, k + 1, k + 2, ..., 4n + k - 2}.

So f is a k – super mean labeling.

Hence H_n is a k – super mean graph.

Example 2.2
25-super mean graph of H_3 is given in figure 2.1:

![Figure 2.1: 25-SML of H_3](image)

Theorem 2.3
The graph H_n ⊕ mK_1 is a k – super mean graph for all positive integers m ≥ 1 and n ≥ 1.

Proof
Let V(H_n ⊕ mK_1) = {u_i; 1 ≤ i ≤ n} ∪ {v_i; 1 ≤ i ≤ n} ∪ {x_i; 1 ≤ l ≤ m, 1 ≤ i ≤ n} ∪ {y_i; 1 ≤ l ≤ m, 1 ≤ i ≤ n} be the vertices of H_n ⊕ mK_1.
The graph $AD(T_4)$ is a $k$-Super mean graph, for $n \geq 2$.

Proof

Let $\mathcal{V}(AD(T_n)) = \{u_i, u_i', v_i, w_i; 1 \leq i \leq n\}$, and

$E(AD(T_n)) = \{e_i = (u_i, u_i'); 1 \leq i \leq n) \cup \{e_i' = (v_i, u_i); 1 \leq i \leq n) \cup \{e_i'' = (v_i, w_i); 1 \leq i \leq n) \cup \{e_i''' = (u_i, w_i); 1 \leq i \leq n) \}$

be the vertices and edges of $AD(T_n)$ respectively. Define

$f: V(AD(T_n)) \to \{k, k+1, k+2, ..., 5n + k - 2\}$ by

$f(u_i) = 10(i + 1 - 10); 1 \leq i \leq \frac{n}{2}$

$f(v_1) = 10(i + 1 - k - 2); 1 \leq i \leq \frac{n}{2}$

$f(u_i) = 10(i + 1 - k - 8); 1 \leq i \leq \frac{n}{2}$

$f(w_i) = 10(i + 1 - k - 4); 1 \leq i \leq \frac{n}{2}$

Now the induced edge labels are as follows:

$f^*(e_i) = 5i + k - 1; 1 \leq i \leq n - 1$

$f^*(e_i') = 10i + k - 9; 1 \leq i \leq \frac{n}{2}$

$f^*(e_i'') = 10i + k - 5; 1 \leq i \leq \frac{n}{2}$

$f^*(e_i''') = 10i + k - 7; 1 \leq i \leq \frac{n}{2}$

Here $p = 2n$ and $q = 3n - 1$. Clearly, $f(V) \cup \{f^*(e): e \in E(AD(T_n))\} = \{k, k + 1, ..., 5n + k - 2\}$.

So $f$ is a $k$–super mean labeling. Hence $AD(T_n)$ is a $k$–super mean graph, for $n \geq 2$.

Example 2.6

40-super mean graph of $AD(T_6)$ is given in figure 2.3.

Figure 2.3: 40-SML of $AD(T_6)$

Theorem 2.7

The graph $n(P_2 \odot K_2)$ is a $k$-Super mean graph, for $n \geq 1$.

Proof

Let $\mathcal{V}(n(P_2 \odot K_2)) = \{v_i, v_i', v_i'', 1 \leq i \leq n\} \cup \{u_i, u_i', u_i''; 1 \leq i \leq n\}$ and

$E(n(P_2 \odot K_2)) = \{e_i = (v_i, v_i'); 1 \leq i \leq n\} \cup \{e_i' = (v_i, v_i''); 1 \leq i \leq n\} \cup \{e_i'' = (v_i', u_i''); 1 \leq i \leq n\} \cup \{e_i''' = (u_i', u_i''); 1 \leq i \leq n\} \cup \{e_i'''' = (u_i', u_i'''); 1 \leq i \leq n\}$

be the vertices and edges of $n(P_2 \odot K_2)$ respectively. Define

$f: V(n(P_2 \odot K_2)) \to \{k, k+1, k+2, ..., 11n + k - 1\}$ by

$f(u_i) = 11i + k - 11; 1 \leq i \leq n$

$f(v_i) = 11i + k - 7; 1 \leq i \leq n$

$f(v_i') = 11i + k - 9; 1 \leq i \leq n$

$f(u_i') = 11i + k - 5; 1 \leq i \leq n$

$f(u_i'') = 11i + k - 1; 1 \leq i \leq n$

$f(u_i''') = 11i + k - 3; 1 \leq i \leq n$

Now the induced edge labels are as follows:

