A Study on Anti- Fuzzy HX Bi-Ideal of a HX Ring

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Abstract: In this paper, we introduce the notion of an anti-fuzzy HX bi-ideal of a HX ring. We discussed some related properties of an anti-fuzzy HX bi-ideal of a HX ring. We also discussed homomorphic image and pre-image of an anti-fuzzy HX bi-ideal of a HX ring.

Keywords: HX ring, fuzzy HX ring, HX bi- ideal, anti-fuzzy HX bi- ideal

1. Introduction

In 1965, Zadeh [16] introduced the concept of fuzzy sets and studied their properties. In 1967, Rosenfeld [13] defined the idea of fuzzy subgroups and gave some of its properties. Li Hong Xing [7] introduced the concept of HX group. In 1982 Wang-jin Liu [9] introduced the concept of fuzzy ring and fuzzy ideal. Kuroki [5] introduced the notion of fuzzy biideals in semi groups. In 1988 Professor Li Hong Xing [8] proposed the concept of HX ring and derived some of its properties, then Professor Zhong [1, 2] gave the structures of HX ring on a class of ring. Lajos and Szasz [6] initiated the idea of bi-ideals in a ring. T.K.Mukherjee and M.K.Sen [11] fuzzified certain results on rings and ideals.

2. Preliminaries

In this section, we site the fundamental definitions that will be used in the sequel. Throughout this paper, $R = (R, +, \cdot)$ is a Ring, e is the additive identity element of R and xy we mean $x \cdot y$.

3. Anti-fuzzy HX bi-ideal

In this section we define anti fuzzy HX bi-ideal of a HX ring and discuss some of their properties.

3.1 Definition

Let μ be a fuzzy set defined on R. Let $\mathfrak{R} \subset 2^{\mathbb{R}} - \{\phi\}$ be a HX ring. A fuzzy subset λ_{μ} of \mathfrak{R} is said to be an anti-fuzzy HX biideal or anti-fuzzy bi-ideal induced by μ of a HX ring \mathfrak{R} if the following conditions are satisfied. For all A, B, C $\in \mathfrak{R}$,

i. $\lambda_{\mu}(A-B) \leq \max \{ \lambda_{\mu}(A), \lambda_{\mu}(B) \}$

ii. $\lambda_{\mu}(AB) \leq \max \{ \lambda_{\mu}(A), \lambda_{\mu}(B) \}$

iii. λ_{μ} (ABC) $\leq \max \{ \lambda_{\mu} (A), \lambda_{\mu} (C) \}$

where, $\lambda_{\mu}(A) = \min\{ \mu(x) \mid \text{ for all } x \in A \subseteq R \}.$

That is, Let μ be a fuzzy set defined on R. Let $\Re \subset 2^R - \{\phi\}$ be a HX ring. An anti-fuzzy HX subring λ_{μ} of \Re is said to be an anti-fuzzy HX bi-ideal or an anti-fuzzy bi-ideal induced by μ of a HX ring \Re if the following condition is satisfied. For all A, B, C $\in \Re$,

 $\begin{array}{l} \lambda_{\mu}\left(ABC\right) \leq \mbox{ max } \left\{ \begin{array}{l} \lambda_{\mu}\left(A\right),\lambda_{\mu}\left(C\right) \right. \right\} \\ \mbox{where, } \lambda_{\mu}\left(A\right) = \min\{ \ \mu(x) \ / \ \mbox{for all } x \in A \subseteq R \ \right\}. \end{array}$

3.2 Theorem

If μ is an anti-fuzzy bi- ideal of a ring R then the fuzzy subset λ_{μ} is an anti-fuzzy HX bi- ideal of a HX ring \Re .

