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Study on Classification of Image Group

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Abstract: ***Image of the irrep(irreducible representation) of the space group has the important significance in the study of the spontaneous symmetry breaking arising in the crystal. In this paper we defined the characteristic quantity of I-Group.And using it we established the procedure for finding the equivalent I-Group. We classified the irreps of the star channel group of the wave vector onto 142 image groups.***

Keywords: representation; spacegroup; symmetry breaking; image

1. Introduction

An irrep of a space group G maps the elements $g \in G$ onto matrices $D(g)$.

The set of distinct matrices is called the image of the irrep. Gufan [1] defined I-Group the and emphasized the role of it in the structural phase transition research and suggested the method for finding the IRBI (Integral Rational Basis of Invariant) and classified irreps corresponding 80 different Lifshitz stars onto 106 L-Groups.

Stokes [2] formed 4 777 irreps corresponding 80 different Lifshitz stars using the irreducible representation table of ref [3] and classified onto 132 inequivalent images.

Such difference is related to the choice of the origin at the coordinate system and the method of forming the physically irreducible representation.

I-Group (Image-Group) are the set of distinct matrices onto which the space group elements are mapped by the irrep.

But there exist the I-Groups which are identical as the conjugate correct degree in the structural viewpoint between I-Groups of which the dimension of the representation's matrices and the order of the I-Group are the same. Since they are equivalent in the structural viewpoint, but the types of the matrices are different they were classified onto the distinct I-Groups.

For example, Let's Consider the distinct I-Gs($D32\mu$, $D32\tau$, $D32\theta$) of which the dimension is 4 and the order is 32.

The matrix types of each I-Groups are same as following.(Table 1,2,3)

Where $\sigma = \exp(i\pi/4)$, $\overline{\sigma} = \exp(-i\pi/4)$.

Table 1: $D32\mu\left(D_4^4(16/1\oplus 2)\right)$

$\begin{pmatrix}1&&&\\&1&&\\&&1&\\&&&1\end{pmatrix}$	$\begin{pmatrix}1&&&\\&1&&\\&&&-1\\&&1&\end{pmatrix}$	$\begin{pmatrix}&-1&&\\1&&&\\&&-1&\\&&&-1\end{pmatrix}$	$\begin{pmatrix}&1&&\\-1&&&\\&&&1\\&&-1&\end{pmatrix}$	$\begin{pmatrix}&&&-1\\&&1&\\&1&&\\-1&&&\end{pmatrix}$	$\begin{pmatrix}&&&1\\&&-1&\\1&&&\\&1&&\end{pmatrix}$	$\begin{pmatrix}&&1&\\&&&1\\&1&&\\-1&&&\end{pmatrix}$	$\begin{pmatrix}&&1&\\&&&1\\-1&&&\\&-1&&\end{pmatrix}$
$\begin{pmatrix}1&&&\\&1&&\\&&-1&\\&&&-1\end{pmatrix}$	$\begin{pmatrix}1&&&\\&1&&\\&&&1\\&&-1&\end{pmatrix}$	$\begin{pmatrix}&-1&&\\1&&&\\&&1&\\&&&1\end{pmatrix}$	$\begin{pmatrix}&1&&\\-1&&&\\&&&-1\\&&1&\end{pmatrix}$	$\begin{pmatrix}&&&-1\\&&1&\\&-1&&\\1&&&\end{pmatrix}$	$\begin{pmatrix}&&&1\\&&-1&\\-1&&&\\&-1&&\end{pmatrix}$	$\begin{pmatrix}&&1&\\&&&1\\&-1&&\\1&&&\end{pmatrix}$	$\begin{pmatrix}&&1&\\&&&1\\1&&&\\&1&&\end{pmatrix}$
$\begin{pmatrix}-1&&&\\&-1&&\\&&1&\\&&&1\end{pmatrix}$	$\begin{pmatrix}-1&&&\\&-1&&\\&&&-1\\&&1&\end{pmatrix}$	$\begin{pmatrix}&1&&\\-1&&&\\&&-1&\\&&&-1\end{pmatrix}$	$\begin{pmatrix}&-1&&\\1&&&\\&&&1\\&&-1&\end{pmatrix}$	$\begin{pmatrix}&&&1\\&&-1&\\&1&&\\-1&&&\end{pmatrix}$	$\begin{pmatrix}&&&-1\\&&1&\\-1&&&\\&-1&&\end{pmatrix}$	$\begin{pmatrix}&&-1&\\&&&-1\\&1&&\\-1&&&\end{pmatrix}$	$\begin{pmatrix}&&-1&\\&&&-1\\-1&&&\\&-1&&\end{pmatrix}$
$\begin{pmatrix}-1&&&\\&-1&&\\&&-1&\\&&&-1\end{pmatrix}$	$\begin{pmatrix}-1&&&\\&-1&&\\&&&1\\&&-1&\end{pmatrix}$	$\begin{pmatrix}&1&&\\-1&&&\\&&1&\\&&&1\end{pmatrix}$	$\begin{pmatrix}&-1&&\\1&&&\\&&&-1\\&&1&\end{pmatrix}$	$\begin{pmatrix}&&&1\\&&-1&\\&-1&&\\1&&&\end{pmatrix}$	$\begin{pmatrix}&&&-1\\&&1&\\-1&&&\\&-1&&\end{pmatrix}$	$\begin{pmatrix}&&-1&\\&&&-1\\&-1&&\\1&&&\end{pmatrix}$	$\begin{pmatrix}&&-1&\\&&&-1\\1&&&\\&1&&\end{pmatrix}$

