Common Fixed Point Theorem in Intuitionistic Fuzzy Metric Space Satisfying Integral Type Inequality

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Abstract: The aim of this paper is to present some common fixed point theorem in Intuitionistic fuzzy metric space satisfying integral type inequality for E.A. Property.

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1. Introduction

The theory of fixed point equations in one of the basic tools to handle various physical formulations. Fuzzy set was defined by Zadeh [1]. Kramosil and Michalek [2] introduced fuzzy metric space. many authors extend their views. George and Veermanyam [3], modified the notion of fuzzy metric spaces with the help of continuous t-norms Grabiec[4].

In the recent year, several common fixed point theorems for contractive type mappings have been proved by several authors. Branciari [5], gave a fixed point result for a single mapping satisfying Banach’s contraction principle for an integral type inequality.

Aliouche[6] established a common fixed point theorem for weakly compatible mappings in symmetric spaces satisfying a contractive condition of integral type and property (E.A.) introduced by Aamri and El. Moutawakil [7].

Boikanyo and Choudhary [8] prove some common fixed point theorem for pointwise R-weakly commuting mappings in symmetric space.


In this paper, we obtain common fixed point theorem in Intuitionistic fuzzy metric space using E.A. property.

2. Preliminary

Definition 2.1 [11] A binary operation *: [0,1] x [0,1] → [0,1] is a continuous t–norm if it satisfies the following conditions :

(i) a*b = a, for all a ∈ [0,1]
(ii) a*b = c*d, whenever a ≤ c and b ≤ d, for all a,b,c,d ∈ [0,1].

Definition 2.2 [11] A binary operation ◊: [0,1] x [0,1] → [0,1] is a continuous t–conorm if it satisfies the following conditions :

(i) ◊is associative and commutative
(ii) ◊is continuous,
(iii) a◊1 = a, for all a ∈ [0,1]
(iv) a◊b ≤ c◊d, whenever a ≤ c and b ≤ d, for all a,b,c,d ∈ [0,1].

Definition 2.3 [11] A 5-tuple (X, M, N, *, ◊) is said to be an intuitionistic fuzzy metric space if X is an arbitrary set, * is a continuous t-norm, ◊ is a continuous t-conorm and M,N are fuzzy sets on X × [0,∞) satisfying the following conditions.

(i) M(x,y,t) + N(x,y,t) ≤ 1, for all x,y ∈ X and t > 0,
(ii) M(x,y,0) = 0, for all x,y ∈ X,
(iii) M(x,y,t) = 1, for all x,y ∈ X and t > 0, if x = y,
(iv) M(x,y,t) = M(y,x,t), for all x,y ∈ X and t > 0,
(v) M(x,y,t)*M(y,z,s) ≤ M(x,z,t+s), for all x,y ∈ X and t,s > 0,
(vi) for all x,y ∈ X, M(x,y,.) : [0,∞) → [0,1] is left continuous,
(vii) lim_{t→∞} M(x,y,t) = 1, for all x,y ∈ X and t > 0,
(viii) N(x,y,0) = 1, for all x,y ∈ X,
(ix) N(x,y,t) = 0, for all x,y ∈ X and t > 0, if x = y,
(x) N(x,y,t) = N(y,x,t), for all x,y ∈ X and t > 0,
(xi) N(x,y,t)*N(y,z,s) ≤ N(x,z,t+s), for all x,y ∈ X and t,s > 0,
(xii) for all x,y ∈ X, N(x,y,.) : [0,∞) → [0,1] is right continuous,
(xiii) lim_{t→∞} N(x,y,t) = 0, for all x,y ∈ X.

Remark 2.1[13]
In intuitionistic metric fuzzy space (X, M, N, *) is an intuitionistic fuzzy space of the form (X, M, 1-M, *, ◊).
such that t-norm * and t-conorm ◊ are associated as x ◊ y = 1-
((1-x) * (1-y)) for all x,y ∈ X.

Remark 2.2[13]
In intuitionistic fuzzy metric space (X ,M ,N ,*,◊) , M(x,y,*) is non-decreasing and N(x,y,◊) is Non-increasing for all x,y ∈ X.

