Performance Review of Successive Cancellation Decoding Methods of Polar Codes

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Abstract: The first provably capacity achieving codes with low complexity named polar codes were discovered recently and the successive cancellation (SC) decoding is widely known decoding algorithm for polar codes. There are many techniques like folded SC and permuted SC available for improve the SC decoding algorithm. In this paper, we study the different methods of SC decoding for polar codes which are recently developed and give a comparison based on bit error rate (BER) and decoding complexity.

Keywords: Polar codes; SC decoding; List Decoding; Decoding Latency; Channel Capacity; BER; SNR; Folded Structure.

1. Introduction

Polar codes, the first provable class of channel capacity achieving codes introduced by Arikan [Arikan, 2009] since Shannon presented the noisy channel coding theorem [Shannon, 1948]. The standard SC decoding algorithm presented in [Tal, 2015] have $O(N \log N)$ decoding complexity, where N is the code length. But for achieving channel capacity it requires large size of block length so in that way to reduce block length and hence reduced decoding complexity and improve BER many methods [Li, 2012], [Chan, 2013] and [Haung, 2013] have been introduced.

In this paper some latest methods of SC decoding [Kahraman, 2014], [Vangala, 2014] and [Liu, 2015] are presented and compared for decoding complexity and BER performances.

2. Background

Polar Codes

Let,
$$F = \begin{bmatrix} 0 & 1 \\ 1 & 1 \end{bmatrix}$$
, $F^{\otimes n}$ is a $N \times N$ matrix, where

 $N = 2^n$, $\otimes n$ denotes the *n* th Kronecker power, and $F^{\otimes n} = F \otimes F^{\otimes (n-1)}$. Let the *n*-bit binary representation of integer *i* be $b_{n-1}, b_{n-2}, ..., b_0$. The *n*- bit binary representation $b_0, b_1, ..., b_{n-1}$ is a bit-reversal order of *i*. The generator matrix of polar code is defined as $G_N = B_N F^{\otimes n}$, where B_N is a bit-reversal permutation matrix. The polar code is generated by

$$x_1^N = u_1^N G_N = u_1^N B_N F^{\otimes n} \qquad \text{Eq. 1}$$

where $x_1^N = (x_1, x_2, ..., x_n)$ is the encoded bit sequence, and $u_1^N = (u_1, u_2, ..., u_N)$ is the encoding bit sequence. The bit indexes of u_1^N are divided into two subsets: the one containing the information bits represented by A and the other containing the frozen bits represented by A^c . The polar code can be further expressed as

$$x_1^N = u_A G_N(A) \oplus u_{A^c} G_N(A^c)$$
 Eq. 2

where $G_N(A)$ denotes the submatrix of G_N formed by the rows with indices in A, and $G_N(A^c)$ denotes the submatrix of G_N formed by the rows with indices in A^c . u_A are the information bits, and u_{A^c} are the frozen bits. Polar codes can be decoded with the very efficient SC decoder, which has a decoding complexity of $O(N \log N)$ and can achieve capacity when N is very large.

3. Conventional SC Decoding

Consider a polar code with parameters (N, K, A, u_{A^c}) [Arikan, 2009]. where N, K, u, and u_{A^c} denote the code length, information length, set of information bits, and frozen bit values, respectively. The estimation is $\widehat{u}_1^N = (\widehat{u}_1, ..., \widehat{u}_N)$. If \widehat{u}_i is a frozen bit, $\widehat{u}_i = 0$. Otherwise, if u_i is an information bit, then

$$\widehat{u}_{i} = \begin{cases} 0, if W_{n}^{(i)}(y_{1}^{N}, \widehat{u}_{1}^{i-1}/0) \ge W_{N}^{i}(y_{1}^{N}, \widehat{u}_{1}^{i-1}/1), \\ 1, if W_{n}^{(i)}(y_{1}^{N}, \widehat{u}_{1}^{i-1}/0) < W_{N}^{i}(y_{1}^{N}, \widehat{u}_{1}^{i-1}/1), \end{cases} \text{Eq. 3}$$

Where $W_N^i(y_1^N, \hat{u}_1^{i-1})$ is the channel transition probability or likelihood probability. For better robust-ness and lower complexity, S decoder over logarithm domain is more preferred. Define the log likelihood probability as

$$L_N^{(i)}(y_1^N, \hat{u}_1^{i-1}/u_i) \Box \ln W_N^{(i)}(y_1^N, \hat{u}_1^{i-1}/u_i)$$
 Eq. 4