Table 2: $D32\tau\left(C_{4h}^{6}(12/1\oplus 4)\right)$

$\begin{pmatrix}1&&&\\&1&&\\&&1&\\&&&1\end{pmatrix}$	$\begin{pmatrix}1&&&\\&-1&&\\&&-1&\\&&&1\end{pmatrix}$	$\begin{pmatrix}&&-i&\\&&&1\\1&&&\\&i&&\end{pmatrix}$	$\begin{pmatrix}&&i&\\&&&1\\1&&&\\&-i&&\end{pmatrix}$	$\begin{pmatrix}&&1&\\&&&1\\1&&&\\&1&&\end{pmatrix}$	$\begin{pmatrix}&&-1&\\&&&1\\1&&&\\&-1&&\end{pmatrix}$	$\begin{pmatrix}1&&&\\&i&&\\&&-i&\\&&&1\end{pmatrix}$	$\begin{pmatrix}1&&&\\&-i&&\\&&i&\\&&&1\end{pmatrix}$
$\begin{pmatrix}-1&&&\\&-1&&\\&&-1&\\&&&-1\end{pmatrix}$	$\begin{pmatrix}-1&&&\\&1&&\\&&1&\\&&&-1\end{pmatrix}$	$\begin{pmatrix}&&i&\\&&&-1\\-1&&&\\&-i&&\end{pmatrix}$	$\begin{pmatrix}&&-i&\\&&&-1\\-1&&&\\&i&&\end{pmatrix}$	$\begin{pmatrix}&&-1&\\&&&-1\\-1&&&\\&-1&&\end{pmatrix}$	$\begin{pmatrix}&&1&\\&&&-1\\-1&&&\\&1&&\end{pmatrix}$	$\begin{pmatrix}-1&&&\\&-i&&\\&&i&\\&&&-1\end{pmatrix}$	$\begin{pmatrix}-1&&&\\&i&&\\&&-i&\\&&&-1\end{pmatrix}$
$\begin{pmatrix}-i&&&\\&-1&&\\&&-1&\\&&&i\end{pmatrix}$	$\begin{pmatrix}-i&&&\\&-i&&\\&&i&\\&&&i\end{pmatrix}$	$\begin{pmatrix}-i&&&\\&i&&\\&&-i&\\&&&i\end{pmatrix}$	$\begin{pmatrix}&&-1&\\&&&-i\\i&&&\\&-1&&\end{pmatrix}$	$\begin{pmatrix}&&1&\\&&&-i\\i&&&\\&1&&\end{pmatrix}$	$\begin{pmatrix}&&-i&\\&&&-i\\i&&&\\&i&&\end{pmatrix}$	$\begin{pmatrix}&&i&\\&&&-i\\i&&&\\&-i&&\end{pmatrix}$	$\begin{pmatrix}-i&&&\\&1&&\\&&1&\\&&&i\end{pmatrix}$
$\begin{pmatrix}1&&&\\&1&&\\&&1&\\&&&-i\end{pmatrix}$	$\begin{pmatrix}i&&&\\&i&&\\&&-i&\\&&&-i\end{pmatrix}$	$\begin{pmatrix}i&&&\\&-i&&\\&&i&\\&&&-i\end{pmatrix}$	$\begin{pmatrix}&&1&\\&&&i\\-i&&&\\&1&&\end{pmatrix}$	$\begin{pmatrix}&&-1&\\&&&i\\-i&&&\\&1&&\end{pmatrix}$	$\begin{pmatrix}&&i&\\&&&i\\-i&&&\\&-i&&\end{pmatrix}$	$\begin{pmatrix}&&-i&\\&&&i\\-i&&&\\&i&&\end{pmatrix}$	$\begin{pmatrix}i&&&\\&-1&&\\&&-1&\\&&&-i\end{pmatrix}$

