Reliability Analysis using Weibull Distribution – Kiln of Alsalam Cement Factory as Case Study

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Abstract: In this paper we estimate reliability of a system using Weibull distribution. "Reliability" is one of engineering indicators for describing the performance of an item or a system by probability functions. Reliability is defined as the probability that an item or system is capable of performing, its intended function in a specified time under given working conditions. The Weibull distribution is a generalization of the exponential distribution. The two-parameter Weibull distribution can accommodate increasing, decreasing, or constant failure rates because of the great flexibility expressed through the model's parameter. This distribution is recognized today as industrial standard. This paper presents the maximum likelihood method for estimating the Weibull distribution parameters and focuses on studying and evaluating the reliability of equipment as a basis to study the reliability of systems and the ways of calculating it.

Keywords: Reliability, Weibull distribution, maximum likehood, parameters

1. Introduction

"Reliability" is one of engineering indicators for describing the performance of an item or system by probability functions. Reliability is defined as the probability that an item or system is capable of performing, its intended function in a specified time under given working conditions. Modern industry have property of contrast and fastness of products development, so that the high costs that occur because of failure machines due to failure, Therefore analysis reliability is an important factor from point of view of the factory managers and the costumer [3].

In recent years, Weibull distribution has been one of the most commonly used accepted, recommended distribution to determine the reliability of equipment, also has been recognized as an appropriate model in reliability studies and life testing problems such as time to failure or life length of equipment and also is extremely useful for maintenance planning, particularly reliability centered maintenance. Shape parameter tells the analyst whether or not scheduled inspections and overhauls are needed, If value is less than or equal to one, overhauls are not cost effective. With value greater than one, the overhaul period or scheduled inspection interval is read directly from the plot at an acceptable probability of failure. For wear out failure modes, if the cost of an unplanned failure is much greater than the cost of a planned replacement, there is an optimum replacement interval for minimum cost [1].

Waloddi Weibull invented the Weibull distribution in **1937** and delivered his hallmark American paper on this subject in **1951**. He claimed that his distribution applied to a wide range of problems. Weibull distribution is very flexible and can through an appropriate choice of parameters and model many types of failure rate behaviors[1]. This distribution can be found with two or three parameters; scale, shape and location parameters. Over the years, estimation of the shape and scale parameters for a Weibull distribution function has been approached through number of methods, some are graphical and others are analytical. Graphical methods

include Weibull probability plotting and hazard plotting and analytical methods include Maximum likelihood method (**MLM**), least square method and method of moment. These methods are considered as more accurate and reliable compared to the graphical methods [2]. In this paper an attempt is made to estimate the system reliability using two parameter Weibull distributions by Maximum likelihood method and computation is made using "Minitab 17" software.

This paper focuses on studying and evaluating the reliability of kiln of Alsalam Cement factory as a basis to study the reliability of equipment.

2. Methods & Materials

2.1 Weibull Distribution

parameter Weibull distribution The two requires characteristic life (η) and shape parameters (β) values. Beta (β) determines the shape of the distribution. If β is greater than 1, the failure rate is increasing. If β is less than 1, the failure rate is decreasing. If β is equal to 1, the failure rate is constant. There are several ways to check whether data follows a Weibull distribution, the best choice is to use a Weibull analysis software product. If such a tool is not available, data can be manually plotted on a Weibull probability plot to determine if it follows a straight line. A straight line on the probability plot indicates that the data is following a Weibull distribution. The cumulative % failures versus operating time data are plotted on Weibull graph [4, 5].

2.2 Estimation of Weibull Parameters

The term parameter estimation refers to the process of using sample data (in reliability engineering, usually times-tofailure data) to estimate the parameters of the selected distribution. Several parameter estimation methods are available. We start with the relatively simple method of Probability Plotting and continue with the more sophisticated

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methods of Rank Regression (or Least Squares), Maximum Likelihood Estimation and Bayesian Estimation Methods.

2.2.1 Probability Plotting Method

The least mathematically intensive method for parameter estimation is the method of probability plotting. As the term implies, probability plotting involves a physical plot of the data on specially constructed probability plotting paper. This method is easily implemented by hand, given that one can obtain the appropriate probability plotting paper. The method of probability plotting takes the CDF (Cumulative Density Function) of the distribution and attempts to linearize it by employing a specially constructed paper [6]. The steps are includes:

- Linearize the unreliability function
- Construct the probability plotting paper
- Determine the X and Y positions of the plot points and then using the plot to read any particular time or reliability/unreliability value of interest.

