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Extended Hybrid Block Simpson's Method of the 5th and 6th Step-Sizes for Ordinary Differential Equations

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Abstract: ***We construct a self-starting Simpson's type block hybrid method (BHM) consisting of very closely accurate members each of order p=q+2 as a block. The higher order members of each were obtained by increasing the number k in the multi-step collocation (MC) used to derive the k-step continuous formula (5≤k≤6) through the aid of MAPLE software program. In this paper, we identify a continuous hybrid block schemes (CHBS) through the addition of one off-mesh collocation points in the MC. The (CHBS) is evaluated along with it's first derivative where necessary to give a hybrid block schemes for a simultaneous application to the ordinary differential equations (ODEs}. The results of numerical experiment confirm the reliability of these schemes.***

Keywords: Continuous hybrid block schemes (CHBS), Multi-step collocation (MC), ODEs.

1. Introduction

There is always a conflict between three basic aims in the design of schemes to solve ordinary differential equations, that is, accuracy, stability and efficiency.

Following Onumanyi et-al [5] we identify a continuous hybrid block scheme (CHBS) through the addition of one or more off-mesh collocation points in the multi-step collocation (MC) of the form given by equation (2.1.8) in next section. The (CHBS) is evaluated at some distinct points involving mesh and off-mesh points along with its first derivative, where necessary, to give multiple hybrid block schemes for the treatment of stiff ordinary differential equations.

This paper is classified into sections. In section 2.0 we restate the MC procedure involving off-mesh collocation points for each k and we analyze on its convergence analysis obtained in a block form. We obtained the order and error constants in a block form, the stability regions are also plotted. Section 3.0 is the numerical implementation of the block hybrid schemes on stiff (ODEs) and we give conclusion in section 4.0.

Definition 1.1 Linear Multi-Step Method
A k-step linear multi-step (lmm) is of the form

$$\sum_{j=0}^{k} \alpha_j y_{n+j} = h\sum_{j=0}^{k} \beta_j f_{n+j} \quad 1.1$$

Where

$$f_{n+j} = f(x_{n+j}, y_{n+j}), \;\; y_{n+j} = y(x_{n+j})$$

α_j and β_j are constants and satisfy the constraints

$$\alpha_k \neq 0, \alpha_0^2 + \beta_0^2 > 0$$

(1.1) is explicit if $\beta_k = 0$ and implicit if $\beta_k \neq 0$

Definition 1.2
If a numerical method is forced to be used, in a certain interval of integration, a step length, which is excessively small in relation to the smoothness of the exact solution in that interval, then the problem is said to be stiff in that interval.

Unlike the linear definition of stiffness, our definition allows a single equation, not just a system of equations, to be stiff. It also allows a problem to be stiff in parts, a nonlinear problem may start off non-stiff and become stiff, or vice versa. It may even have alternating stiff and non-stiff internal.

2. Construction of the Methods

2.1 Derivation techniques of MC

Let us consider the first order system of ODEs

$y^1 = f(x,y)$, $a < x < b$, $y,f \in \Re^s$ 2.1.1

where y satisfies a given set of s associated conditions, which are either all initial, all boundary or mixed conditions. The idea of the k – step MC, following Onumanyi et. al {4}, is to find a polynomial U of the form

$$U(x) = \sum_{j=0}^{t-1} \phi_j(x) y_{n+j} + h\sum_{j=0}^{m-1} \varphi_j(x) f(x_j, u(\bar{x}_j)), \qquad x_n \le x \le x_{n+x} \qquad 2.1.2$$

Where t denotes the number of interpolation points x_{n+i}, $i = 0, 1, ---, t-1$, and m denote the number of distinct collocation points $\bar{x}_i \in [x_n, x_{n+k}]$, $i = 0,1,---, m-1$ *the points* $\bar{x}_i$ are chosen from the step x_{n+i} as well as one or more off – step points.

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The following assumptions are made;

1. Although the step size can be variable, for simplicity in our presentation of the analysis in this paper, we assume it is constant $h = x_{n+1} - x_n$, $N = \frac{b-a}{h}$ with the steps given by $\{x_n / x_n = a + nh,\ n = 0,1,---,N\}$,