Proof

Let µ be an anti-fuzzy bi-ideal of R. i.max{ λ_{μ} (A), λ_{μ} (B)}= $\max{\min{\{\mu(x)/\text{for all } x \in A \subseteq R\}},}$ $\min\{\mu(y)/\text{for all } y \in B \subseteq R\}\}$ max { $\mu(x_0), \mu(y_0)$ } $\mu(x_0-y_0)$, since μ is an anti-fuzzy bi-ideal of R \geq \geq min { $\mu(x-y)$ /for all $x-y \in A-B \subseteq R$ } λ_{μ} (A–B) \geq $\lambda_{\mu} (A-B) \leq$ $\max{\{\lambda_{\mu}(A), \lambda_{\mu}(B)\}}$ ii. max { λ_{μ} (A), λ_{μ} (B) } = $\max\{\min\{\mu(x)/\text{for all } x \in A \subseteq R\},\$ $\min\{\mu(y)/\text{for all } y \in B \subseteq R\}\}$ $\max\{ \mu(\mathbf{x}_0), \mu(\mathbf{y}_0) \}$ $\mu(x_0 y_0)$, since μ is an anti-fuzzy bi-ideal of R \geq $\geq \min\{ \mu(xy) / \text{for all } x - y \in A - B \subseteq R \}$ $\geq \lambda_{\mu} (AB)$ λ_{μ} (AB) $\leq \max\{\lambda_{\mu}(A), \lambda_{\mu}(B)\}$ iii.max { λ_{μ} (A), λ_{μ} (C)} = " $\max\{\min\{\mu(x)/\text{for all } x \in A \subseteq R\},\$ $\min\{\mu(z) \mid \text{ for all } z \in C \subseteq R\}\}$ $\max\{ \mu(x_0), \mu(z_0) \}$ = $\mu(x_0y_0 z_0)$, since μ is an anti-fuzzy bi-ideal of R \geq min{ $\mu(xyz)$ /for all $xyz \in ABC \subseteq R$ \geq $\geq \lambda_u (ABC)$ $\lambda_{\mu}(ABC) \leq \max{\{\lambda_{\mu}(A), \lambda_{\mu}(C)\}}$ Hence, λ_{μ} is an anti-fuzzy HX bi-ideal of a HX ring \Re .

3.3 Remark

i. If μ is not an anti-fuzzy bi-ideal of R then the fuzzy subset λ_{μ} of \mathfrak{R} is an anti-fuzzy HX bi-ideal of \mathfrak{R} , provided $|X| \ge 2$ for all $X \in \mathfrak{R}$.

ii. If μ is a fuzzy subset of a ring R and λ_{μ} be an anti-fuzzy HX bi-ideal on $\Re,$ such that

 $\lambda_{\mu} (A) = \min\{\mu(x) \ / \ for \ all \ x \in A \subseteq R \ \}, \ then \ \mu \ may \ or \ may \ not \ be \ an \ anti-fuzzy \ bi-ideal \ of \ R.$

3.4 Theorem

Let μ be a fuzzy subset on R. Let $\Re \subset 2^R - \{\phi\}$ be a HX ring. If λ_{μ} is an anti-fuzzy HX right ideal of a HX ring \Re then λ_{μ} is an anti-fuzzy HX bi-ideal of a HX ring \Re .

Proof

Let λ_{μ} be an anti-fuzzy HX right ideal of a HX ring \mathfrak{R} . Then for all A, B $\in \mathfrak{R}$.

 $\begin{array}{l} \lambda_{\mu} (A-B) &\leq \max \left\{ \lambda_{\mu} (A), \lambda_{\mu} (B) \right\} \\ \lambda_{\mu} (AB) &\leq \lambda_{\mu} (A). \\ \text{Therefore,} \quad \lambda_{\mu} (AB) &\leq \max \left\{ \lambda_{\mu} (A), \lambda_{\mu} (B) \right\} \\ \text{Let } A, B, C \in \mathfrak{R}, \\ \lambda_{\mu} (ABC) &= \lambda_{\mu} (A(BC)) \\ &\leq \lambda_{\mu} (A), \\ &\leq \max \left\{ \lambda_{\mu} (A), \lambda_{\mu} (C) \right\}. \\ \lambda_{\mu} (ABC) &\leq \max \left\{ \lambda_{\mu} (A), \lambda_{\mu} (C) \right\}. \end{array}$

Hence, λ_{μ} is an anti-fuzzy HX bi-ideal of a HX ring \Re .

3.5 Theorem

Let μ be a fuzzy subset on R. Let $\Re \subset 2^{R} - \{\phi\}$ be a HX ring. If λ_{μ} is an anti-fuzzy HX left ideal of a HX ring \Re then λ_{μ} is an anti-fuzzy HX bi-ideal of a HX ring \Re .

Proof

Let λ_{μ} be an anti-fuzzy HX left ideal of a HX ring \mathfrak{N} . Then for all A, B $\in \mathfrak{N}$. $\lambda_{\mu} (A-B) \leq \max \{ \lambda_{\mu} (A), \lambda_{\mu} (B) \}$ $\lambda_{\mu} (AB) \leq \lambda_{\mu} (B)$.

