

Some More Results on k – Super Mean Graphs

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Abstract: Let G be a (p, q) graph and $f: V(G) \rightarrow \{1, 2, 3, \dots, p+q\}$ be an injection. For each edge $e = uv$, let $f^*(e) = \frac{f(u)+f(v)}{2}$ if $f(u) + f(v)$ is even and $f^*(e) = \frac{f(u)+f(v)+1}{2}$ if $f(u) + f(v)$ is odd, then f is called **super mean labeling** if $f(V) \cup \{f^*(e): e \in E(G)\} = \{1, 2, 3, \dots, p+q\}$. A graph that admits a super mean labeling is called **Super mean graph**. Let G be a (p, q) graph and $f: V(G) \rightarrow \{k, k+1, k+2, \dots, p+q+k-1\}$ be an injection. For each edge $e = uv$, let $f^*(e) = \frac{f(u)+f(v)}{2}$ if $f(u) + f(v)$ is even and $f^*(e) = \frac{f(u)+f(v)+1}{2}$ if $f(u) + f(v)$ is odd, then f is called **k-super mean labeling** if $f(V) \cup \{f^*(e): e \in E(G)\} = \{k, k+1, \dots, p+q+k-1\}$. A graph that admits a k-super mean labeling is called **k-Super mean graph**. In this paper we investigate k – super mean labeling of H_n , $H_n \odot mK_1$, $A(DT_n)$, $n(P_1 \odot \overline{K_2})$ and $KP(r, s, l)$.

Keywords: k-Super mean labeling, k-Super mean graph, H_n , $H_n \odot mK_1$, $A(DT_n)$, $n(P_1 \odot \overline{K_2})$ and $KP(r, s, l)$.

1. Introduction

All graphs in this paper are finite, simple and undirected. Terms not defined here are used in the sense of Harary [7]. The symbols $V(G)$ and $E(G)$ will denote the vertex set and edge set of a graph G .

Graph labeling was first introduced in the late 1960's. Many studies in graph labeling refer to Rosa's research in 1967 [11]. Labeled graphs serve as useful models for a broad range of applications such as X-ray, crystallography, radar, coding theory, astronomy, circuit design and communication network addressing. Particularly interesting applications of graph labeling can be found in [1-4].

The concept of mean labeling was introduced and studied by S. Somasundaram and R. Ponraj [12].

The concept of super mean labeling was introduced and studied by D. Ramya et al. [11]. Further some results on super mean graphs are discussed in [8,9,10,13,15].

B. Gayathri and M. Tamilselvi [5-6,14] extended super mean labeling to k-super mean labeling.

In this paper we investigate k – Super mean labeling of H_n , $H_n \odot mK_1$, $A(DT_n)$, $n(P_1 \odot \overline{K_2})$ and $KP(r, s, l)$. For brevity, we use k-SML for k-super mean labeling.

Definition 1.1

Let G be a (p, q) graph and $f: V(G) \rightarrow \{1, 2, 3, \dots, p+q\}$ be an injection. For each edge $e = uv$, let $f^*(e) = \frac{f(u)+f(v)}{2}$ if $f(u) + f(v)$ is even and $f^*(e) = \frac{f(u)+f(v)+1}{2}$ if $f(u) + f(v)$ is odd, then f is called **Super mean labeling** if $f(V) \cup \{f^*(e): e \in E(G)\} = \{1, 2, 3, \dots, p+q\}$. A graph that admits a super mean labeling is called **Super mean graph**.

Definition 1.2

Let G be a (p, q) graph and

$f: V(G) \rightarrow \{k, k+1, k+2, \dots, p+q+k-1\}$ be an injection. For each edge $e = uv$, let $f^*(e) = \frac{f(u)+f(v)}{2}$ if $f(u) + f(v)$ is even and $f^*(e) = \frac{f(u)+f(v)+1}{2}$ if $f(u) + f(v)$ is odd, then f is called **k-Super mean labeling** if $f(V) \cup \{f^*(e): e \in E(G)\} = \{k, k+1, \dots, p+q+k-1\}$. A graph that admits a k-super mean labeling is called **k-Super mean graph**.

Definition 1.3

The H – graph of a path P_n , denoted by H_n is the graph obtained from two copies of P_n with vertices v_1, v_2, \dots, v_n and u_1, u_2, \dots, u_n by joining the vertices $\frac{v_n+1}{2}$ and $\frac{u_{n+1}}{2}$; if n is odd and the vertices $\frac{v_n}{2+1}$ and $\frac{u_n}{2}$; if n is even.