2. That (2.1.1) has a unique solution and the coefficients $\phi_j(x)$, $\varphi_j(x)$ in (2.1.2) can be represented by polynomials of the form

$$\phi_j(x) = \sum_{i=0}^{t+m-1} \phi_{j,i+1} x^i, \qquad j\varepsilon\{0,1,---,t-1\}$$

h

$$\varphi_j(x) = \sum_{i=0}^{t+m-1} \varphi_{j,i+1} x^i \qquad j\varepsilon\{0,1,---,m-1\}$$

with constant coefficients $\phi_{j,i+1}$, $h\varphi_{j,i+1}$ to be determined using the interpolation and collocation conditions:

$$u(x_{n+i}) = y_{n+i},\ i\varepsilon\ \{0,1,---,t-1\} \qquad 2.1.5$$

$$u^1(\bar{x}_i) = f(\bar{x}_i, u(\bar{x}_j)),\ \ j\varepsilon\{0,1,---,m-1\}$$

With this assumptions we obtain an MC polynomial, following [4, 5], in the form

$$u(x) = \sum_{i=0}^{t+m-1} a_i x^i,\ \ a^i = \sum_{j=0}^{t-1} c_{i+1,j+1} + \sum_{j=0}^{m-1} c_{i+1,j+t+1} f_{n+j} \qquad 2.1.3$$

Where $x_n \le x \le x_{n+k}$ and c_{ij} $i,j\varepsilon\{1,2,---,t+m\}$ are constants given by the elements of the inverse matrix $C = D^{-1}$. The MC matrix D is a nonsingular (m+1) square matrix of the type 2.1.4

$$D = \begin{bmatrix} 1 & x_n & x_n^2 & - & - & - & x_n^{t+m-1} \\ 1 & x_{n+1} & x_{n+1}^2 & - & - & - & x_{n+1}^{t+m-1} \\ . & . & . & . & & & . \\ . & . & . & & . & & . \\ . & . & . & & & . & . \\ 1 & x_{n+t-1} & x_{n+t-1}^2 & - & - & - & x_{n+t-1}^{t+m-1} \\ 0 & 1 & 2\bar{x}_0 & - & - & - & (t+m-1)\bar{x}_0^{t+m-1} \\ 0 & 1 & 2\bar{x}_1 & - & - & - & (t+m-1)\bar{x}_1^{t+m-1} \\ . & . & . & . & & & . \\ . & . & . & & . & & . \\ . & . & . & & & . & . \\ 0 & 1 & 2\bar{x}_{m-1} & - & - & - & (t+m-1)\bar{x}_{m-1}^{t+m-1} \end{bmatrix} \qquad 2.1.8$$

2.2 Five steps Block Hybrid Simpson's Method with one off-step point.

The parameters required for equation (2.1.8) are k=5, t=1, m= k+2; (x_n, x_{n+1}) ;

$$\left(\bar{x}_0 = x_n,\ \bar{x}_1 = x_{n+1},\ \bar{x}_2 = x_{n+2},\ \bar{x}_3 = x_{n+3},\ \bar{x}_4 = x_{n+4},\ \bar{x}_{\frac{9}{2}} = x_{n+\frac{9}{2}},\ \bar{x}_5 = x_{n+5}\right)$$

Hence the matrix (2.1.8) takes the following shape.

$$D = \begin{bmatrix} 1 & x_n & x_n^2 & x_n^3 & x_n^4 & x_n^5 & x_n^6 & x_n^7 \\ 0 & 1 & 2x_n & 3x_n^2 & 4x_n^3 & 5x_n^4 & 6x_n^5 & 7x_n^6 \\ 0 & 1 & 2x_{n+1} & 3x_{n+1}^2 & 4x_{n+1}^3 & 5x_{n+1}^4 & 6x_{n+1}^5 & 7x_{n+1}^6 \\ 0 & 1 & 2x_{n+2} & 3x_{n+2}^2 & 4x_{n+2}^3 & 5x_{n+2}^4 & 6x_{n+2}^5 & 7x_{n+2}^6 \\ 0 & 1 & 2x_{n+3} & 3x_{n+3}^2 & 4x_{n+3}^3 & 5x_{n+3}^4 & 6x_{n+3}^5 & 7x_{n+3}^6 \\ 0 & 1 & 2x_{n+4} & 3x_{n+4}^2 & 4x_{n+4}^3 & 5x_{n+4}^4 & 6x_{n+4}^5 & 7x_{n+4}^6 \\ 0 & 1 & 2x_{n+\frac{9}{2}} & 3x^2_{n+\frac{9}{2}} & 4x^3_{n+\frac{9}{2}} & 5x^4_{n+\frac{9}{2}} & 6x^5_{n+\frac{9}{2}} & 7x^6_{n+\frac{9}{2}} \\ 0 & 1 & 2x_{n+5} & 3x_{n+5}^2 & 4x_{n+5}^3 & 5x_{n+5}^4 & 6x_{n+5}^5 & 7x_{n+5}^6 \end{bmatrix} \qquad 2.1.9$$