 $\begin{array}{ll} Therefore, & \lambda_{\mu}(AB) \leq \max \left\{ \begin{array}{l} \lambda_{\mu}\left(A\right), \lambda_{\mu}\left(B\right) \right\} \\ Let \ A, \ B, \ C \in \mathfrak{R}, \\ & \lambda_{\mu}\left(ABC\right) & = \lambda_{\mu}(\ (AB)C) \\ & \leq \lambda_{\mu}(\ C), \\ & \leq \max \left\{ \begin{array}{l} \lambda_{\mu}\left(A\right), \lambda_{\mu}\left(C\right) \right\}. \end{array} \end{array}$

 λ_{μ} (ABC) $\leq \max \{ \lambda_{\mu} (A), \lambda_{\mu} (C) \}.$ Hence, λ_{μ} is an anti-fuzzy HX bi-ideal of a HX ring \Re .

3.6 Remark

Every anti-fuzzy (right or left) HX ideal of a HX ring \Re is an anti-fuzzy HX bi-ideal of a HX ring \Re .

3.7 Theorem

Let μ and η be two fuzzy sets defined on R. Let λ_{μ} and γ_{η} be any two anti-fuzzy HX bi-ideals of a HX ring \Re then $\lambda_{\mu} \cap \gamma_{\eta}$ is also anti-fuzzy HX bi-ideal of a HX ring \Re .

Proof

Let A, B, C $\in \Re$ i. $(\lambda_{\mu} \cap \gamma_{\eta})(A-B) = \min \{\lambda_{\mu} (A-B), \gamma_{\eta} (A-B)\}$ $\leq \min \{\max \{\lambda_{\mu} (A), \lambda_{\mu} (B)\}, \max \{\gamma_{\eta} (A), \gamma_{\eta} (B)\}\}$ $\leq \max \{\min \{\lambda_{\mu} (A), \gamma_{\eta} (A)\}, \min \{\lambda_{\mu} (B), \gamma_{\eta} (B)\}\}$ $\leq \max \{(\lambda_{\mu} \cap \gamma_{\eta})(A), (\lambda_{\mu} \cap \gamma_{\eta}) (B)\}.$ Therefore, $(\lambda_{\mu} \cap \gamma_{\eta})(A-B) \leq \max \{(\lambda_{\mu} \cap \gamma_{\eta})(A), (\lambda_{\mu} \cap \gamma_{\eta}) (B)\}.$

ii.
$$(\lambda_{\mu} \cap \gamma_{\eta})(AB) = \min \{ \lambda_{\mu}(AB), \gamma_{\eta}(AB) \}$$

 $\leq \min \{ \max \{ \lambda_{\mu} (A), \lambda_{\mu} (B) \}, \max \{ \gamma_{\eta} (A), \gamma_{\eta} (B) \} \}$ $\leq \max \{ \min \{ \lambda_{\mu} (A), \gamma_{\eta} (A) \}, \min \{ \lambda_{\mu} (B), \gamma_{\eta} (B) \} \}$ $\geq \max \{ (\lambda_{\mu} \cap \gamma_{\eta})(A), (\lambda_{\mu} \cap \gamma_{\eta}) (B) \}$ refore.

Therefore,

 $\begin{array}{ll} (\lambda_{\mu} \cap \gamma_{\eta})(AB) & \geq \max\{(\lambda_{\mu} \cap \gamma_{\eta})(A), (\lambda_{\mu} \cap \gamma_{\eta})(B)\}.\\ & \text{iii.} & (\lambda_{\mu} \cap \gamma_{\eta})(ABC) = \min\{\lambda_{\mu}(ABC), \gamma_{\eta}(ABC)\}\\ & \leq \min\{\max\{\lambda_{\mu}(A), \lambda_{\mu}(C)\}, \max\{\gamma_{\eta}(A), \gamma_{\eta}(C)\}\}\\ & \leq \max\{\min\{\lambda_{\mu}(A), \gamma_{\eta}(A)\}, \min\{\lambda_{\mu}(C), \gamma_{\eta}(C)\}\}\\ & \leq \max\{(\lambda_{\mu} \cap \gamma_{\eta})(A), (\lambda_{\mu} \cap \gamma_{\eta})(C)\}\\ & \text{Therefore,} \end{array}$

 $\begin{array}{l} (\lambda_{\mu} \cap \gamma_{\eta})(ABC) \leq \mbox{ max}\{(\lambda_{\mu} \cap \gamma_{\eta})(A), \, (\lambda_{\mu} \cap \gamma_{\eta}) \, (C)\}. \\ \mbox{Hence, } \lambda_{\mu} \cap \gamma_{\eta} \mbox{ is an anti-fuzzy HX bi-ideal of a HX ring } \mathfrak{R}. \end{array}$

3.8 Theorem

Let λ_{μ} be an anti-fuzzy HX (right or left) ideal and γ_{η} be an anti-fuzzy HX (right or left) ideal then $\lambda_{\mu} \cap \gamma_{\eta}$ is an anti-fuzzy HX bi-ideal of \mathfrak{R} .