The **CDF** (also the unreliability Q(t)) is given by:

$$F(t) = Q(t) = 1 - e^{-(\frac{t}{\eta})^{\beta}}$$
(1)

This function can then be linearized (i.e., put in the common

form of
$$y = m'x + b$$
 format) as follows

$$Q(t) = 1 - e^{-\left(\frac{t}{\eta}\right)\beta}$$

$$\ln(1 - Q(t)) = \ln\left[e^{-\left(\frac{t}{\eta}\right)\beta}\right]$$
(2)
(3)

$$\ln(1 - Q(t)) = -\left(\frac{t}{\eta}\right)^{\beta}$$

$$\ln(-\ln(1-Q(t))) = \beta(\ln(-))$$

$$\eta$$
(5)

$$\ln(\ln(\frac{1}{1-Q(t)})) = \beta \ln(t) - \beta(\eta)$$
(6)

Then by setting:

$$y = \ln(\ln(\frac{1}{1 - Q(t)})) \tag{7}$$

$$x = \ln(t) \tag{8}$$

The equation can then be rewritten as:

y

$$=\beta x - \beta \ln(\eta) \tag{9}$$

 $m = \beta$ and an intercept of:

$$b = -\beta . \ln(n) \tag{11}$$

2.2.2 Maximum Likelihood Method (MLE)

The most commonly methods used to estimate the parameters are the least square method (LSM), the weighted least square method (WLSM), the maximum likelihood method (MLM) and the method of moments (MOM).All these methods are the analytical methods. In this paper we used the maximum likelihood method which is used by researchers to estimate parameters of Weibull distribution.

The method of maximum likelihood is a commonly used procedure because it has very desirable properties [10]. Let $x_1, x_2, x_3, \dots, x_n$ is be a random sample of size *n* drawn from a probability density function $f(x;\theta)$ where θ is an unknown parameter. The likelihood function of this random sample is the joint density of the *n* random variables and is a function of the unknown parameter. Thus

$$L = \prod_{i=1}^{n} fx_i(x_i, \theta)$$
(12)

is the likelihood function. The maximum likelihood estimator (MLE) of θ , say , $\hat{\theta}$

is the value of θ that maximizes L or, equivalently, the logarithm of L. Often, but

not always, the MLE of θ is a solution of

$$\frac{d\log L}{d\theta} = 0 \tag{13}$$

where solutions that are not functions of the sample values $x_1, x_2, x_3, \ldots, x_n$ are not admissible, nor are solutions which are not in the parameter space. Now, we are going to apply the MLE to estimate the Weibull parameters, namely the shape and the scale parameters. Then likelihood function will be:

$$L(x_1,\dots,x_n;\beta,\eta) = \prod_{i=1}^n (\frac{\beta}{\eta}) \left(\frac{x_i}{\eta}\right)^{\beta-1} e^{-(\frac{x_i}{\eta})\beta}$$
(14)

Differentiating with respect to β and η in turn and equating to zero, we obtain the estimating equations

$$\frac{\partial \ln L}{\partial \beta} = \frac{n}{\beta} + \sum_{i=1}^{n} \ln x_i - \frac{1}{\eta} \sum_{i=1}^{n} x_i^{\beta} \ln x_i = 0$$
(15)

$$\frac{\partial \ln L}{\partial \mu} = -\frac{n}{\eta} + \frac{1}{\eta^2} \sum_{i=1}^n x_i^{\ \beta} = 0 \tag{16}$$

On eliminating η between these two equations and simplifying, we have:

$$\frac{\sum_{i=1}^{n} x_i^{\beta} \ln x_i}{\sum_{i=1}^{n} x_i^{\beta}} - \frac{1}{\beta} - \frac{1}{n} \sum_{i=1}^{n} \ln x_i = 0$$
(17)

which may be solved to get the estimate of $\hat{\eta}_k = \beta$.