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Using the maple software environment to evaluate (2.1.9) at the grid points

$$x=x_{n+1}\,,x=x_{n+2}\,,x=x_{n+3}\,,x=x_{n+4}\,,\; x=x_{n+\frac{9}{2}}\,,\; x=x_{n+5}$$

We obtain the six discrete schemes, namely,

$$y_{n+1}=y_n+h\left[\frac{28199}{90720}f_n+\frac{78553}{70560}f_{n+1}-\frac{4519}{5040}f_{n+2}+\frac{13691}{15120}f_{n+3}-\frac{9841}{10080}f_{n+4}+\frac{13808}{19845}f_{n+\frac{9}{2}}-\frac{1537}{10080}f_{n+5}\right]$$

$$y_{n+2}=y_n+h\left[\frac{169}{567}f_n+\frac{6691}{4410}f_{n+1}-\frac{5}{63}f_{n+2}+\frac{517}{945}f_{n+3}-\frac{206}{315}f_{n+4}+\frac{9472}{19845}f_{n+\frac{9}{2}}-\frac{67}{630}f_{n+5}\right]$$

$$y_{n+3}=y_n+h\left[\frac{1013}{3360}f_n+\frac{11601}{7840}f_{n+1}+\frac{45}{112}f_{n+2}+\frac{689}{560}f_{n+3}-\frac{1017}{1120}f_{n+4}+\frac{464}{735}f_{n+\frac{9}{2}}-\frac{153}{1120}f_{n+5}\right]$$

$$y_{n+4}=y_n+h\left[\frac{170}{567}f_n+\frac{3296}{2205}f_{n+1}+\frac{104}{315}f_{n+2}+\frac{1664}{945}f_{n+3}-\frac{63}{315}f_{n+4}+\frac{8192}{19845}f_{n+\frac{9}{2}}-\frac{32}{315}f_{n+5}\right]$$

$$y_{n+\frac{9}{2}}=y_n+h\left[\frac{2151}{7168}f_n+\frac{374463}{250880}f_{n+1}+\frac{1215}{3584}f_{n+2}+\frac{31077}{17920}f_{n+3}+\frac{3159}{35840}f_{n+4}+\frac{162}{245}f_{n+\frac{9}{2}}-\frac{4131}{35840}f_{n+5}\right]$$

$$y_{n+5}=y_n+h\left[\frac{5435}{18144}f_n+\frac{21125}{14112}f_{n+1}+\frac{325}{1008}f_{n+2}+\frac{5375}{3024}f_{n+3}-\frac{125}{2016}f_{n+4}+\frac{4400}{3969}f_{n+\frac{9}{2}}+\frac{115}{2016}f_{n+5}\right]$$

2.2.0

2.3 Six steps Block Hybrid Simpson's Method with one off-step point.

The parameters required for equation (2.1.8) are k=6, t=1, m= k+2; $(x_n\,,x_{n+1})$,

$$\left(\bar{x}_0=x_n\,,\;\bar{x}_1=x_{n+1}\,,\;\bar{x}_2=x_{n+2}\,,\;\bar{x}_3=x_{n+3}\,,\;\bar{x}_4=x_{n+4}\,,\;\bar{x}_5=x_{n+5}\,,\;\bar{x}_{\frac{11}{2}}=x_{n+\frac{11}{2}}\,,\;\bar{x}_6=x_{n+6}\right)$$

Hence the matrix (2.1.8) takes the following shape.

$$D=\begin{bmatrix}
1 & x_n & x_n^2 & x_n^3 & x_n^4 & x_n^5 & x_n^6 & x_n^7 & x_n^8\\
0 & 1 & 2x_n & 3x_n^2 & 4x_n^3 & 5x_n^4 & 6x_n^5 & 7x_n^6 & 8x_n^7\\
0 & 1 & 2x_{n+1} & 3x_{n+1}^2 & 4x_{n+1}^3 & 5x_{n+1}^4 & 6x_{n+1}^5 & 7x_{n+1}^6 & 8x_{n+1}^7\\
0 & 1 & 2x_{n+2} & 3x_{n+2}^2 & 4x_{n+2}^3 & 5x_{n+2}^4 & 6x_{n+2}^5 & 7x_{n+2}^6 & 8x_{n+2}^7\\
0 & 1 & 2x_{n+3} & 3x_{n+3}^2 & 4x_{n+3}^3 & 5x_{n+3}^4 & 6x_{n+3}^5 & 7x_{n+3}^6 & 8x_{n+3}^7\\
0 & 1 & 2x_{n+4} & 3x_{n+4}^2 & 4x_{n+4}^3 & 5x_{n+4}^4 & 6x_{n+4}^5 & 7x_{n+4}^6 & 8x_{n+4}^7\\
0 & 1 & 2x_{n+5} & 3x_{n+5}^2 & 4x_{n+5}^3 & 5x_{n+5}^4 & 6x_{n+5}^5 & 7x_{n+5}^6 & 8x_{n+5}^7\\
0 & 1 & 2x_{n+\frac{11}{2}} & 3x^2_{n+\frac{11}{2}} & 4x^3_{n+\frac{11}{2}} & 5x^4_{n+\frac{11}{2}} & 6x^5_{n+\frac{11}{2}} & 7x^6_{n+\frac{11}{2}} & 8x^7_{n+\frac{11}{2}}\\
0 & 1 & 2x_{n+6} & 3x_{n+6}^2 & 4x_{n+6}^3 & 5x_{n+6}^4 & 6x_{n+6}^5 & 7x_{n+6}^6 & 8x_{n+6}^7
\end{bmatrix} \quad 2.2.1$$