Proof

It is clear.

3.9 Theorem

Let μ and η be any two fuzzy sets of R. Let $\mathfrak{R} \subset 2^R - \{\phi\}$ be a HX ring and if λ_{μ} and γ_{η} are any two anti-fuzzy HX biideals of \mathfrak{R} then $(\lambda_{\mu} \cup \gamma_{\eta})$ is also an anti-fuzzy HX biideal of a HX ring \mathfrak{R} .

Proof It is clear.

3.10 Definition

Let μ and η be fuzzy subsets of the rings R_1 and R_2 respectively. Let λ_{μ} and γ_{η} be two anti-fuzzy HX bi-ideals of the HX rings \Re_1 and \Re_2 then the cartesian anti-product of λ_{μ} and γ_{η} is defined as $(\lambda_{\mu} \times \gamma_{\eta})$ (A, B) = max { λ_{μ} (A), γ_{η} (B)} for every (A, B) $\in \Re_1 \times \Re_2$.

3.11 Theorem

If λ_{μ} and γ_{η} are any two anti-fuzzy HX bi-ideals of HX rings \Re_1 and \Re_2 then $\lambda_{\mu} \times \gamma_{\eta}$ is also an anti-fuzzy HX bi-ideal of $\Re_1 \times \Re_2$.

Proof

Let λ_{μ} and γ_{η} be any two anti-fuzzy HX bi-ideals of \Re_1 and \Re_2 respectively

Let A, B, C $\in \mathfrak{R}_1 \times \mathfrak{R}_2$ where A = (D, E), B = (F, G), C =(H, I). i. $(\lambda_\mu \times \gamma_\eta) (A - B) = (\lambda_\mu \times \gamma_\eta) ((D, E) - (F, G))$ $= (\lambda_\mu \times \gamma_\eta) (D - F, E - G)$ $= \max\{\lambda_\mu (D - F), \gamma_\eta (E - G)\}$ $\leq \max\{\max\{\lambda_\mu (D), \lambda_\mu (F)\}, \max\{\gamma_\eta (E), \gamma_\eta (G)\}\}$ $\leq \max\{\max\{\lambda_\mu (C), \gamma_\eta (E)\}, \max\{\lambda_\mu (F), \gamma_\eta (G)\}\}$ $\leq \max\{(\lambda_\mu \times \gamma_\eta) (C, E), (\lambda_\mu \times \gamma_\eta) (F, G)\}$

= max{ $(\lambda_{\mu} \times \gamma_{\eta})$ (A), $(\lambda_{\mu} \times \gamma_{\eta})$ (B) } $(\lambda_{\mu} \times \gamma_{\eta})$ $(A-B) \leq \max\{ (\lambda_{\mu} \times \gamma_{\eta}) (A), (\lambda_{\mu} \times \gamma_{\eta}) (B) \}.$ ii. $(\lambda_{\mu} \times \gamma_{\eta})(AB) = (\lambda_{\mu} \times \gamma_{\eta})((D, E) (F, G))$ $= (\lambda_{u} \times \gamma_{n})(DF, EG)$ $= \max{\{\lambda_{\mu}(DF), \gamma_{\eta}(EG)\}}$ $\leq \max\{\max\{\lambda_{\mu}(D), \lambda_{\mu}(F)\},\$ $\max\{\gamma_{\eta}(E), \gamma_{\eta}(G)\}\}$ $= \max\{\max\{\lambda_{\mu}(D), \gamma_{\eta}(E)\},\$ $\max{\{\lambda_{\mu}(F), \gamma_{\eta}(G)\}}$ $\leq \max\{(\lambda_{\mu} \times \gamma_{\eta})(D, E), (\lambda_{\mu} \times \gamma_{\eta})(F, G)\}$ = max{ $(\lambda_{\mu} \times \gamma_{\eta})(A), (\lambda_{\mu} \times \gamma_{\eta})(B)$ } $(\lambda_{\mu} \times \gamma_{\mu})(AB) \le \max\{(\lambda_{\mu} \times \gamma_{\mu})(A), (\lambda_{\mu} \times \gamma_{\mu})(B)\}$ iii. $(\lambda_{\mu} \times \gamma_{\eta})(ABC) = (\lambda_{\mu} \times \gamma_{\eta})(DFH, EGI)$ $= \max{\{\lambda_{\mu}(DFH), \gamma_{n}(EGI)\}}$ $\leq \max\{\max\{\lambda_{\mu}(D), \lambda_{\mu}(H)\}\}$, max{ $\gamma_{\eta}(E), \gamma_{\eta}(I)$ } $= \max\{\max\{\lambda_{\mu}(D), \gamma_{\eta}(E)\},\$ $max\{\lambda_{\mu}(H), \gamma_{\eta}(I)\}\}$ $\leq \max\{(\lambda_{\mu} \times \gamma_{\eta})(D, E), (\lambda_{\mu} \times \gamma_{\eta})(H, I)\}$ $(\lambda_{\mu} \times \gamma_{\eta})(ABC) \leq \max\{(\lambda_{\mu} \times \gamma_{\eta})(A), (\lambda_{\mu} \times \gamma_{\eta})(C)\}$ Hence, $\lambda_{\mu} \times \gamma_{\eta}$ is an anti-fuzzy HX bi-ideal of $\Re_1 \times \Re_2$.