Definition 1.4

A triangular snake (T_n) is obtained from a path v_1, v_2, \dots, v_n by joining v_i and v_{i+1} to a new vertex w_i for $i = 1, 2, \dots, n-1$.

Definition 1.5

A double triangular snake (DT_n) consists of two triangular snakes that have a common path. That is, a double triangular snake is obtained from a path v_1, v_2, \dots, v_n by joining v_i and v_{i+1} to a new vertex w_i for $i = 1, 2, \dots, n-1$ and to a new vertex u_i for $i = 1, 2, \dots, n-1$.

Definition 1.6

A double alternate triangular snake $AD(T_n)$ consists of alternate triangular snakes that have a common path. That is, double triangular snake is obtained from the path u_1, u_2, \dots, u_n by joining u_i and u_{i+1} (alternatively) to two new vertices v_i and w_i .

Definition 1.7

If G has order n , the corona of G with H , $G \odot H$ is the graph obtained by taking one copy of G and n copies of H and joining the i th vertex of G with an edge to every vertex in the i th copy of H .

Definition 1.8

For any graph G , the graph mG denotes the disjoint union of m copies of G .

Definition 1.9

A Kayak paddle $KP(r, s, l)$ is obtained from two cycles C_r and C_s that are joined together by a path of length l .

2. Main Results

Theorem 2.1

The graph H_n is a k -Super mean graph for $n \geq 1$.

Proof

Let $V(H_n) = \{u_i; 1 \leq i \leq n\} \cup \{v_i; 1 \leq i \leq n\}$ be the vertices of H_n and

$$E(H_n) = \left\{ e = \begin{cases} (v_{\frac{n+1}{2}}, u_{\frac{n+1}{2}}); & \text{if } n \text{ is odd} \\ (v_{\frac{n}{2}+1}, u_{\frac{n}{2}}); & \text{if } n \text{ is even} \end{cases} \right\} \cup$$

$$\begin{cases} \{e_i = (u_i, u_{i+1}); 1 \leq i \leq n-1\} \cup \\ \{e_i = (v_i, v_{i+1}); 1 \leq i \leq n-1\} \end{cases}$$

be the edges of H_n .

Define $f: V(H_n) \rightarrow \{k, k+1, k+2, \dots, 4n+k-2\}$ by

$$f(u_i) = 2i + k - 2; 1 \leq i \leq n,$$

$$f(v_i) = 2n + 2i + k - 2; 1 \leq i \leq n.$$

Now the induced edge labels as follows:

$$f^*(e_i) = 2i + k - 1; 1 \leq i \leq n-1,$$

$$f^*(e_i) = 2n + 2i + k - 1; 1 \leq i \leq n-1,$$

$$f^*(e) = \begin{cases} 2n + k - 1, & \text{if } n \text{ is odd} \\ 2n + k - 1, & \text{if } n \text{ is even} \end{cases}$$

Here $p = 2n$ and $q = 2n-1$.

$$\text{Clearly, } f(V) \cup \{f^*(e); e \in E(H_n)\} = \{k, k+1, k+2, \dots, 4n+k-2\}.$$

So f is a k -super mean labeling.

Hence H_n is a k -super mean graph.

Example 2.2

25-super mean graph of H_5 is given in figure 2.1:

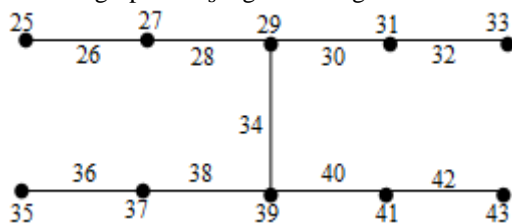


Figure 2.1: 25-SML of H_5

Theorem 2.3

The $H_n \odot mK_1$ is a k -super mean graph for all positive integers $m \geq 1$ and $n \geq 1$.