Using the maple software environment to evaluate (2.2.1) at the grid points

$$x=x_{n+1}\,,x=x_{n+2}\,,x=x_{n+3}\,,x=x_{n+4}\,,\; x=x_{n+5}\,,\; x=x_{n+\frac{11}{2}}\,,\; x=x_{n+6}$$

We obtain the seven discrete schemes, namely,

$$y_{n+1}=y_n+h\left[\frac{4553}{15120}f_n+\frac{107293}{90720}f_{n+1}-\frac{3727}{3360}f_{n+2}+\frac{19001}{15120}f_{n+3}-\frac{4271}{3780}f_{n+4}+\frac{3559}{3360}f_{n+5}-\frac{400}{567}f_{n+\frac{11}{2}}+\frac{4381}{30240}f_{n+6}\right]$$

$$y_{n+2}=y_n+h\left[\frac{4027}{13860}f_n+\frac{4454}{2835}f_{n+1}-\frac{103}{420}f_{n+2}+\frac{52}{63}f_{n+3}-\frac{3047}{3780}f_{n+4}+\frac{82}{105}f_{n+5}-\frac{16384}{31185}f_{n+\frac{11}{2}}+\frac{137}{1260}f_{n+6}\right]$$

$$y_{n+3}=y_n+h\left[\frac{361}{1232}f_n+\frac{345}{224}f_{n+1}+\frac{243}{1120}f_{n+2}+\frac{859}{560}f_{n+3}-\frac{36}{35}f_{n+4}+\frac{1053}{1120}f_{n+5}-\frac{48}{77}f_{n+\frac{11}{2}}+\frac{143}{1120}f_{n+6}\right]$$

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$$y_{n+4}=y_n+h\left[\frac{3034}{10395}f_n+\frac{880}{567}f_{n+1}+\frac{16}{105}f_{n+2}+\frac{1952}{945}f_{n+3}-\frac{386}{945}f_{n+4}+\frac{16}{21}f_{n+5}-\frac{16384}{31185}f_{n+\frac{11}{2}}+\frac{104}{945}f_{n+6}\right]$$

$$y_{n+5}=y_n+h\left[\frac{295}{1008}f_n+\frac{28025}{18144}f_{n+1}+\frac{125}{672}f_{n+2}+\frac{1975}{1008}f_{n+3}+\frac{125}{756}f_{n+4}+\frac{955}{672}f_{n+5}-\frac{400}{567}f_{n+\frac{11}{2}}+\frac{275}{2016}f_{n+6}\right]$$

$$y_{n+\frac{11}{2}}=y_n+h\left[\frac{905773}{3096576}f_n+\frac{7180745}{4644864}f_{n+1}+\frac{310123}{1720320}f_{n+2}+\frac{7643933}{3870720}f_{n+3}+\frac{2019127}{15482880}f_{n+4}+\frac{1476079}{860160}f_{n+5}-\frac{4213}{9072}f_{n+\frac{11}{2}}+\frac{1925957}{15482880}f_{n+6}\right]$$

$$y_{n+6}=y_n+h\left[\frac{41}{140}f_n+\frac{54}{35}f_{n+1}+\frac{27}{140}f_{n+2}+\frac{68}{35}f_{n+3}+\frac{27}{140}f_{n+4}+\frac{54}{35}f_{n+5}+\frac{41}{140}f_{n+6}\right] \quad 2.2.2$$

2.5 The Order and Error constants of the A-stable Block Hybrid Methods.

The hybrid block methods which are obtained in a block form with the help of maple software have the following order and error constants for each case.