3.12 Theorem

Let μ be a fuzzy set defined on R. Let λ^{μ} is a fuzzy HX biideal of \Re if and only if $(\lambda^{\mu})^{c}$ is an anti-fuzzy HX bi-ideal of \Re .

Proof

Let λ^{μ} be a fuzzy HX bi-ideal of \Re . Let A, $B \in \Re$ $i.\lambda^{\mu}(A-B) \geq \min\{\lambda^{\mu}(A),\lambda^{\mu}(B)\}$ \Leftrightarrow $1 - (\lambda^{\mu})^{c}(A-B) \geq \min \{ (1 - (\lambda^{\mu})^{c}(A)), (1 - (\lambda^{\mu})^{c}(B)) \}$ $(\lambda^{\mu})^{c}(A-B) \leq 1 - \min \{ (1 - (\lambda^{\mu})^{c}(A)), (1 - (\lambda^{\mu})^{c}(B)) \}$ \Leftrightarrow max { $(\lambda^{\mu})^{c}(A), (\lambda^{\mu})^{c}(B)$ } \Leftrightarrow $(\lambda^{\mu})^{c}(A-B) \leq \Delta^{\mu}$ ii. $\lambda^{\mu}(AB) \geq$ min{ $\lambda^{\mu}(A), \lambda^{\mu}(B)$ } $1 - (\lambda^{\mu})^{c}(AB) \ge \min \{ (1 - (\lambda^{\mu})^{c}(A)), (1 - (\lambda^{\mu})^{c}(B)) \}$ \Leftrightarrow $(\lambda^{\mu})^{c}(AB) \leq 1 - \min \{ (1 - (\lambda^{\mu})^{c}(A)), (1 - (\lambda^{\mu})^{c}(B)) \}$ \Leftrightarrow $(\lambda^{\mu})^{c}(AB) \leq \max \{ (\lambda^{\mu})^{c}(A), (\lambda^{\mu})^{c}(B) \}$ \Leftrightarrow iii. $\lambda^{\mu}(ABC) \geq \min\{\lambda^{\mu}(A), \lambda^{\mu}(C)\}$ $1 - (\lambda^{\mu})^{c}(ABC) \geq \min\{(1 - (\lambda^{\mu})^{c}(A)), (1 - (\lambda^{\mu})^{c}(C))\}$ $(\lambda^{\mu})^{c}(ABC) \leq 1 - \min \{ (1 - (\lambda^{\mu})^{c}(A)), (1 - (\lambda^{\mu})^{c}(C)) \}$ \Leftrightarrow $\Leftrightarrow (\lambda^{\mu})^{c}(ABC) \leq$ max { $(\lambda^{\mu})^{c}(A), (\lambda^{\mu})^{c}(C)$ } Hence, $(\lambda^{\mu})^{c}$ is an anti-fuzzy HX bi-ideal on \Re .

4. Homomorphism and anti homomorphism of an anti-fuzzy HX bi-ideal of a HX ring **R**

In this section, we discuss the properties of fuzzy HX bi-ideal of a HX ring \Re under homomorphism and anti homomorphism.