4. Conventional SC List Decoder

For each step of SC decoder, only the most likely bit decision survived. Whenever certain bit is incorrectly

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decoded, the corresponding decoding fails. In order to deal with this error propagation, an improved SC decoder, named SCL decoder, is proposed. SCL decoder can be regarded as a breadth first search (BFS) version of SC decoder. Similar to *K*-best detection for multi-input multi-output (MIMO) systems, SCL decoder expands and selects paths level-by-level on the full binary-tree. At each level, SCL decoder expands paths and computes path metrics, then selects the *l* paths with largest metrics instead of only keeping the best path. In addition, the list of *l* candidate paths $P \subseteq [GF(2)]^{N \times 2l}$, are stored for further processing. At the leaf nodes, the path with largest matric wins. The decoded code is then output. Details of SCL decoder are listed in algorithm 1.

Algorithm: 1 Conventional SC List Decoding

Require: $x_1^N = [x_1, x_2, ..., x_N]$ $P^{(0)} = [0_1^N, 0_1^N, ..., 0_1^N], L^{(0)} = [0, 0, ..., 0]$

for i=1:N do

Paths expansion:

 $P^{(i)} = \overline{\{(p_1^{i-1}, p) / p_1^{i-1} \in P^{(i-1)}, p \in GF(2)\}}$

Metric update: iterative computation

if
$$|P^{(i)}| \leq L$$
 then

Continue

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else
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Keep the l candidate paths with largest metric end if

if i = N then

 $\hat{u}_1^N = \arg\max_{p_1^N \in P} (L^{(N)}), L^{(N)} \in L^{(N)}$

end if end for

Fig. 1 illustrates the search process of SCL decoder with l=2. It is noted that the dominated computational complexity of each level lies in the 2l path metric comparisons. Therefore, although larger l provides better performance, the increasing complexity makes SCL decoder with large l less favorable.

5. Different SC Decoding Methods

Folded SC Decoder

SC is a suboptimal decoder so it has quasi-linear complexity $N(1 + \log N)$ in the code length N. In [Kahraman, 2014], a new non-binary SC decoder with low complexity $\frac{N}{2}(1 + \log \frac{N}{2})$ was proposed. It named as folded SC decoder. This method is based on folded decoding tree

decoder. This method is based on folded decoding tree structure, given in [Kahraman, 2013] to implement an efficient ML decoder of polar codes. By applying once the folding operation, a conventional SC decoder can be redesigned as a non-binary half-length SC decoder. In this

method conditional probabilities of bit-pairs in $1 + \log \frac{N}{2}$

steps of the folded SC decoder were considered instead of the likelihood ratios used in the $1 + \log N$ steps of conventional SC decoder. In addition the folding operation enables to construct $\log N$ alternative pairings of bits, which giver the better error performance with the same complexity.



Figure 1: Search process of conventional SCL polar decoder with l = 2.

6. Improved Multiple Folded SC Decoder

Folding is a technique to modify the decoder graph of polar codes based on successive cancellation decoding (SCD). In previous method [Kahraman, 2014] shows folding only for $k \leq 3$ due to rapidly increasing complexity. In this method [Vangala, 2014], an alternative implementation of the multiple-folding SC decoder proposed, to significantly reduce its complexity. The complexity of this algorithm is only slightly higher than that of the original SCD for polar codes. Hence, the algorithm enables to decode longer codes with larger k s, which exhibit significant performance gains, in addition to the latency gain.

A Stage-Reduced Low-Latency SC Decoder

In today's world the large code length required by practical applications cause high decoding latency because the conventional SC decoder decodes bits serially. In this method [Liu, 2015] an improved SC decoding method is given which achieves almost 50% latency reduction with no performance degradation.

This method eliminates the last stage of the conventional SC decoder with a hard decision module. This decoder can reduce the decoding latency by N clock cycles without performance degradation. A simple 1-stage reduced decoding algorithm is given below.