The method k=5 is of order 7 as a block and has error constants

$$C_8=\left(\frac{1759}{211680},\frac{337}{52920},\frac{57}{7840},\frac{44}{6615},\frac{54351}{8028160},\frac{275}{42336}\right)^T$$

The method k=6 is of order 8 as a block and has error constants

$$C_9=\left(-\frac{209749}{29030400},-\frac{653}{11340},-\frac{2277}{35840},-\frac{169}{28350},-\frac{7325}{1161216},-\frac{46301497}{731782400},-\frac{9}{1400}\right)^T$$

2.6 Stability Regions of the Block Hybrid Simpson's Methods

To compute and plot the absolute stability regions of the block hybrid Simpson's methods, the methods are reformulated as general linear methods expressed as;

$$\begin{bmatrix} Y \\ -- \\ y_{i+1}\end{bmatrix}=\begin{bmatrix} A & / & U \\ - & - & - \\ B & / & V\end{bmatrix}\begin{bmatrix} hf(Y) \\ \\ y_{i-1}\end{bmatrix}$$

where,

$$A=\begin{bmatrix} a_{11} & . & . & . & a_{1s} \\ . & . & & & . \\ . & & . & & . \\ . & & & . & . \\ a_{s1} & . & . & . & a_{ss}\end{bmatrix} \quad B=\begin{bmatrix} b_{11} & . & . & . & b_{1s} \\ . & . & & & . \\ . & & . & & . \\ . & & & . & . \\ b_{k1} & . & . & . & b_{ks}\end{bmatrix}$$

$$Y=\begin{bmatrix} y_n \\ y_{n+1} \\ . \\ . \\ . \\ y_{n+k}\end{bmatrix} \quad y_{i+1}=\begin{bmatrix} y_{n+k} \\ . \\ . \\ y_{n+k-1}\end{bmatrix} \quad y_{i-1}=\begin{bmatrix} y_{n+k-1} \\ . \\ . \\ y_{n+k-2}\end{bmatrix}$$

and the elements of the matrices A,B,U and V are obtained from the interpolation and collocation and collocation points.

The elements of the matrices A, B, U and V are substituted into the stability matrix

$$M(z)=B_2+zA_2(I-zA_1)^{-1}B_1 \quad where \quad A_1=A, A_2=B, B_1=U, B_2=V$$

and the stability function

$$\rho(\eta,z)=\det(\eta I-M(z))$$

Computing the stability function with Maple yields the stability polynomial of the method which is plotted in Matlab to produce the required absolute stability region of the method.

2.6.1 Absolute stability region of the block hybrid method K=5

The block hybrid methods (2.2.0) with one off-grid point are arranged as shown below;

The coefficients of these methods expressed in tabular form bellow gives the coefficients of the new method.

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$$\begin{bmatrix} y_n \\ y_{n+1} \\ y_{n+2} \\ y_{n+3} \\ y_{n+4} \\ y_{n+\frac{9}{2}} \\ y_{n+5} \\ \hdashline y_{n+5} \\ y_{n+4} \\ y_{n+3} \\ y_{n+2} \\ y_{n+1} \end{bmatrix} = \left[\begin{array}{ccccccc|c|ccccc} 0 & 0 & 0 & 0 & 0 & 0 & 0 & . & 0 & 0 & 0 & 0 & 1 \\ \frac{28199}{90720} & \frac{78553}{70560} & -\frac{4519}{5040} & \frac{13691}{15120} & -\frac{9841}{10080} & \frac{13808}{19845} & -\frac{1537}{10080} & . & 0 & 0 & 0 & 0 & 1 \\ \frac{169}{567} & \frac{6691}{4410} & -\frac{5}{63} & \frac{517}{945} & -\frac{206}{315} & \frac{9472}{19845} & -\frac{67}{630} & . & 0 & 0 & 0 & 0 & 1 \\ \frac{1013}{3360} & \frac{11601}{7840} & \frac{45}{112} & \frac{689}{560} & -\frac{1017}{1120} & \frac{464}{735} & -\frac{153}{1120} & . & 0 & 0 & 0 & 0 & 1 \\ \frac{170}{567} & \frac{3296}{2205} & \frac{104}{315} & \frac{1664}{945} & -\frac{62}{315} & \frac{8192}{19845} & -\frac{32}{315} & . & 0 & 0 & 0 & 0 & 1 \\ \frac{2151}{7168} & \frac{374463}{250880} & \frac{1215}{3584} & \frac{31077}{17920} & \frac{3159}{35840} & \frac{162}{245} & -\frac{4131}{35840} & . & 0 & 0 & 0 & 0 & 1 \\ \frac{5435}{18144} & \frac{21125}{14112} & \frac{325}{1008} & \frac{5375}{3024} & -\frac{125}{2016} & \frac{4400}{3969} & \frac{115}{2016} & . & 0 & 0 & 0 & 0 & 1 \\ \hdashline \frac{5435}{1814} & \frac{21125}{14112} & \frac{325}{1008} & \frac{5375}{3024} & -\frac{125}{2016} & \frac{4400}{3969} & \frac{115}{2016} & . & 0 & 0 & 0 & 0 & 1 \\ \frac{170}{567} & \frac{3296}{2205} & \frac{104}{315} & \frac{1664}{945} & -\frac{62}{315} & \frac{8192}{19845} & -\frac{32}{315} & . & 0 & 0 & 0 & 0 & 1 \\ \frac{1013}{3360} & \frac{11601}{7840} & \frac{45}{112} & \frac{689}{560} & -\frac{1017}{1120} & \frac{464}{735} & -\frac{153}{1120} & . & 0 & 0 & 0 & 0 & 1 \\ \frac{169}{567} & \frac{6691}{4410} & -\frac{5}{63} & \frac{517}{945} & -\frac{206}{315} & \frac{9472}{19845} & -\frac{67}{630} & . & 0 & 0 & 0 & 0 & 1 \\ \frac{28199}{90720} & \frac{78553}{70560} & -\frac{4519}{5040} & \frac{13691}{15120} & -\frac{9841}{10080} & \frac{13808}{19845} & -\frac{1537}{10080} & . & 0 & 0 & 0 & 0 & 1 \end{array}\right] \begin{bmatrix} f_n \\ f_{n+1} \\ f_{n+2} \\ f_{n+3} \\ f_{n+4} \\ f_{n+\frac{9}{2}} \\ f_{n+5} \\ \hdashline y_{n+4} \\ y_{n+3} \\ y_{n+2} \\ y_{n+1} \\ y_n \end{bmatrix}$$