Proof

$$\text{Let } V(H_n \odot mK_1) = \{u_i; 1 \leq i \leq n\} \cup \{v_i; 1 \leq i \leq n\} \cup \{x_{i,l}; 1 \leq l \leq m, 1 \leq i \leq n\} \cup \{y_{i,l}; 1 \leq l \leq m, 1 \leq i \leq n\}$$

be the vertices of $H_n \odot mK_1$.

$$E(H_n \odot mK_1) = \left\{ e = \begin{cases} (v_{\frac{n+1}{2}}, u_{\frac{n+1}{2}}); & \text{if } n \text{ is odd} \\ (v_{\frac{n}{2}+1}, u_{\frac{n}{2}}); & \text{if } n \text{ is even} \end{cases} \right\} \cup$$

$$\{e_i = (u_i, u_{i+1}); 1 \leq i \leq n-1\} \cup$$

$$\{e_i = (v_i, v_{i+1}); 1 \leq i \leq n-1\} \cup$$

$$\{e_{i,l} = (u_i, x_{i,l}); 1 \leq l \leq m, 1 \leq i \leq n\} \cup$$

$$\{e_{i,l} = (v_i, y_{i,l}); 1 \leq l \leq m, 1 \leq i \leq n\}$$

be the edges of $H_n \odot mK_1$.

Define

$$f: V(H_n \odot mK_1) \rightarrow \{k, k+1, \dots, 4n(m+1) + k - 2\}$$

For $1 \leq i \leq n$,

$$f(u_i) = \begin{cases} 2(m+1)(i-1) + k; & i \text{ is odd} \\ 2(m+1)i + k - 2; & i \text{ is even} \end{cases}$$

$$f(v_i) = \begin{cases} f(u_i) + 2n(m+1) + 2m; & i \text{ is odd and } n \text{ is odd} \\ f(u_i) + 2n(m+1) - 2m; & i \text{ is even and } n \text{ is odd} \\ f(u_i) + 2n(m+1); & n \text{ is even} \end{cases}$$

For $1 \leq i \leq n$ and $1 \leq l \leq m$,

$$f(x_{i,l}) = \begin{cases} 2(m+1)(i-1) + 4l + k - 2; & i \text{ is odd} \\ 2(m+1)(i-2) + 4l + k; & i \text{ is even} \end{cases}$$

$$f(y_{i,l}) = \begin{cases} f(x_{i,l}) + 2n(m+1) - 2m; & i \text{ is odd and } n \text{ is odd} \\ f(x_{i,l}) + 2n(m+1) + 2m; & i \text{ is even and } n \text{ is odd} \\ f(x_{i,l}) + 2n(m+1); & n \text{ is even} \end{cases}$$

Now the induced edge labels are as follows:

For $1 \leq i \leq n-1$,

$$f^*(e_i) = 2i(m+1) + k - 1,$$

$$f^*(e_i) = f^*(e_i) + 2n(m+1).$$

For $1 \leq i \leq n$ and $1 \leq l \leq m$,

$$f^*(e_{i,l}) = 2(m+1)(i-1) + k - 1 + 2l,$$

$$f^*(e_{i,l}) = f^*(e_{i,l}) + 2n(m+1),$$

$$f^*(e) = 2n(m+1) + k - 1; \text{ if } n \text{ is odd,}$$

$$f^*(e) = 2n(m+1) + k - 1; \text{ if } n \text{ is even.}$$

Here $p = 2n(m+1)$ and $q = 2n(m+1)-1$.

Clearly,

$$f(V) \cup \{f^*(e); e \in E(H_n \odot mK_1)\} = \{k, k+1, \dots, 4n(m+1) + k - 2\}.$$

So f is a k -super mean labeling.

Hence $H_n \odot mK_1$ is a k -super mean graph.

Example 2.4

50-super mean graph of $H_4 \odot 2K_1$ is given in figure 2.2:

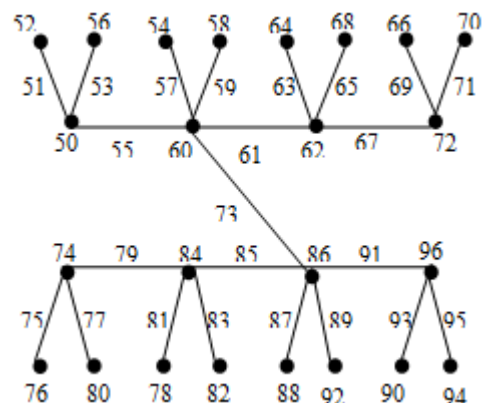


Figure 2.2: 50-SML of $H_4 \odot 2K_1$

Theorem 2.5

The graph $AD(T_n)$ is a k -Super mean graph, for $n \geq 2$.