Algorithm 2: The 1-Stage-Reduced Decoding Algorithm

for i = 1: N/2case 1: $2i - 1 \in A^c \& \& 2i \in A^c$ then $\hat{u}_{2i-1} = 0, \hat{u}_{2i} = 0$ case 2: $2i - 1 \in A^c \& \& 2i \in A$

then
$$\hat{u}_{2i-1} = 0$$
,
 $\hat{u}_{2i} = \begin{cases} 0, if L_{N/2}^{(i)}(y_1^{N/2}, \hat{a}_1^{i-1}) + L_{N/2}^{(i)}(y_{N/2+1}^{N}, \hat{b}_1^{i-1}) \ge 0\\ 1, else \end{cases}$

case 3:
$$2i - 1 \in A \& \& 2i \in A^{c}$$

then $\widehat{u}_{2i} = 0$,
 $\widehat{u}_{2i-1} = \begin{cases} 0, if \operatorname{sgn}[L_{N/2}^{(i)}(y_{1}^{N/2}, \widehat{a}_{1}^{i-1})]\operatorname{sgn}[L_{N/2}^{(i)}(y_{N/2+1}^{N}, \widehat{b}_{1}^{i-1})] \ge 0\\ 1, else \end{cases}$

case 4: $2i - 1 \in A \& \& 2i \in A$ then

$$\hat{u}_{2i-1} = \begin{cases} 0, if \operatorname{sgn}[L_{N/2}^{(i)}(y_1^{N/2}, \hat{a}_1^{i-1})]\operatorname{sgn}[L_{N/2}^{(i)}(y_{N/2+1}^N, \hat{b}_1^{i-1})] \ge 0\\ 1, else \end{cases}$$

$$\hat{u}_{2i} = \begin{cases} 0, if L_{N/2}^{(i)}(y_{N/2+1}^N, \hat{b}_1^{i-1}) \ge 0\\ 1, else \end{cases}$$
end case
$$\hat{a} = \hat{u} \quad \bigoplus \hat{u} \quad \hat{b} = \hat{u}$$

1, else end case

then \hat{u}

 $\widehat{a}_i = \widehat{u}_{2i-1} \oplus \widehat{u}_{2i}, \widehat{b}_i = \widehat{u}_{2i}$ end for

7. Results

In this section we give the simulated results of these three SC decoding methods described in previous section for BER performance. Also give the complexity comparison of that three SC decoding methods.

Fig. 2 shows the BER performance of SC and folded SC decoder for polar code (128,64) at rate 0.5 and $E_{h}/N_{0} = 1$. We can see that the BER performance of folded SC decoder is way better than conventional SC decoder. In paper [Kahraman, 2014] more results are shown. In fig. 3 the performance of the multiple folded SC decoder is shown. Fig. 4 shows the performance of 2-stage reduced SC decoder. For folded SC polar decoding of code length N = 8192 and rate R = 0.8 considered and Additive White Gaussian Noise (AWGN) channel and BPSK modulation taken. For stage reduced SC decoder code length N = 1024and half rate considered.

Folded SC gives approax 10^{-5} BER at 5 dB SNR. Improved multiple folded SC gives 10^{-5} BER at 9.7 dB SNR and stage reduced SC gives approax 10^{-5} BER at 3.5 dB SNR. We can see from the results that in all methods BER performance is better than the conventional SC decoding method also decoding complexity is reduced. These methods are applicable at



Figure 2: BER performance of SC and folded SC for polar code (128,64)



Figure 3: BER performance of SC and multiple folded SC for polar code



Figure 4: BER and FER performance of SC and 2-stage reduced SC for polar codes

Different applications as per required block length, i.e. for short block length codes folded SC is useful and large code length stage reduced SC is very handy.

Also here we mansion complexity view of these decoding methods. These three methods have reduced the decoding complexity than the conventional SC decoding method.

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Decoding complexity of conventional SC decoder is $O(N \log N)$ where as decoding complexities of folded, improved multiple folded and Stage reduced SC decoders are slight less than the conventional SC decoder. In addition decoding latency of stage reduced SC decoder is 50% less than conventional SC decoder.

8. Conclusion

In this brief, the overview of recent SC decoding methods is given and comparison of these methods with conventional SC decoding method is described on the basis of their bit error rate, decoding complexity and decoding latency. In short all these methods are better than the Conventional SC decoding method in all aspects i.e. BER, complexity and decoding latency. Furthermore we can conclude from this short brief that all these methods are best in their applications i.e. short block length or large block length.

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