4.1 Definition

Let R_1 and R_2 be any two rings. Let $\mathfrak{R}_1 \subset 2^{R_1} - \{\phi\}$ and $\mathfrak{R}_2 \subset 2^{R_2} - \{\phi\}$ be any two HX rings defined on R_1 and R_2 respectively. Let μ and α be any two fuzzy subsets in R_1 and R_2 respectively. Let λ_{μ} and η_{α} be anti-fuzzy HX bi-ideals defined on \mathfrak{R}_1 and \mathfrak{R}_2 respectively induced by μ and α . Let f:

 $\mathfrak{R}_1 \rightarrow \mathfrak{R}_2$ be a mapping then the anti-image of λ_{μ} denoted as f (λ_{μ}) is a fuzzy subset of \mathfrak{R}_2 defined as for each $U \in \mathfrak{R}_2$, (f (λ_{μ})) (U) =

$$\begin{cases} \inf \{\lambda_{\mu}(X): X \in f^{-1}(U)\}, & \text{if } f^{-1}(U) \neq \phi \\ 1, & \text{otherwise} \end{cases}$$

Also the anti-pre-image of η^{α} denoted as $f^{-1}(\eta_{\alpha})$ under f is a fuzzy subset of \mathfrak{R}_1 defined as for each $X \in \mathfrak{R}_1$, $(f^{-1}(\eta_{\alpha}))(X) = \eta_{\alpha}(f(X)).$

4.2 Theorem

Let \Re_1 and \Re_2 be any two HX rings on the rings R_1 and R_2 respectively. Let $f: \Re_1 \to \Re_2$ be a homomorphism onto HX rings. Let λ_μ be an anti-fuzzy HX bi-ideal of \Re_1 then $f(\lambda_\mu)$ is an anti-fuzzy HX bi-ideal of \Re_2 , if λ_μ has an infimum property and λ_μ is f-invariant.

Proof

Let μ be a fuzzy subset of R_1 and λ_{μ} is an anti-fuzzy HX bi-ideal of \mathfrak{R}_1 .

There exist X, Y, Z $\in \Re_1$ such that f(X), f(Y), f (Z) $\in \Re_2$, i. $(f(\lambda_{\mu}))(f(X) - f(Y)) = (f(\lambda_{\mu}))(f(X-Y)),$ $\lambda_{\mu}(X-Y)$ $\leq \max \{\lambda_{\mu}(X), \lambda_{\mu}(Y)\}$ $= \max \{ (f(\lambda_{\mu})) (f(X)), (f(\lambda_{\mu})) (f(Y)) \} \}$ Therefore, $(f(\lambda_{\mu}))(f(X) - f(Y)) \leq$ $\max \{ (f(\lambda_{\mu}))(f(X)), (f(\lambda_{\mu}))(f(Y)) \}.$ $(f(\lambda_u))(f(X) f(Y)) = (f(\lambda_u))(f(XY)),$ ii. $\lambda_{u}(XY)$ $\leq \max \{\lambda_{\mu}(X), \lambda_{\mu}(Y)\}$ max {(f (λ_{μ})) (f(X)), (f (λ_{μ})) (f(Y))} Therefore, $(f(\lambda_{\mu}))(f(X)f(Y))$ $\leq \max \{ (f(\lambda_{\mu})) (f(X), (f(\lambda_{\mu})) (f(Y)) \}.$ iii.($f(\lambda_{\mu})$) (f(X)f(Y)f(Z))= ($f(\lambda_{\mu})$) (f(XYZ)) $= \lambda_{\mu} (XYZ)$ $\leq \max \{\lambda_{\mu}(X), \lambda_{\mu}(Z)\}$ $= \max\{(f(\lambda_u))(f(X)), (f(\lambda_u))(f(Z))\}$ $(f(\lambda_{\mu})) (f(X)f(Y)f(Z)) \leq max\{(f(\lambda_{\mu})) (f(X)), (f(\lambda_{\mu})) (f(Z))\}$

Hence, $(f(\lambda_{\mu}))$ is an anti-fuzzy HX bi- ideal on \Re_2 .

4.3 Theorem

Let \mathfrak{R}_1 and \mathfrak{R}_2 be any two HX rings on R_1 and R_2 respectively. Let $f: \mathfrak{R}_1 \to \mathfrak{R}_2$ be a homomorphism on HX rings. Let η_α be an anti-fuzzy HX bi-ideal of \mathfrak{R}_2 then $f^{-1}(\eta_\alpha)$ is an anti-fuzzy HX bi- ideal of \mathfrak{R}_1 .

