in this section. An assumption for the proposed method to work is that, in the capture-mark-recapture method, the markings should be numbered so that, on the second sampling occasion, the information will not be limited to the number m_2 of individuals that got captured on both the occasions. An additional information on their serial numbers $t_1, t_2, ..., t_{m_2}$ will also be available, where each of the t_i's can be any positive integer between 1 and n_1. Using the standard notation, let $t_{(1)}, t_{(2)}, ..., t_{(m_2)}$ be the ordered values of $t_1, t_2, ..., t_{m_2}$. Writing $t_{m_2} = m$ the German tank estimate is given by $\widehat{n_1} = (1 + \frac{1}{n_2})\, m - 1$

It is interesting to note that n_1 is already known and it is therefore possible to know whether the sample obtained on the second occasion overestimates or underestimates n_1. This information will then be useful in making an appropriate improvement in $\widehat{N}$ to make it more accurate.

Suppose we take the ratio $\frac{n_1}{\widehat{n_1}}$ as the correctional factor and define $\widehat{N}^* = \frac{n_1}{\widehat{n_1}}\widehat{N}$. The new estimator will reduce the value of $\widehat{N}$ whenever $n_1 < \widehat{n_1}$. On the other hand, the new estimator will enlarge the value of $\widehat{N}$ whenever $n_1 > \widehat{n_1}$. When $n_1 < \widehat{n_1}$, $\widehat{n_1}$ overestimates n_1 and so does $\widehat{N}$. Similarly, when $n_1 > \widehat{n_1}$, $\widehat{n_1}$ underestimates n_1 so does $\widehat{N}$. This is how the correctional factor is justified.

As an aside, it is also interesting to note that one more estimate of the population size N can be constructed by combining the samples obtained on the two occasions. The number of distinct individuals in the two samples taken together is
$r = m_2 + (n_1 - m_2) + (n_2 - m_2) = n_1 + n_2 - m_2$

Further, the probability that an individual is included in at least one of the two samples is given by $p^* = 1 - (1 - p_1)(1 - p_2)$

Hence, an estimate of N is given by
$$\widehat{N} = \frac{r}{p^*} = \frac{n_1+n_2-m_2}{1-(1-p_1)(1-p_2)}.$$

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