Table 3: $D32\theta\left(D_{2d}^{12}(12/1\times 2)\right)$

$\begin{pmatrix}1&&&\\&1&&\\&&1&\\&&&1\end{pmatrix}$	$\begin{pmatrix}1&&&\\&-i&&\\&&1&\\&&&i\end{pmatrix}$	$\begin{pmatrix}1&&&\\&-1&&\\&&1&\\&&&-1\end{pmatrix}$	$\begin{pmatrix}1&&&\\&i&&\\&&1&\\&&&-i\end{pmatrix}$	$\begin{pmatrix}-1&&&\\&-1&&\\&&-1&\\&&&-1\end{pmatrix}$	$\begin{pmatrix}-1&&&\\&i&&\\&&-1&\\&&&-i\end{pmatrix}$	$\begin{pmatrix}-1&&&\\&1&&\\&&-1&\\&&&1\end{pmatrix}$	$\begin{pmatrix}-1&&&\\&-i&&\\&&-1&\\&&&i\end{pmatrix}$
$\begin{pmatrix}-i&&&\\&-i&&\\&&i&\\&&&i\end{pmatrix}$	$\begin{pmatrix}-i&&&\\&-1&&\\&&i&\\&&&-1\end{pmatrix}$	$\begin{pmatrix}-i&&&\\&i&&\\&&i&\\&&&-i\end{pmatrix}$	$\begin{pmatrix}-i&&&\\&1&&\\&&i&\\&&&1\end{pmatrix}$	$\begin{pmatrix}i&&&\\&i&&\\&&-i&\\&&&-i\end{pmatrix}$	$\begin{pmatrix}i&&&\\&1&&\\&&-i&\\&&&1\end{pmatrix}$	$\begin{pmatrix}i&&&\\&-i&&\\&&-i&\\&&&i\end{pmatrix}$	$\begin{pmatrix}i&&&\\&-1&&\\&&-i&\\&&&-1\end{pmatrix}$
$\begin{pmatrix}&-i\vec{\sigma}&&\\-i\vec{\sigma}&&&\\&&&i\sigma\\&&i\sigma&\end{pmatrix}$	$\begin{pmatrix}&-\vec{\sigma}&&\\-i\vec{\sigma}&&&\\&&&-\sigma\\&&i\sigma&\end{pmatrix}$	$\begin{pmatrix}&i\vec{\sigma}&&\\-i\vec{\sigma}&&&\\&&&-i\sigma\\&&i\sigma&\end{pmatrix}$	$\begin{pmatrix}&\vec{\sigma}&&\\-i\vec{\sigma}&&&\\&&&\sigma\\&&i\sigma&\end{pmatrix}$	$\begin{pmatrix}&i\vec{\sigma}&&\\i\vec{\sigma}&&&\\&&&-i\sigma\\&&-i\sigma&\end{pmatrix}$	$\begin{pmatrix}&\vec{\sigma}&&\\i\vec{\sigma}&&&\\&&&\sigma\\&&-i\sigma&\end{pmatrix}$	$\begin{pmatrix}&-i\vec{\sigma}&&\\i\vec{\sigma}&&&\\&&&i\sigma\\&&-i\sigma&\end{pmatrix}$	$\begin{pmatrix}&-\vec{\sigma}&&\\i\vec{\sigma}&&&\\&&&-\sigma\\&&-i\sigma&\end{pmatrix}$
$\begin{pmatrix}&-\vec{\sigma}&&\\-\vec{\sigma}&&&\\&&&-i\sigma\\&&-i\sigma&\end{pmatrix}$	$\begin{pmatrix}&i\vec{\sigma}&&\\-\vec{\sigma}&&&\\&&&-i\sigma\\&&-\sigma&\end{pmatrix}$	$\begin{pmatrix}&\vec{\sigma}&&\\-\vec{\sigma}&&&\\&&&\sigma\\&&-\sigma&\end{pmatrix}$	$\begin{pmatrix}&-i\vec{\sigma}&&\\-\vec{\sigma}&&&\\&&&i\sigma\\&&\sigma&\end{pmatrix}$	$\begin{pmatrix}&\vec{\sigma}&&\\\vec{\sigma}&&&\\&&&\sigma\\&&\sigma&\end{pmatrix}$	$\begin{pmatrix}&-i\vec{\sigma}&&\\\vec{\sigma}&&&\\&&&i\sigma\\&&\sigma&\end{pmatrix}$	$\begin{pmatrix}&-\vec{\sigma}&&\\\vec{\sigma}&&&\\&&&-\sigma\\&&\sigma&\end{pmatrix}$	$\begin{pmatrix}&i\vec{\sigma}&&\\\vec{\sigma}&&&\\&&&-i\sigma\\&&\sigma&\end{pmatrix}$