Proof

Let μ be a fuzzy subset of R_1 and η_α is an anti-fuzzy HX biideal of $\mathfrak{R}_1.$

There exist X, Y, Z $\in \mathfrak{R}_1$ such that f(X), f(Y), f (Z) $\in \mathfrak{R}_2$, i. $(f^{-1}(\eta_\alpha))(X-Y) = \eta_\alpha (f(X-Y))$ $= \eta_\alpha (f(X) - f(Y))$ $\leq \max \{\eta_\alpha (f(X)), \eta_\alpha (f(Y))\}$ $= \max \{(f^{-1}(\eta_\alpha))(X), (f^{-1}(\eta_\alpha))(Y)\}$

Therefore, $(f^{-1}(\eta_{\alpha}))(X-Y) \leq$

 $\max\{(f^{-1}(\eta_{\alpha}))(X), (f^{-1}(\eta_{\alpha}))(Y)\}$ $(f^{-1}(\eta_{\alpha}))(XY)$ ii. $\eta_{\alpha}(f(XY))$ = $\eta_{\alpha}\left(f(X)\;f(Y)\right)$ = max { η_{α} (f(X)), η_{α} (f(Y))} \leq max {($f^{-1}(\eta_{\alpha})$) (X), ($f^{-1}(\eta_{\alpha})$) (Y)} Therefore, $(f^{-1}(\eta_{\alpha}))(XY) \leq$ $\max\{(f^{-1}(\eta_{\alpha}))(X), (f^{-1}(\eta_{\alpha}))(Y)\}.$ iii. $(f^{-1}(\eta_{\alpha}))(XYZ) =$ $\eta_{\alpha}(f(XYZ))$ $= \eta_{\alpha} (f(X) f(Y) f(Z))$ $\max \{\eta_{\alpha} (f(X)), \eta_{\alpha} (f(Z))\}$ \leq = max {($f^{-1}(\eta_{\alpha})$) (X), ($f^{-1}(\eta_{\alpha})$) (Z)} $\max\{(f^{-1}(\eta_{\alpha}))(X), (f^{-1}(\eta_{\alpha}))(Z)\}$ $\max\{(f^{-1}(\eta_{\alpha}))(X), (f^{-1}(\eta_{\alpha}))(Z)\}\$ $(f^{-1}(\eta_{\alpha}))(XYZ)$ \leq Therefore, f⁻¹(η_{α}) is an anti-fuzzy HX bi- ideal of \Re_1 .

4.4 Theorem

Let \Re_1 and \Re_2 be any two HX rings on the rings R_1 and R_2 respectively. Let $f: \Re_1 \rightarrow \Re_2$ be an anti-homomorphism onto HX rings. Let λ_{μ} be an anti-fuzzy HX bi-ideal of \Re_1 then f (λ_{μ}) is an anti-fuzzy HX bi-ideal of \Re_2 , if λ_{μ} has an infimum property and λ_{μ} is f-invariant.

Proof

Let μ be a fuzzy subset of R_1 and λ_{μ} is an anti-fuzzy HX biideal of \Re_1 .