This can be accomplished by the use of standard iterative procedures (i.e., Newton-Raphson method). Once β is determined, η can be estimated using equation (16) as:

$$\eta = \frac{\sum_{i=1}^{n} x_i^{\beta}}{n}$$
(18)

2.3 Reliability analysis

The reliability function for the two-parameter Weibull distribution is given as:

(10)

$$R(t) = e^{-\left(\frac{t}{\eta}\right)^{\beta}}$$
(19)

The Weibull failure rate function (Hazard Rate Function) is defined as the number of failures per unit time that can be expected to occur for the product. It is given as [8].

$$\lambda(t) = \frac{\beta}{\eta} \times \left(\frac{t}{\eta}\right)^{\beta - 1}$$
(20)

The two-parameter Weibull probability density function f(t) is given as:

$$\mathbf{PDF} = f(t) = \frac{\beta}{\eta} \times (\frac{t}{\eta})^{\beta-1} \times e^{-(\frac{t}{\eta})^{\beta}}$$
(21)

$$\mathbf{CDF} = F(t) = 1 - e^{-\left(\frac{t}{\eta}\right)^{\beta}}$$
(22)

The mean life is the average time of failure (MTTF or MTBF) -free operation up to a failure event. The MTTF of the Weibull Probability density Function is given as calculated from a homogeneous lot of equipments under operation. The MTTF or MTBF of the Weibull PDF (Probability Density Function) is given as:

$$\mathbf{MTTF} = \eta \times \Gamma\left(\frac{1}{\beta} + 1\right) \tag{23}$$

3. Results & Discussion

3.1 Case Study

Estimation of reliability, Probability Density Function (PDF), Hazard function , Mean time between failure (MTBF) or Expected time to system failure E (T) and Cumulative density function (CDF) of Kiln of Alsalam cement factory using data of time to failure of system collecting from factory records.

The time to failure data (in hours) were as given below: 528,13,19,576,72,120,504,96,48,24,480,16,11,48,13&48



Using the software tool 'Minitab 17' as shown in figures (1,2,3,4)

Weibull parameters are:

$$\beta = 0.736142$$
 , $\eta = 132.547$ hrs

Estimation of Mean time To failure (MTTF) or Expected time to system failure

$$E(T) = \eta \times \Gamma\left(\frac{1}{\beta} + 1\right) =$$

$$132.547 \times \Gamma\left(\frac{1}{0.736142} + 1\right) = 160.31 \ hr$$

$$Re \ liability = R(t) = e^{-\left(\frac{t}{\eta}\right)^{\beta}} = e^{-\left(\frac{160.31}{132.547}\right)^{0.736142}} = 0.32$$

Cummulative DensityFunction(CDF) = 1 - R(t) = 1 - 0.32 = 0.68

Failure rate =
$$\lambda(t) = \frac{\beta}{\eta} \times \left(\frac{t}{\eta}\right)^{\beta-1} =$$

$$\frac{0.736142}{132.547} \times \left(\frac{160.31}{132.547}\right)^{0.736142 - 1} = 0.005$$
Pr obability Density Function = PDF = $\lambda(t) \times R(t) = 0.32 \times 0.005$
= 0.0017

4. Conclusion

Weibull distribution parameters are estimated using 'Minitab 17' software tool very easily and statistical computation & charts are presented in fig (1, 2,3 and 4) the Fig(1): Presented the Probability Density Function & Time to failure of kiln. Fig (2): presents the Weibull probability plot with parameters are estimated & Time to failure of kiln . Fig.(3) shows the Reliability Function & Time to failure of kiln, Fig (4) shows Hazard Function vs Time to failure plot of kiln .

By using estimated parameters of Weibull distribution $\beta = 0.736142$, $\eta = 132.547$ hrs E (T), R(t), CDF, λ (t) and PDF are calculated. Reliability function R(t) value is 0.32 very low that indicate there is a problem in maintenance program or design and installation of kiln.

This paper regarding the Weibull parameter estimation using 'Minitab 17' software tool and estimation of the reliability of kiln, The empirical approximation of functions was taken and it showed that the Weibull distribution parameters are $\beta = 0.736142$, $\eta = 132.547$ approximates well the reliability of the of kiln, and that the expected time of failure-free function E (T) = 160.31 hr. Starting from the established fact that the kiln has an decreasing rate of failure, and that failure causes may be different in nature from designing or fatigue of material, to wear and corrosion, it was necessary to determine the individual failure rates of kiln parts (subsystems) and their contribution to overall reliability and failure rate.

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