$f^*(e_i) = 11i + k - 10; 1 \leq i \leq n$

$f^*(e_i') = 11i + k - 8; 1 \leq i \leq n$

$f^*(e_i'') = 11i + k - 6; 1 \leq i \leq n$

$f^*(e_i''') = 11i + k - 4; 1 \leq i \leq n$

$f^*(e_i''''') = 11i + k - 2; 1 \leq i \leq n$

Here $p = 6n$ and $q = 5n$. Clearly, $f(V) \cup \{f^*(e): e \in E(n(P_2 \odot K_2))\} = \{k, k + 1, ..., 11n + k - 1\}$.

So $f$ is a $k$–super mean labeling. Hence $n(P_2 \odot K_2)$ is a $k$–super mean graph, for $n \geq 1$.

Example 2.8

35-super mean graph of $4(P_2 \odot K_2)$ is given in figure 2.4:

Figure 2.4: 35-SML of $4(P_2 \odot K_2)$

Theorem 2.9

The graph $KP(r, s, l)$ is a $k$–Super mean graph, for all $r, s \geq 3, r, s \neq 4$ and $l \geq 1$.

Proof

Case 1: $r, s$ is odd

Let $r, s = 2n + 1$ for all $n \geq 1$.

Let $\mathcal{V}(KP(r, s, l)) = \{v_i, 1 \leq i \leq r) \cup \{u_i, 1 \leq i \leq l + 1) \cup \{v_i, 1 \leq i \leq s\}$

be the vertices of $KP(r, s, l)$ and

$E(KP(r, s, l)) = \{e_i = (v_i, v_{i+1}), 1 \leq i \leq r\} \cup \{e_i' = (v_i, v_{i+1}), 1 \leq i \leq s\} \cup \{e_i'' = (u_i, u_{i+1}), 1 \leq i \leq l\}$

be the edges of $KP(r, s, l)$.

Define

$f: V(KP(r, s, l)) \to \{k, k + 1, ..., 2(r + s + l) + k - 2\}$ by

$f(u_i) = 2i + k - 2; 1 \leq i \leq n + 1$

$f(v_{n+1+i}) = 2(n + 1 + i) + k - 1; 1 \leq i \leq n$

$f(u_i) = f(v_{i+1}) + 2(i - 1); 2 \leq i \leq l + 1$

$f(v_i) = f(u_{i+1}) + 2(i - 1); 2 \leq i \leq n + 1$

$f(v_{n+1+i}) = f(v_i) + 2(n + i + 1); 1 \leq i \leq n$

Now the induced edge labels are as follows:

$f^*(e_i) = 2i + k - 1; 1 \leq i \leq n$

$f^*(e_{n+1+i}) = 2(n + i) + 2k; 1 \leq i \leq n$

$f^*(e_{2n+1+i}) = 2k + 2n + 1$

$f^*(e_i''') = f(u_i) + 2i - 1; 1 \leq i \leq l$,

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\[ f^*(e_i) = f(v_i) + 2i - 1; 1 \leq i \leq n, \]
\[ f^*(e_i) = f(v_i) + 2(n + 1); 1 \leq i \leq n, \]
\[ f^*(e_{2n+1}) = f(v_i) + 2n + 1. \]

Here \( p = r + s + l - 1 \), \( q = r + s + l \).

Clearly, \( f(V) \cup \{f^*(e): e \in E(KP(r,s,l))\} = \{k, k + 1, ..., 2(r + s + l) + k - 2\} \).