Proof

Let $V(AD(T_n)) = \{u_i, u'_i, v_i, w_i; 1 \leq i \leq \frac{n}{2}\}$, and
 $E(AD(T_n)) = \{e_i = (u_i, u'_i); 1 \leq i \leq n-1\} \cup$
 $\{e'_i = (v_i, u'_i); 1 \leq i \leq \frac{n}{2}\} \cup$
 $\{e''_i = (v_i, u'_i); 1 \leq i \leq \frac{n}{2}\} \cup$
 $\{e'''_i = (u_i, w_i); 1 \leq i \leq \frac{n}{2}\} \cup$
 $\{e^{iv}_i = (u'_i, w_i); 1 \leq i \leq \frac{n}{2}\}$

be the vertices and edges of $AD(T_n)$ respectively.

Define

$f: V(AD(T_n)) \rightarrow \{k, k+1, k+2, \dots, 5n+k-2\}$ by

$f(u_i) = 10i + k - 10; 1 \leq i \leq \frac{n}{2},$

$f(u'_i) = 10i + k - 2; 1 \leq i \leq \frac{n}{2},$

$f(v_i) = 10i + k - 8; 1 \leq i \leq \frac{n}{2},$

$f(w_i) = 10i + k - 4; 1 \leq i \leq \frac{n}{2}.$

Now the induced edge labels are as follows:

$f^*(e_i) = 5i + k - 1; 1 \leq i \leq n-1,$

$f^*(e'_i) = 10i + k - 9; 1 \leq i \leq \frac{n}{2},$

$f^*(e''_i) = 10i + k - 5; 1 \leq i \leq \frac{n}{2},$

$f^*(e'''_i) = 10i + k - 7; 1 \leq i \leq \frac{n}{2},$

$f^*(e^{iv}_i) = 10i + k - 3; 1 \leq i \leq \frac{n}{2}.$

Here $p=2n$ and $q=3n-1$.

Clearly,

$f(V) \cup \{f^*(e); e \in E(AD(T_n))\} =$
 $\{k, k+1, \dots, 5n+k-2\}.$

So f is a k -super mean labeling.

Hence $AD(T_n)$ is a k -super mean graph, for $n \geq 2$.

Example 2.6

40-super mean graph of $AD(T_6)$ is given in figure 2.3:

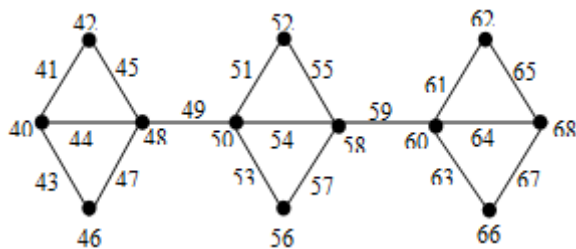


Figure 2.3: 40-SML of $AD(T_6)$

Theorem 2.7

The graph $n(P_2 \odot \overline{K_2})$ is a k -Super mean graph, for $n \geq 1$.

Proof

Let $V(n(P_2 \odot \overline{K_2})) = \{v_i, v'_i, v''_i; 1 \leq i \leq n\} \cup$
 $\{u_i, u'_i, u''_i; 1 \leq i \leq n\}$ and
 $E(n(P_2 \odot \overline{K_2})) = \{e_i = (v_i, v'_{i+1}); 1 \leq i \leq n\} \cup$
 $\{e'_i = (v'_i, v''_{i+1}); 1 \leq i \leq n\} \cup$
 $\{e''_i = (v''_i, u_i); 1 \leq i \leq n\} \cup$
 $\{e'''_i = (u'_i, u''_i); 1 \leq i \leq n\} \cup$
 $\{e^{iv}_i = (u'_i, u''_i); 1 \leq i \leq n\}$

be the vertices and edges of $n(P_2 \odot \overline{K_2})$ respectively.

Define

$f: V(n(P_2 \odot \overline{K_2})) \rightarrow \{k, k+1, k+2, \dots, 11n+k-1\}$ by

$f(v_i) = 11i + k - 11; 1 \leq i \leq n,$

$f(v'_i) = 11i + k - 7; 1 \leq i \leq n,$

$f(v''_i) = 11i + k - 9; 1 \leq i \leq n,$

$f(u_i) = 11i + k - 5; 1 \leq i \leq n,$

$f(u'_i) = 11i + k - 1; 1 \leq i \leq n,$

$f(u''_i) = 11i + k - 3; 1 \leq i \leq n.$

Now the induced edge labels are as follows:

$f^*(e_i) = 11i + k - 10; 1 \leq i \leq n,$

$f^*(e'_i) = 11i + k - 8; 1 \leq i \leq n,$

$f^*(e''_i) = 11i + k - 6; 1 \leq i \leq n,$

$f^*(e'''_i) = 11i + k - 4; 1 \leq i \leq n,$

$f^*(e^{iv}_i) = 11i + k - 2; 1 \leq i \leq n.$

Here $p=6n$ and $q=5n$.