where,

$$A = \begin{bmatrix} 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ \frac{28199}{90720} & \frac{78553}{70560} & -\frac{4519}{5040} & \frac{13691}{15120} & -\frac{9841}{10080} & \frac{13808}{19845} & -\frac{1537}{10080} \\ \frac{169}{567} & \frac{6691}{4410} & -\frac{5}{63} & \frac{517}{945} & -\frac{206}{315} & \frac{9472}{19845} & -\frac{67}{630} \\ \frac{1013}{3360} & \frac{11601}{7840} & \frac{45}{112} & \frac{689}{560} & -\frac{1017}{1120} & \frac{464}{735} & -\frac{153}{1120} \\ \frac{170}{567} & \frac{3296}{2205} & \frac{104}{315} & \frac{1664}{945} & -\frac{62}{315} & \frac{8192}{19845} & -\frac{32}{315} \\ \frac{2151}{7168} & \frac{374463}{250880} & \frac{1215}{3584} & \frac{31077}{17920} & \frac{3159}{35840} & \frac{162}{245} & -\frac{4131}{35840} \\ \frac{5435}{18144} & \frac{21125}{14112} & \frac{325}{1008} & \frac{5375}{3024} & -\frac{125}{2016} & \frac{4400}{3969} & \frac{115}{2016} \end{bmatrix}, \quad U = \begin{bmatrix} 0 & 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 & 1 \end{bmatrix}$$

$$B = \begin{bmatrix} \frac{5435}{18144} & \frac{21125}{14112} & \frac{325}{1008} & \frac{5375}{3024} & -\frac{125}{2016} & \frac{4400}{3969} & \frac{115}{2016} \\ \frac{170}{567} & \frac{3296}{2205} & \frac{104}{315} & \frac{1664}{945} & -\frac{62}{315} & \frac{8192}{19845} & -\frac{32}{315} \\ \frac{1013}{3360} & \frac{11601}{7840} & \frac{45}{112} & \frac{689}{560} & -\frac{1017}{1120} & \frac{464}{735} & -\frac{153}{1120} \\ \frac{169}{567} & \frac{6691}{4410} & -\frac{5}{63} & \frac{517}{945} & -\frac{206}{315} & \frac{9472}{19845} & -\frac{67}{630} \\ \frac{28199}{90720} & \frac{78553}{70560} & -\frac{4519}{5040} & \frac{13691}{15120} & -\frac{9841}{10080} & \frac{13808}{19845} & -\frac{1537}{10080} \end{bmatrix}, \quad V = \begin{bmatrix} 0 & 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 & 1 \end{bmatrix}$$

Substituting the values of A, B, U and V into the stability matrix and the stability function and using Maple software yields the stability polynomial of the block method.

Using a Matlab program, we obtained the stability region of the block hybrid Simpson's method for K= 5 as shown in figure 1. The stability region of the block method shows that it is A-stable.