Rep (representation) $D_4^4(16/1\oplus 2)$ is the direct sum of the first rep T_1 and the second rep T_2 corresponding Lifshitz star $\vec{K}_{16}$ of the space group $P4_12_12(D_4^4)$.

Rep $C_{4h}^{6}(12/1\oplus 4)$ is the direct sum of the first rep T_1 and the fourth rep T_4 corresponding Lifshitz star $\vec{K}_{12}$ of the space group $I4_1/a(C_{4h}^{6})$.

Rep $D_{2d}^{12}(12/1\times 2)$ is the direct sum of the twice first rep T_1 corresponding Lifshitz star $\vec{K}_{12}$ of the space group $I\bar{4}2d(D_{2d}^{12})$.

Since the types of the above representation matrices are distinct, they were classified onto the distinct I-Groups $D32\tau$, $D32\mu$, $D32\theta$.

By the following unitary transformation

$$D32\mu(g) = U_1 D32\tau(g) U_1^{-1}$$

$$D32\mu(g) = U_2 D32\theta(g) U_2^{-1} \qquad (1)$$

The matrices of $D32\tau$, $D32\theta$ coincide with the type of matrix of $D32\mu$ and they are become an I-Group. Where the unitary transformation matrices U_1 and U_2 are the same as

$$U_1 = \frac{1}{\sqrt{2}}\begin{pmatrix} & -i & -1 & \\ & -1 & -i & \\ -1 & & & -i \\ -i & & & -1 \end{pmatrix}, \quad U_2 = \frac{1}{\sqrt{2}}\begin{pmatrix} & 1 & & -i \\ & -i & & 1 \\ -i\bar{\sigma} & & i\bar{\sigma} & \\ -\bar{\sigma} & & -\bar{\sigma} & \end{pmatrix}$$

It is difficult to find the unitary transformation matrices coinciding the I-Groups of which the dimension and the order is the same . There is no practical method to find the such unitary transformation matrices.

Furthermore, it is very difficult problem to find the unitary transformation matrices coinciding with the equivalent I-Groups of which the dimension is 12 and the order is 1536.

It needs the new method to find the equivalent I-Group.

Otherwise the different physical phenomena arising in the solid crystals are related to not only all the starlines of the star of the wave vector but also some starlines of it.

In the [7] they found the irrep of the star channel group of the wave vector and suggested a three components order parameter model to describe the successive phase transitions due to condensation of several soft modes.

There must be established the procedure for classifying I-Group to find correctly the I-Groups of the irrep of the star channel group that is the important mathematical tool in the structural phase transition research of the crystal.

2. The characteristic quantity of I-Group

1) The trace of I-Group matrix

Denote the trace of the I-Group matrix $M(g)$ by $\chi(g)$ .This is the sum of the diagonal elements of matrix in the arbitrary base.[5]

$$\chi(g) = \sum_{i=1}^{s} M_{ii}(g) \qquad (2)$$

Where g is the number of I-Group matrices and it equals the order of I-G.

S is the dimension of the matrix.

The traces of I-Group matrices are invariant on the resemblance transition.
In fact, if

$$M'(g) = UM(g)U^{-1} \qquad (3)$$

Then

$$\chi'(g) = \sum_i M'_{ii}(g) = \sum_{ijk} U_{ij} M_{jk}(g) U_{ki}^{-1} = \\ = \sum_{ijk} M_{jk}(g)(U^{-1}U)_{kj} = \sum_j M_{jj}(g) = \chi(g) \qquad (4)$$

Since the trace of I-Group matrix is invariant on the resemblance transition, the trace of the equivalent I-Group's matrix must be equal.