Clearly, $f(V) \cup \{f^*(e); e \in E(n(P_2 \odot \overline{K_2}))\} =$
 $\{k, k+1, \dots, 11n+k-1\}.$

So f is a k -super mean labeling.

Hence $n(P_2 \odot \overline{K_2})$ is a k -super mean graph, for $n \geq 1$.

Example 2.8

35-super mean graph of $4(P_2 \odot \overline{K_2})$ is given in figure 2.4:

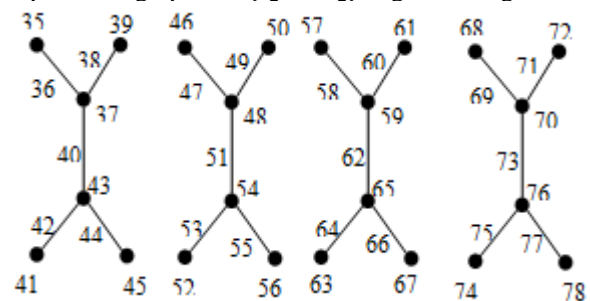


Figure 2.4: 35-SML of $4(P_2 \odot \overline{K_2})$

Theorem 2.9

The graph $KP(r, s, l)$ is a k -Super mean graph, for all $r, s \geq 3, r, s \neq 4$ and $l \geq 1$.

Proof

Case 1: r, s is odd

Let $r, s = 2n+1$ for all $n \geq 1$.

Let $V(KP(r, s, l)) = \{v_i, 1 \leq i \leq r\} \cup$
 $\{u_i, 1 \leq i \leq l+1\} \cup$
 $\{v'_i, 1 \leq i \leq s\}$

be the vertices of $KP(r, s, l)$ and

$E(KP(r, s, l)) = \{e_i = (v_i, v'_{i+1}); 1 \leq i \leq r\} \cup$
 $\{e'_i = (v'_i, v'_{i+1}); 1 \leq i \leq s\} \cup$
 $\{e''_i = (u_i, u_{i+1}); 1 \leq i \leq l\}$

be the edges of $KP(r, s, l)$.

Define

$f: V(KP(r, s, l)) \rightarrow \{k, k+1, \dots, 2(r+s+l)+k-2\}$ by

$f(v_i) = 2i + k - 2; 1 \leq i \leq n+1,$

$f(v_{n+1+i}) = 2(n+1+i) + k - 1; 1 \leq i \leq n,$

$f(u_1) = f(v_r),$

$f(u_i) = f(v_r) + 2(i-1); 2 \leq i \leq l+1,$

$f(v'_1) = f(u_{l+1}),$

$f(v'_i) = f(v'_1) + 2(i-1); 2 \leq i \leq n+1,$

$f(v'_{n+1+i}) = f(v'_1) + 2(n+i+1); 1 \leq i \leq n.$

Now the induced edge labels are as follows:

$f^*(e_i) = 2i + k - 1; 1 \leq i \leq n,$

$f^*(e_{n+i}) = 2(n+i) + k; 1 \leq i \leq n,$

$f^*(e_{2n+1}) = k + 2n + 1,$

$f^*(e''_i) = f(v_r) + 2i - 1; 1 \leq i \leq l,$

$f^*(e'_i) = f(v'_1) + 2i - 1; 1 \leq i \leq n,$
 $f^*(e_i) = f(v_1) + 2(n + i); 1 \leq i \leq n,$
 $f^*(e_{2n+1}) = f(v'_1) + 2n + 1.$
 Here $p = r + s + l - 1, q = r + s + l.$
 Clearly, $f(V) \cup \{f^*(e): e \in E(KP(r, s, l))\} =$
 $\{k, k + 1, \dots, 2(r + s + l) + k - 2\}.$
 So f is a k – super mean labeling.
 Hence $KP(r, s, l)$ is a k – super mean graph, when r, s is odd.