Example 2.1 – Let (X,d) be a metric space . Define a*b = ab and a ◊ b = min (a,b) for all a,b ∈ {0,1} and let M_0 and N_0 be a fuzzy sets on X x (0,∞) defined as
M_0(x,y,t) = t / t + d(x,y) , N_0(x,y,t) = d(x,y) / t + d(x,y)
Then (X , M_0 , N_0 ,*,◊) is an intuitionistic fuzzy metric space.

Definition 2.4[12]
Let (X , M , N ,*,◊) be an intuitionistic fuzzy metric space . Then
(1) A sequence {x_n} in X is set to be convergent to a point x in X iff Lim n→∞ \( M(x_n, x,t) \) = 1 and Lim n→∞ \( N(x_n, x,t) \) = 0, for all t > 0.

Lemma 2.1[12]
Let (X , M , N ,*,◊) be an intuitionistic fuzzy metric space . If for all x,y ∈ X and t > 0 with positive number k ∈ (0,1) and M(x,y,kt) ≥ M(x,y,t) and N(x,y,kt) ≤ N(x,y,t), then x = y.

Definition 2.5[13]
A pair of self mappings ( P , Q ) of an intuitionistic fuzzy metric space (X , M , N ,*,◊) is said to be compatible if \( \lim_{n→∞} M(PQx_n , QPx_n, t) = 1 \) and \( \lim_{n→∞} N(PQx_n , QPx_n, t) = 0 \), for all t > 0.

Definition 2.6[14]
A pair of self mappings ( P , Q ) of an intuitionistic fuzzy metric space (X , M , N ,*,◊) is said to be semi compatible if \( \lim_{n→∞} PQx_n = Qx_n \) and \( \lim_{n→∞} QPx_n = Qx_n \), for all x ∈ X.

Definition 2.7[15]
A pair of self mappings ( P , Q ) of an intuitionistic fuzzy metric space (X , M , N ,*,◊) is said to satisfy the E.A. property if there exist a sequence \( \{x_n\} \) in X such that \( \lim_{n→∞} x_n = z \) and \( x_n \in X \) for some x ∈ X.

Definition 2.8[15]
Mapping A, B, S and T on an intuitionistic fuzzy metric space (X , M , N ,*,◊) are said to satisfy the common E.A. property if there exist a sequence \( \{x_n\} \) and \( \{y_n\} \) in X such that
\( \lim_{n→∞} A_Sx_n = Ty_n = S_A x_n = x_n \) for some z ∈ X.

Theorem 3.1
Let (X,M,N*,◊) be an Intuitionistic fuzzy metric space with continuous t-norm * and continuous t-conorm ◊ . Let P,Q,S and T be self mappings on X, satisfying the following properties :
1. pair (P,S) and (Q,T) share the common property E.A.
2. S(X) and T(X) are closed subset of X.
3. For any x,y ∈ X and for all t>0 there exist k∈ (0,1) such that
\[ \int_0^{M(Px,Py,kt)} \phi(t)dt \geq \max\{M(Sx,Ty,kt),(M(Sx,Px,kt)+M(St,Py,kt))\} \phi(t)dt \]
\[ \int_0^{N(Px,Py,kt)} \phi(t)dt \geq \min\{N(Sx,Ty,kt),(N(Sx,Px,kt)+N(St,Py,kt))\} \phi(t)dt \]

For all x,y ∈ X, where \( \phi : R⁺ → R⁺ \) is a Lesbegue integrable mapping which is summable satisfying for each 0 < ϵ < 1,
\[ 0 < \int_0^{M(Px,Py,kt)} \phi(t)dt < 1, \int_0^{N(Px,Py,kt)} \phi(t)dt = 1 \]
Then each of pair (P,S) and (Q,T) have a point of coincidence . If the pairs (P,S) and (Q,T) are semi compatible , then P, Q, S and T have a unique common fixed point.