2) The order of I-Group matrix

One property of the representation matrix is that the unit matrix is get by multiplating itself limitedness

$$M^n(g) = E \qquad (5)$$

This n is called the order of the **I-Group matrix**
The order of I-Group matrices are invariant on the resemblance transition.

$$M'(g) = UM(g)U^{-1}$$

$$M'^n(g) = \left(UM(g)U^{-1}\right)^n = UM(g)U^{-1} \cdot UM(g)U^{-1} \cdots \\ \cdots UM(g)U^{-1} = UM^n(g)U^{-1} = UEU^{-1} = E \qquad (6)$$

As the order of I-Group's matrices is invariant on the resemblance transition, the order of the equivalent I-Group's matrices must be equal.

Though the trace and the order of matrix of I-G are the same, the number of matrices corresponding them can be different. It is related to the kernel of irreducible representation.

3) The dimension of subspace of I-Group

By the Landau theory [6], the change $\Delta\rho(\vec{r})$ of the dencity function $\rho(\vec{r})$ which find in the spontaneous symmetry breaking phase transition of the crystal can be expanded by the basis functions $\varphi_m(\vec{r})$ of the some irreducible representation T^ν of the symmetry group G_0.

$$\Delta\rho(\vec{r}) = \sum_m \vec{\eta}_m \varphi_m(\vec{r}) \qquad (7)$$

The coefficients $\vec{\eta}_m$ formed an m-dimensional vector $\vec{\eta}$ called the order parameter. The symmetry group G_0 obtained from the distortion consists of all element $g \in G_0$ which leave the distortion invariant. Such a group is called an isotropy subgroup of G_0.

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Because I-G acts as the abstracting point group in the space ε_n, the space ε_n is disjointed by the orbits, which is the set of the different vectors that are get operating all the elements of the group to the given vector $\vec{\eta}$.

The dimension of the invariant subspace to the subgroup H_α that leave invariant the nonzero subspace of the orderparameter's space is

$$r = \frac{1}{[G_{f\alpha}]}\sum \chi[T_f(g_i)] = \frac{1}{[H_\alpha]}\sum_i \chi(h_i) \quad (8)$$

where $g_i \in G_{f\alpha}$, $h_i \in H_\alpha$ and $\chi[T_f(g_i)]$ is the trace of $T_f(g_i)$. $\chi(h_i)$ is the sum of the diagonal elements corresponding the element h_i, r is the number of the independent part of the order parameter $\vec{\eta}$ that characterize the isotropy subgroup. Actually, the number of H_α of the equivalent I-Group must be equal.

3. The condition for existence of the unitary matrix for finding the equivalent I-G

We formulate the existence condition of the unitary transformation of the I-Group as following.

Given two I-Gs I_1, I_2

We denote their matrices and subgroups by $M_1(g)$, $M_2(g)$, $H_{1\alpha}$, $H_{2\alpha}$ and the order by $n(M(g))$, the number of subgroups by $r(H_\alpha)$.

If the following relation stands between two I-Gs ,

$$\chi(M_1(g)) = \chi(M_2(g)) \quad (8)$$

$$n(M_1(g)) = n(M_2(g))$$

$$r_\alpha(H_{1\alpha}) = r_\alpha(H_{2\alpha})$$

there exists surely the unitar's transformation matrix U satisfied

$$M_2(g) = UM_1(g)U^+ \quad (9)$$

and two I-Groups are equivalent.

For example, let's consider the I-Groups $C24\alpha$, $C24\beta$, $C24\gamma$ with the dimension 3 and the order 24. In the bellow table we presented the characteristic quantity of each the I-Groups.

Table 4: Characteristic quantity of $C24\alpha$, $C24\beta$, $C24\gamma$

No	I-G	Irrep	$a(x)$	b_n	c
1	$C24\alpha$	$C_3^1(14/1)$	$a(0)=16, a(1)=3, a(-1)=3,$ $a(3)=1, a(-3)=1,$	$b_1=8, b_2=8,$ $b_5=8$	$c_1 =$ $c_2 =$
2	$C24\beta$	$D_3^1(12/1)$	$a(0)=8, a(1)=6,$ $a(-1)=9, a(3)=1$	$b_1=10, b_2=8,$ $b_3=6$	//
3	$C24\gamma$	$D_3^1(12/2)$	//	//	$c_1 =$

In the irrep column is only indicated a representation of the space group corresponding to I-G. Where rep $C_3^1(14/1)$ is the first rep of the Lifshitz star $\vec{K}_{14}$ of the spacegroup C_3^1.