Case 2: r, s is even

Let $r, s = 2n$ for all $n \geq 3.$

Let $V(KP(r, s, l)) = \{v_i, 1 \leq i \leq r\} \cup$
 $\{u_i, 1 \leq i \leq l + 1\} \cup$
 $\{v_i, 1 \leq i \leq s\}$

be the vertices of $KP(r, s, l)$ and

$E(KP(r, s, l)) = \{e_i = (v_i, v_{i+1}), 1 \leq i \leq r\} \cup$
 $\{e_i = (v_i, v_{i+1}), 1 \leq i \leq s\} \cup$
 $\{e_i = (u_i, u_{i+1}), 1 \leq i \leq l\}$

be the edges of $KP(r, s, l).$

Define

$f: V(KP(r, s, l)) \rightarrow \{k, k + 1, \dots, 2(r + s + l) + k - 2\}$ by

$f(v_1) = k$
 $f(v_i) = 4i + k - 6; 2 \leq i \leq n,$
 $f(v_{n+j}) = 4n - 3j + k + 2; 1 \leq j \leq 2,$
 $f(v_{n+j+2}) = 4(n - j) + k - 3; 1 \leq j \leq n - 2,$
 $f(u_1) = f(v_{n+1}),$
 $f(u_i) = f(v_{n+1}) + 2(i - 1); 2 \leq i \leq l + 1,$
 $f(v'_1) = f(u_{l+1}),$
 $f(v'_i) = f(v'_1) + 4i - 6; 2 \leq i \leq n,$
 $f(v'_{n+j}) = f(v'_1) + 4n - 3j + 2; 1 \leq j \leq 2,$
 $f(v'_{n+j+2}) = f(v'_1) + 4(n - j) - 3; 1 \leq j \leq n - 2.$

Now the induced edge labels are as follows:

$f^*(e_1) = k + 1,$
 $f^*(e_i) = k + 4(i - 1); 2 \leq i \leq n - 1,$
 $f^*(e_n) = 4n + k - 3,$
 $f^*(e_{n+1}) = 4n + k - 2,$
 $f^*(e_{n+1+j}) = 4(n - j) + k - 1; 1 \leq j \leq n - 1,$
 $f^*(e'_i) = f(v_{n+1}) + 2i - 1; 1 \leq i \leq l,$
 $f^*(e'_1) = f(v'_1) + 1,$
 $f^*(e'_i) = f(v'_1) + 4(i - 1); 2 \leq i \leq n - 1,$
 $f^*(e'_n) = f(v'_1) + 4n - 3,$
 $f^*(e'_{n+1}) = f(v'_1) + 4n - 2,$
 $f^*(e'_{n+1+j}) = f(v'_1) + 4(n - j) - 1; 1 \leq j \leq n - 1.$

Here $p = r + s + l - 1, q = r + s + l.$

Clearly,

$f(V) \cup \{f^*(e): e \in E(KP(r, s, l))\} =$
 $\{k, k + 1, \dots, 2(r + s + l) + k - 2\}.$

So f is a k – super mean labeling.

Hence $KP(r, s, l)$ is a k – super mean graph, when r, s is even.

Example 2.10

10-super mean graph of $KP(3, 5, 2)$ is given in figure 2.5:

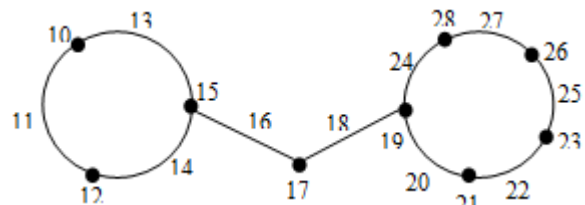


Figure 2.5: 10-SML of $KP(3, 5, 2)$

Remark 2.11

$KP(4, 4, l)$ and $KP(r, 4, l)$ are not k -super mean graph because C_4 is not a k -super mean graph.

Remark 2.12

$KP(4, s, l)$ is a k -super mean graph, for $s \geq 3, s \neq 4$ and $l \geq 2.$

Example 2.13

$KP(4, 5, 2)$ is a 5-super mean graph is given in figure 2.6:

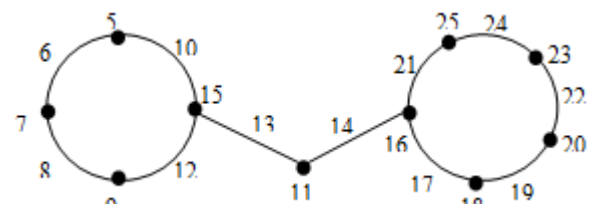


Figure 2.6: 5-SML of $KP(4, 5, 2)$

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