Hence expanding the exponent of the wave function ψ_0 in the powers of y we get

$$\psi_0(y) = \text{const}\exp\{-(c_2y^2 + c_4y^4 + c_6y^6 + ...)\} \quad (6)$$

where

$$c_2 = a_1^{\frac{1}{2}},\ c_4 = \frac{1}{2}a_2a_1^{-\frac{1}{2}},\ c_6 = \frac{\lambda}{16}a_1^{-\frac{1}{2}} - \frac{1}{8}a_2^2a_1^{-\frac{3}{2}}, \text{etc.} \quad (7)$$

Therefore, near the origin we get

$$\frac{1}{2}\frac{d^2}{dy^2}\psi_0(y)\{-c_2 + 2(c_2^2 - 3c_4)y^2 - (15c_6 - 8c_2c_4)y^4 + ...\} \quad (8)$$

Substituting equation (7) into equation (2) and equating the coefficients of y^0, y^2, y^4 to zero, we get the following equations:

$$a_1^{\frac{1}{2}} - \varepsilon = 0,$$

$$-3a_2a_1^{-\frac{1}{2}} + 2a_1 - \frac{1}{2} = 0 \quad (8)$$

$$\text{and}\ \ 4a_2 - \frac{15}{2}a_1^{\frac{1}{2}}\left\{\frac{a_3}{a_1} - \frac{1}{4}\left(\frac{a_2}{a_1}\right)^2\right\} - \mu = 0$$

a_1 and a_2 can be solved in terms of a_3, where $a_3 = \frac{\lambda}{8}$. Hence we get the following characteristic equation for ε:

$$1008\varepsilon^4 - 312\varepsilon^2 - 288\mu\varepsilon - 270\lambda + 15 = 0 \quad (9)$$

4. The Energy eigenvalues

Equation (9) can be solved using Mathematica [17] for different chosen values of the anharmonic coefficients μ and λ. The results are tabulated in Table 1.

The ground state Energy eigenvalues $\varepsilon\left(\frac{E}{\hbar\omega}\right)$ of a sextic anharmonic oscillator including quartic anharmonicity

λ	$\mu = 0$	$\mu = 0.1$	$\mu = 0.5$	$\mu = 1.0$	$\mu = 10$	$\mu = 100$	$\mu = 1000$
0	0.500000	0.559512	0.695777	0.801717	1.49006	3.09068	6.60199
0.01	0.512985	0.567431	0.699199	0.803799	1.49035	3.09072	6.60199
0.1	0.586616	0.621538	0.727012	0.8215461	1.49288	3.09099	6.60202
1.0	0.82528	0.835627	0.88996	0.945755	1.51736	3.09374	6.6023
10	1.33946	1.34381	1.36104	1.3822	1.704	3.12067	6.60509
100	2.30892	2.3103	2.31581	2.32268	2.44259	3.35012	6.6328
1000	4.06465	4.06509	4.0683	4.06901	4.10805	4.47766	6.88755
10000	7.20484	7.20498	7.20553	7.20622	7.21863	7.34153	8.45782
20000	8.56431	8.5644	8.56479	8.56528	8.57406	8.66134	9.48539

5. Results and Discussions

For λ and μ both vanishing we clearly get the harmonic oscillator eigenvalue for ε, namely, 0.5, as we should. As λ and μ increase the ground state energies increase slowly. The method of evaluating the ground state energies is rather simple compared to other methods cited in the introduction and is very much valid for small values of μ and λ. By including more number of parameters in the interpolative wave function it may be possible to improve the numerical accuracy of the energy eigenvalues. Further, one can use this method to estimate the excited state energies of anharmonic oscillators by using a wave function that is a product of a suitable polynomial with the ground state wave function used in this article.

Acknowledgements

The authors thank Dr. Sathyanarayana, Department of mathematics, Vijaya College, Bangalore for his assistance in computations using Mathematica.

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