$a(x)$ is the number of matrices having the trace of the matrix x, b_n is the number of matrices having the order n, c_r is the number of r dimension subspace.

Table1 shows that the traces and the orders of $C24\beta$ and $C24\gamma$ are the identical, but the number of the subspace are not identical, the number of the subspace of $C24\alpha$ and $C24\beta$ are identical.

Because of the three conditions don't satisfy, the above I-Groups are classified the non-equivalent I-Groups.

For example, Let's consider 4 dimension I-Gs $D48\chi$, $D48\delta$, $D48\gamma$, $D48\beta$, $D48\alpha$.

Table 5: Characteristic quantity of $D48\chi$, $D48\delta$, $D48\gamma$, $D48\beta$, $D48\alpha$

No	I-G	Irrep	$a(x)$	b_n	c_r
1	$D48\chi$	$O_h^9(10/3)$	$a(2\varepsilon+2\varepsilon^2)=2, a(0)=42, a(4)=1,$ $a(-2\varepsilon-2\varepsilon^2)=2, a(-4)=1$	$b_1=24, b_2=2, b_3=8,$ $b_5=10, b_{11}=4$	$c_1=4,$ $c_2=6$
2	$D48\delta$	$D_{6h}^4(17/3)$	//	//	//
3	$D48\gamma$	$D_{6h}^4(15/1)$	//	//	//
4	$D48\beta$	$D_{6h}^3(17/3)$	//	//	//
5	$D48\alpha$	$D_{6h}^2(15/1)$	//	//	//

Since the above I-Gs's characteristic quantities are the same, they are the equivariant I-Gs

There exist the unitar's matrices $U_1 \sim U_4$ that satisfies the following expression

$$D48\alpha(g)=U_1 D48\chi(g)U_1^+$$
$$D48\alpha(g)=U_2 D48\delta(g)U_2^+$$
$$D48\alpha(g)=U_3 D48\gamma(g)U_3^+ \quad (10)$$
$$D48\alpha(g)=U_4 D48\beta(g)U_4^+$$

They are the same as follows.

$$U_1=\frac{1}{\sqrt{2}}\begin{pmatrix} 1 & & 1 & \\ 1 & & -1 & \\ & 1 & & 1 \\ & 1 & & -1 \end{pmatrix} \quad U_2=\frac{1}{\sqrt{2}}\begin{pmatrix} 1 & & 1 & \\ 1 & & -1 & \\ & 1 & & -1 \\ & -1 & & -1 \end{pmatrix}$$

$$U_3=\begin{pmatrix} 1 & & & \\ & 1 & & \\ & & & i \\ & & i & \end{pmatrix} \quad U_4=\frac{1}{\sqrt{2}}\begin{pmatrix} 1 & & -i & \\ 1 & & i & \\ & 1 & & i \\ & -1 & & i \end{pmatrix}$$

4. Procedure for the Classification of Image-Group.

We establish the procedure for the classification of I-Group of the irreducible representation corresponding to the different Lifshitz's stars of the different spacegroup G_0 on the based the definition of the characteristic quantity of I-G and the condition for existence of unitary transformation.

The procedure consisted of two stages.

1)The stage finding I-Groups (①~③)

① Find the homomorphic kernel $\ker T$ of the full irreducible representation of the spacegroup G_0.

② Find the factor group $G_f = G_0/\ker T$ by $\ker T$.

③ Find the correct representation T_f of G_f.

2)The stage classifying onto non-equivalent I-Groups.(④~⑦)

④ Find the traces, orders, the number of subspace among the I-Groups that have the same dimensions and orders.

⑤ Decide the identical I-Group of the I-Groups that have the same characteristic quantity.

⑥ Find the unitar's transformation matrices that coincide with the same I-Gs.

⑦ Coincide with the equivalent I-Groups operating unitar's transformation $I_1(g)=UI_2(g)U^+$ to all the matrices of I-Group.

Above classified I-Groups are the non-equivalent

We presented the number of the full irreducible representation of 230 space group corresponding to I-G in the table 6.