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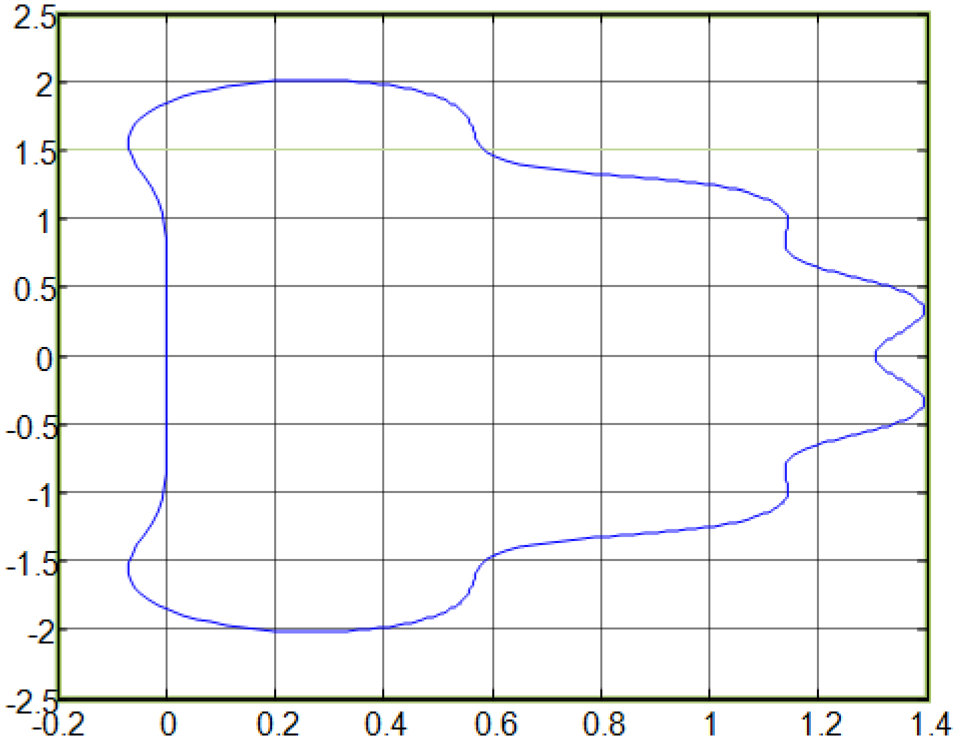

Figure 1: Stability region of the block hybrid Simpson's method K=5

Computing the stability function with Maple software yield, the stability polynomial of the method which is then plotted in MATLAB environment to produce the required absolute stability region of the methods, as shown by the figures 2 :

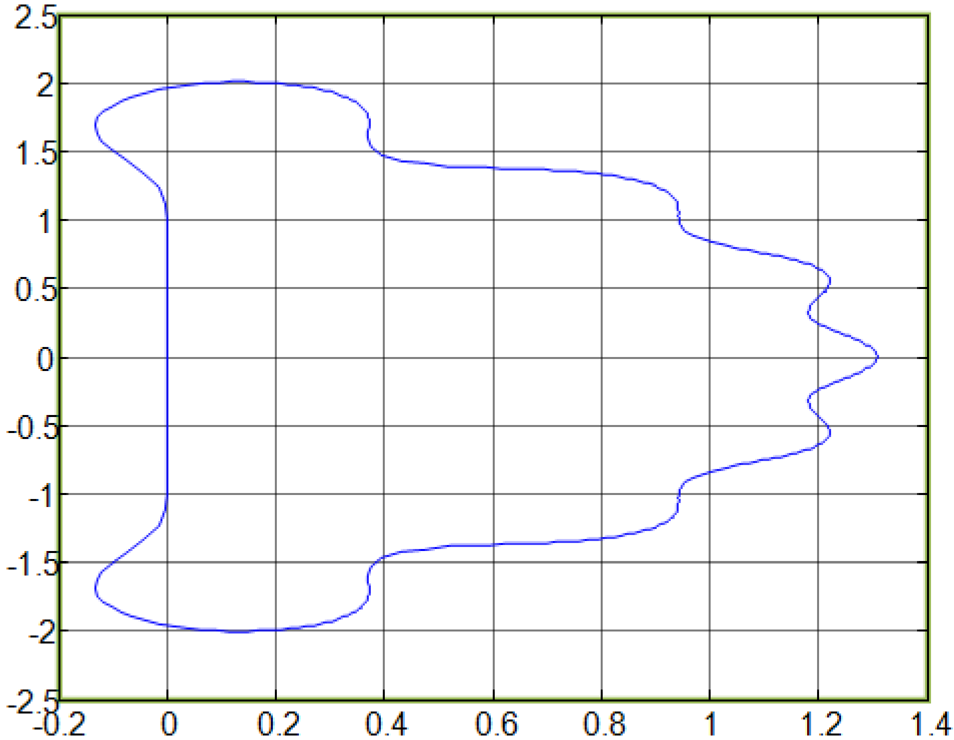

Figure 2: Stability region of the block hybrid Simpson's method K=6

3. Numerical Implementation

To study the efficiency of the block hybrid method for $5 \leq k \leq 6$, we present some numerical examples as follows:

$$Experiment1 \quad y^{1} = -1000000y, \quad where\ h = 0.1, \quad x\varepsilon[0,1.8]$$

$$Exact\ solution \quad y(x) = e^{-1000000x}$$

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Experiment 2

$$y' = \begin{pmatrix} -21 & 19 & -20 \\ 19 & -21 & 20 \\ 40 & -40 & -40 \end{pmatrix} y, \quad y(0) = \begin{pmatrix} 1 \\ 0 \\ -1 \end{pmatrix}, \quad 0 \le x \le 1, \; h = 0.1$$

$$y(x) = \tfrac{1}{2} \begin{pmatrix} e^{-2x} + e^{-40x}(\cos(40x) + \sin(40x)) \\ e^{-2x} - e^{-40x}(\cos(40x) + \sin(40x)) \\ 2e^{-40x}(\sin(40x) - \cos(40x)) \end{pmatrix}$$

Table of absolute errors for experiment 1
ABSOLUTE ERRORS OF A-STABLE NEW BHSM AT x $\in$[0, 1.0] $\lambda = -1000000$

Table 1

Y	A-stable Hybrid Block Simpson's K=5	A-stable Hybrid Block Simpson's K=6
0.1	1.56×10^{-1}	1.36×10^{-1}
0.2	5.56×10^{-2}	4.24×10^{-2}
0.3	3.33×10^{-2}	2.27×10^{-2}
0.4	2.22×10^{-2}	1.82×10^{-2}
0.5	1.11×10^{-1}	1.52×10^{-2}
0.6	1.73×10^{-2}	9.09×10^{-2}
0.7	6.17×10^{-3}	1.24×10^{-2}
0.8	3.70×10^{-3}	3.86×10^{-3}
0.9	2.47×10^{-3}	2.07×10^{-3}
1.0	1.23×10^{-2}	1.65×10^{-3}

Table of absolute errors for experiment 2
ABSOLUTE ERRORS OF NEW BHSM AT x $\in$[0, 1.0]

Table 2

Y	A-stable Hybrid Block Simpson's K=5			A-stable Hybrid Block Simpson's K=6		
	y_1	y_2	y_3	y_1	y_2	y_3
0.1	8.08×10^{-3}	8.08×10^{-3}	1.78×10^{-1}	5.09×10^{-3}	5.09×10^{-3}	1.59×10^{-1}
0.2	9.53×10^{-5}	9.53×10^{-5}	5.23×10^{-2}	3.45×10^{-4}	3.45×10^{-4}	4.07×10^{-2}
0.3	1.33×10^{-3}	1.33×10^{-3}	2.67×10^{-2}	1.81×10^{-3}	1.81×10^{-3}	1.81×10^{-2}
0.4	1.11×10^{-3}	1.11×10^{-3}	1.88×10^{-2}	1.31×10^{-3}	1.31×10^{-3}	1.27×10^{-2}
0.5	9.84×10^{-3}	9.84×10^{-3}	1.81×10^{-2}	1.20×10^{-3}	1.20×10^{-3}	1.15×10^{-2}
0.6	1.69×10^{-3}	1.69×10^{-3}	6.53×10^{-4}	7.92×10^{-3}	7.92×10^{-3}	1.31×10^{-2}
0.7	4.90×10^{-3}	4.90×10^{-3}	3.23×10^{-5}	1.16×10^{-3}	1.16×10^{-3}	3.05×10^{-4}
0.8	2.51×10^{-4}	2.51×10^{-4}	7.17×10^{-5}	2.91×10^{-4}	2.91×10^{-4}	6.12×10^{-5}
0.9	1.77×10^{-4}	1.77×10^{-4}	5.70×10^{-5}	1.29×10^{-4}	1.29×10^{-4}	5.91×10^{-5}
1.0	1.63×10^{-4}	1.63×10^{-4}	3.86×10^{-4}	9.07×10^{-5}	9.07×10^{-5}	5.56×10^{-5}

4. Conclusion and Acknowledgement

It is evident from the above tables that our proposed methods are indeed accurate, and can handle stiff equations. Also in terms of stability analysis, the methods are A-stables and the schemes have also been shown to be of good order. Finally, we are indebted to the referees for a number of constructive comments.

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