So \( f \) is a \( k \) – super mean labeling.

Hence \( KP(r, s, l) \) is a \( k \) – super mean graph, when \( r, s \) is even.

**Case 2: \( r, s \) is even**

Let \( r, s = 2n \) for all \( n \geq 3 \).

Let \( V(KP(r, s, l)) = \{v_i, 1 \leq i \leq r\} \cup \{u_i, 1 \leq i \leq l + 1\} \cup \{v_i, 1 \leq i \leq s\} \)

be the vertices of \( KP(r, s, l) \) and

\[ E(KP(r, s, l)) = \{e_i = (v_i, v_{i+1}), 1 \leq i \leq r\} \cup \{e_i = (v_i, v_{i+1}), 1 \leq i \leq s\} \cup \{e_i = (u_i, u_{i+1}), 1 \leq i \leq l\} \]

be the edges of \( KP(r, s, l) \).

Define

\[ f: V(KP(r, s, l)) \rightarrow \{k, k + 1, ..., 2(r + s + l) + k - 2\} \]

by

\[ f(v_i) = k \]
\[ f(v_i) = 4i + k - 6; 2 \leq i \leq n, \]
\[ f(v_{n+j}) = 4n - 3j + k + 2; 1 \leq j \leq 2, \]
\[ f(v_{n+j+2}) = 4(n - j) + k - 3; 1 \leq j \leq n - 2, \]
\[ f(u_i) = f(v_{n+1}) \]
\[ f(u_i) = f(v_{n+1}) + 2(i - 1); 2 \leq i \leq l + 1, \]
\[ f(v_i) = f(u_i) + 1 \]
\[ f(v_i) = f(v_i) + 4i - 6; 2 \leq i \leq n, \]
\[ f(v_{n+j}) = f(v_i) + 4n - 3j + 2; 1 \leq j \leq 2, \]
\[ f(v_{n+j+2}) = f(v_i) + 4(n - j) - 3; 1 \leq j \leq n - 2. \]

Now the induced edge labels are as follows:

\[ f^*(e_i) = k + 1, \]
\[ f^*(e_i) = k + 4(i - 1); 2 \leq i \leq n - 1, \]
\[ f^*(e_n) = 4n + k - 3, \]
\[ f^*(e_{n+1}) = 4n + k - 2, \]
\[ f^*(e_{n+j}) = 4(n - j) + k - 1; 1 \leq j \leq n - 1, \]
\[ f^*(e_i) = f(v_{n+1}) + 2i - 1; 1 \leq i \leq l, \]
\[ f^*(e_i) = f(v_i) + 1, \]
\[ f^*(e_i) = f(v_i) + 4(i - 1); 2 \leq i \leq n - 1, \]
\[ f^*(e_i) = f(v_i) + 4n - 3, \]
\[ f^*(e_i) = f(v_i) + 4n - 2, \]
\[ f^*(e_{n+j}) = f(v_i) + 4(n - j) - 1; 1 \leq j \leq n - 1. \]

Here \( p = r + s + l - 1, q = r + s + l \).

Clearly,

\[ f(V) \cup \{f^*(e): e \in E(KP(r,s,l))\} = \{k, k + 1, ..., 2(r + s + l) + k - 2\}. \]

So \( f \) is a \( k \) – super mean labeling.

Hence \( KP(r, s, l) \) is a \( k \) – super mean graph, when \( r, s \) is even.

**Example 2.10**

A 10-super mean graph of \( KP(3,5,2) \) is given in figure 2.5:

**Remark 2.11**

\( KP(4,4,l) \) and \( KP(4,4,l) \) are not \( k \)-super mean graph because \( C_s \) is not a \( k \)-super mean graph.

**Remark 2.12**

\( KP(4,4,l) \) is a \( k \)-super mean graph, for \( s \geq 3, s \neq 4 \) and \( l \geq 2 \).

**Example 2.13**

A 5-super mean graph is given in figure 2.6:

**References**


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