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Table 6: The number of the Irrep corresponding to I-G

№	I-G	n	№	I-G	n	№	I-G	n
1	*A1a*	230	25	*D24b*	3	49	*D72d*	12
2	*A2a*	1848	26	*D24c*	5	50	*D96a*	3
3	*B3a*	46	27	*D24d*	3	51	*D96b*	2
4	*B4a*	273	28	*D24e*	20	52	*D96c*	1
5	*B6a*	109	29	*D32a*	2	53	*D96d*	2
6	*B6b*	94	30	*D32b*	51	54	*D128a*	4
7	*B8a*	934	31	*D32c*	29	55	*D144a*	2
8	*B8b*	8	32	*D32d*	3	56	*D192a*	2
9	*B12a*	160	33	*D32e*	8	57	*D192b*	8
10	*B12b*	10	34	*D36a*	4	58	*D192c*	2
11	*B16a*	12	35	*D36b*	16	59	*D384a*	4
12	*B24a*	10	36	*D36c*	18	60	*E48a*	4
13	*C12a*	43	37	*D48a*	2	61	*E48b*	9
14	*C24a*	73	38	*D48b*	2	62	*E48c*	2
15	*C24b*	85	39	*D48c*	2	63	*E96a*	2
16	*C24c*	85	40	*D48d*	1	64	*E96b*	4
17	*C48a*	208	41	*D48e*	5	65	*E96c*	6
18	*D8a*	16	42	*D64a*	4	66	*E96d*	12
19	*D12a*	15	43	*D64b*	2	67	*E96e*	8
20	*D16a*	2	44	*D64c*	46	68	*E96f*	12
21	*D16b*	3	45	*D64d*	4	69	*E96g*	1
22	*D16c*	60	46	*D72a*	1	70	*E96h*	2
23	*D18a*	16	47	*D72b*	4	71	*E96i*	2
24	*D24a*	3	48	*D72c*	4	72	*E96j*	2
№	I-G	n	№	I-G	n	№	I-G	n
73	*E96k*	2	93	*F32b*	1	113	*F384b*	1
74	*E192a*	6	94	*F64a*	4	114	*F384c*	1
75	*E192b*	1	95	*F64b*	1	115	*G96a*	1
76	*E192c*	4	96	*F64c*	1	116	*G192a*	1
77	*E192d*	14	97	*F72a*	2	117	*G192b*	1
78	*E192e*	14	98	*F72b*	1	118	*G384a*	1
79	*E192f*	9	99	*F96a*	3	119	*G384b*	1
80	*E192g*	2	100	*F96b*	1	120	*G384c*	1
81	*E192h*	4	101	*F96c*	1	121	*G768a*	1
82	*E192i*	2	102	*F96d*	1	122	*G768b*	2
83	*E192j*	2	103	*F128a*	1	123	*G768c*	1
84	*E384a*	2	104	*F128b*	1	124	*G768d*	1

85	E384b	2	105	F144a	2	125	G1536a	1
86	E384c	8	106	F192a	4	126	G1536b	2
87	E384d	2	107	F192b	2	127	G1536c	1
88	E768a	4	108	F192c	1	128	G1536d	1
89	E768b	4	109	F192d	1	129	H192a	1
90	E768c	2	110	F192e	1	130	H384a	1
91	E1536a	4	111	F192f	1	131	H384b	1
92	F32a	1	112	F384a	2	132	K1536a	1

In thice table the first letter of the I-G denotes the dimension of the matrices (A=1, B=2, C=3, D=4,E=6, F=8, G=12, H=16, K=24) and the number denotes the order of the Image-Group (number of distinct matrices) the final letter is arbitraliy chosen to distinguish images of the same dimension and order.

Concretely, 4 777 irreps corresponding to 80 Lifshitz's stars 0f 230 space groups were classified onto 132 I-Gs; 1 dimensional 2 078 irreps were classified onto 2 I-Gs, 2 dimensional 1 608 irreps were classified onto 10 I-Gs, 3 dimensional 494 irreps were classified onto 5 I-Gs, 4 dimensional 387 irreps were classified onto 42 I-Gs, 6 dimensional 154 irreps were classified onto 32 I-Gs, 8 dimensional 35 irreps were classified onto 23 I-Gs, 12 dimensional 17 irreps were classified onto 14 I-Gs, 16 dimensional 3 irreps were classified onto 3 I-Gs, 24 dimensional one irrep was classified onto one I-G.