There exist X, Y, Z $\in \mathfrak{R}_1$ such that f(X), f(Y), $f(Z) \in \mathfrak{R}_2$, i. $(f(\lambda_{\mu}))(f(X) - f(Y)) = (f(\lambda_{\mu}))(f(Y-X)),$ $\lambda_u (Y-X)$ $\leq \max \{\lambda_{\mu}(\mathbf{Y}), \lambda_{\mu}(\mathbf{X})\}$ $\leq \max \{\lambda_{\mu}(X), \lambda_{\mu}(Y)\}$ $= \max\{(f(\lambda_{u}))(f(X)), (f(\lambda_{u}))(f(Y))\}$ Therefore, $(f(\lambda_{\mu}))(f(X) - f(Y)) \leq$ max {(f (λ_{μ})) (f(X)), (f (λ_{μ})) (f(Y))}. ii. $(f(\lambda_{\mu}))(f(X) f(Y)) = (f(\lambda_{\mu}))(f(YX)),$ $= \lambda_{\mu} (YX)$ $\leq \max \{\lambda_{\mu}(\mathbf{Y}), \lambda_{\mu}(\mathbf{X})\}$ $\leq \max \{\lambda_{\mu}(X), \lambda_{\mu}(Y)\}$ $= \max\{(f(\lambda_{\mu}))(f(X)), (f(\lambda_{\mu}))(f(Y))\}$ Therefore, $(f(\lambda_{\mu}))(f(X)f(Y))$ \leq $\max \{ (f(\lambda_{\mu})) (f(X), (f(\lambda_{\mu})) (f(Y)) \}.$ iii. $(f(\lambda_{\mu}))(f(X)f(Y)f(Z)) = (f(\lambda_{\mu}))(f(ZYX))$ $= \lambda_{\mu} (ZYX)$ $\leq \max \{\lambda_{\mu}(Z), \lambda_{\mu}(X)\}$ $\leq \max \{\lambda_{\mu}(X), \lambda_{\mu}(Z)\}$ $= \max\{(f(\lambda_{\mu}))(f(X)), (f(\lambda_{\mu}))(f(Z))\}$ $(f(\lambda_{\iota\iota})) (f(X)f(Y)f(Z)) \leq max\{(f(\lambda_{\iota\iota})) (f(X)), (f(\lambda_{\iota\iota})) (f(Z))\}$ Hence, (f (λ_{μ})) is an anti-fuzzy HX bi- ideal on \Re_2 .

4.5 Theorem

Let \mathfrak{R}_1 and \mathfrak{R}_2 be any two HX rings on R_1 and R_2 respectively. Let $f : \mathfrak{R}_1 \to \mathfrak{R}_2$ be an anti-homomorphism on HX rings. Let η_{α} be an anti-fuzzy HX bi-ideal of \mathfrak{R}_2 then f^{-1} (η_{α}) is an anti-fuzzy HX bi- ideal of \mathfrak{R}_1 .

Proof

Let μ be a fuzzy subset of R₁ and η_{α} is an anti-fuzzy HX biideal of \Re_1 . There exist X, Y, Z $\in \mathfrak{R}_1$ such that f(X), f(Y), $f(Z) \in \mathfrak{R}_2$, i. $(f^{-1}(\eta_{\alpha}))(X-Y) = \eta_{\alpha}(f(X-Y))$ $= \eta_{\alpha}(f(Y) - f(X))$ $\leq \max \{\eta_{\alpha}(f(Y)), \eta_{\alpha}(f(X))\}$ $\leq \max \{\eta_{\alpha}(f(X)), \eta_{\alpha}(f(Y))\}$ max {($f^{-1}(\eta_{\alpha})$) (X), ($f^{-1}(\eta_{\alpha})$) (Y)} = Therefore, $(f^{-1}(\eta_{\alpha}))(X-Y)$ \leq $\max\{(f^{-1}(\eta_{\alpha}))(X), (f^{-1}(\eta_{\alpha}))(Y)\}$ ii. $(f^{-1}(\eta_{\alpha}))(XY) =$ η_{α} (f(XY)) $\eta_{\alpha} \left(f(Y) f(X) \right)$ = max { $\eta_{\alpha}(f(Y)), \eta_{\alpha}(f(X))$ } \leq max { η_{α} (f(X)), η_{α} (f(Y))} \leq $max \; \{(f^{-1}(\eta_\alpha)) \; (X), \, (f^{-1}(\eta_\alpha)) \; (Y) \}$ Therefore, $(f^{-l}(\eta_{\alpha}))(XY) \leq$ $\max\{(f^{-1}(\eta_{\alpha}))(X), (f^{-1}(\eta_{\alpha}))(Y)\}.$ iii. $(f^{-1}(\eta_{\alpha}))(XYZ) =$ $\eta_{\alpha}(f(XYZ))$ $= \eta_{\alpha} \left(f(Z) f(Y) f(X) \right)$ max { η_{α} (f(Z)), η_{α} (f(X))} \leq max { η_{α} (f(X)), η_{α} (f(Z))} \leq = max { ($f^{-1}(\eta_{\alpha})$) (X), ($f^{-1}(\eta_{\alpha})$) (Z) } $= \max\{(f^{-1}(\eta_{\alpha}))(X), (f^{-1}(\eta_{\alpha}))(Z)\} \\ \le \max\{(f^{-1}(\eta_{\alpha}))(X), (f^{-1}(\eta_{\alpha}))(Z)\}$ $(f^{-1}(\eta_{\alpha}))(XYZ)$ Therefore, $f^{-1}(\eta_{\alpha})$ is an anti-fuzzy HX bi- ideal of \Re_1 .

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