There are I-Groups of which the dimension is 12 and the order is 1536, such as G1536b of the I-Groups to find the equivalent I-Groups

It is very difficult to find the unitary transformation matrix with large dimention and ordre by the experiential method.

So, the new method classifying I-Group is the most successful and correct

All the 8 460 irreps of 2811 star channel group corresponding to 80 Lifshitz's stars 0f 230 space groups were classified onto 142 I-Gs by the above procedure. (Table 7)

1 dimensional 3 965 irreps were classified onto 2 I-Gs, 2 dimensional 3 884irreps were classified onto 11 I-Gs, 3 dimensional 582 irreps were classified onto 5 I-Gs, 4 dimensional 575 irreps were classified onto 47 I-Gs, 6 dimensional 154 irreps were classified onto 32 I-Gs, 8 dimensional 50 irreps were classified onto 27 I-Gs, 12 dimensional 17 irreps were classified onto 14 I-Gs, 16 dimensional 3 irreps were classified onto 3 I-Gs, 24 dimensional one irrep was classified onto one I-G.

The irreps of the star channel group of the wave vector was classified onto more 10 I-Groups than the full irreducible representations of the space group
That is 2 dimensional I-Group B4b, 4 dimensional I-Groups D16d, D32f, D64e, D128b, D128c, 8 dimensional I-Groups F64d, F256a, F256b, F256c.

Table 7: The number of the star channel group's irrep corresponding to I-G

№	I-G	n	№	I-G	n	№	I-G	n
1	A1a	230	31	D32a	2	61	D144a	2
2	A2a	3735	32	D32b	60	62	D192a	2
3	B3a	46	33	D32c	60	63	D192b	8
4	B4a	392	34	D32d	9	64	D192c	2
5	B4b	6	35	D32e	8	65	D384a	4
6	B6a	109	36	D32f	6	66	E48a	4
7	B6b	114	37	D36a	4	67	E48b	9
8	B8a	1973	38	D36b	16	68	E48c	2
9	B8b	8	39	D36c	18	69	E96a	2
10	B12a	1188	40	D48a	2	70	E96b	4
11	B12b	14	41	D48b	2	71	E96c	6
12	B16a	24	42	D48c	2	72	E96d	12

13	B24a	10	43	D48d	1	73	E96e	8
14	C12a	59	44	D48e	5	74	E96f	12
15	C24a	81	45	D64a	4	75	E96g	1
16	C24b	109	46	D64b	2	76	E96h	2
17	C24c	109	47	D64c	70	77	E96i	2
18	C48a	224	48	D64d	4	78	E96j	2
19	D8a	16	49	D64e	6	79	E96k	2
20	D12a	15	50	D72a	1	80	E192a	6
21	D16a	2	51	D72b	4	81	E192b	1
22	D16b	3	52	D72c	4	82	E192c	4
23	D16c	138	53	D72d	12	83	E192d	14
24	D16d	3	54	D96a	3	84	E192e	14
25	D18a	16	55	D96b	2	85	E192f	9
26	D24a	3	56	D96c	1	86	E192g	2
27	D24b	3	57	D96d	2	87	E192h	4
28	D24c	5	58	D128a	4	88	E192j	2
29	D24d	7	59	D128b	6	89	E192i	2
30	D24e	20	60	D128c	6	90	E384a	2
№	I-G	n	№	I-G	n	№	I-G	n
91	E384b	2	109	F96d	1	127	G192b	1
92	E384c	8	110	F128a	1	128	G384a	1
93	E384d	2	111	F128b	1	129	G384b	1
94	E768a	4	112	F144a	2	130	G384c	1
95	E768b	4	113	F192a	4	131	G768a	1
96	E768c	2	114	F192b	2	132	G768b	2
97	E1536a	4	115	F192c	1	133	G768c	1
98	F32a	1	116	F192d	1	134	G768d	1
99	F32b	1	117	F192e	1	135	G1536a	1
100	F64a	4	118	F192f	1	136	G1536b	2
101	F64b	1	119	F256a	6	137	G1536c	1
102	F64c	1	120	F256b	3	138	G1536d	2
103	F64d	3	121	F256c	3	139	H192a	1
104	F72a	2	122	F384a	2	140	H384a	1
105	F72b	1	123	F384b	1	141	H384b	1
106	F96a	3	124	F384c	1	142	K1536a	1
107	F96b	1	125	G96a	1			
108	F96c	1	126	G192a	1			

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