Proof – Since the pairs (P,S) and (Q,T) Share the common property (E.A.), then there exists two sequences \( \{x_n\} \) and \( \{y_n\} \) in X such that
\[ \lim_{n→∞} Px_n = lim_{n→∞} Sx_n = lim_{n→∞} Qy_n = lim_{n→∞} Ty_n = z \] for some z ∈ X.
Since S(X) is closed subset of X ,therefore there exists a point v ∈ X such that z=Sv.
Now, we prove that Pv=z.
By inequality (3), Putting x=v, and y=y_n we get
\[ \int_0^{M(Pv,z,kt)} \phi(t)dt \geq \int_0^{\min\{M(z,v,kt),(M(z,v,kt)+M(z,v,kt))\}} \phi(t)dt \]
Taking let n→∞, we get \( y_n \to v \) .
\[ \int_0^{M(Pv,z,kt)} \phi(t)dt \geq \int_0^{\min\{M(z,v,kt),(M(z,v,kt)+M(z,v,kt))\}} \phi(t)dt \]
Taking \( y_n \to v \), we get
\[ \int_0^{M(Pv,z,kt)} \phi(t)dt \geq \int_0^{\min\{M(z,v,kt),(M(z,v,kt)+M(z,v,kt))\}} \phi(t)dt \]
Similarly,
\[ \int_0^{M(Pv,z,kt)} \phi(t)dt \leq \int_0^{\min\{M(z,v,kt),(M(z,v,kt)+M(z,v,kt))\}} \phi(t)dt \]
Taking let n→∞, we get
\[ \int_0^{M(Pv,z,kt)} \phi(t)dt \leq \int_0^{\min\{M(z,v,kt),(M(z,v,kt)+M(z,v,kt))\}} \phi(t)dt \]
By Lemma 2.1, we conclude that Pv=z.
Since z=Sv and we proved that z=Pv, then from this we get
z=Pv=Sv, which shows that v is a coincidence point of the pair (P,S).

Since T(x) is also closed subset of X. There for lim_n→∞ Ty_n = z in T(X) and hence there exists w ∈ Such that Tw = z = Sv.
Now, we will prove that Qw=z.
By using in equality (3), putting x = v, y = w, we get
\[ \int_0^{M(Pv,z,kt)} \phi(t)dt \geq \int_0^{N(Pv,z,kt)} \phi(t)dt \]
Let \( u \) be another common fixed point of \( P, Q, S \) and \( T \).
Now, by using inequality (3), putting \( x = z \) and \( y = u \),
we have
\[
\int_0^1 \min \{ M(z, w, t) \} \phi(t) dt \geq \int_0^1 \min \{ M(z, w, t) \} \phi(t) dt
\]
Similarly,
\[
\int_0^1 \min \{ M(z, w, t) \} \phi(t) dt \geq \int_0^1 \min \{ M(z, w, t) \} \phi(t) dt
\]
Therefore, from Lemma 2.1, we get \( Qw = z \).
Combining all results we get \( Tw = Qw = z \).
Which shows that \( w \) is the coincidence point of the pair \((Q, T)\).

Corollary 3.2 Let \((X, M, N, *, 0, 1)\) be a complete\nintuitionistic fuzzy metric space and let \( P, Q, S \) and \( T \) be self\nmappings of \( X \) satisfying the conditions of Theorem 3.1 and\nthere exists \( k \in (0, 1) \) such that for all \( x, y \in X \) and \( t > 0 \),
\[
\frac{M(Px, Qy, t)}{M(x, y, t)} \geq \frac{M(Sx, Ty, t)}{M(x, y, t)}
\]
And
\[
\frac{M(Px, Qy, t)}{M(x, y, t)} \leq \frac{M(Sx, Ty, t)}{M(x, y, t)}
\]
Then \( P, Q, S \) and \( T \) have a unique common fixed point in \( X \).

Corollary 3.3 Let \((X, M, N, *, 0, 1)\) be a complete\nintuitionistic fuzzy metric space and let \( P, Q, S \) and \( T \) be self\nmappings of \( X \) satisfying the conditions of Theorem 3.1 and\nthere exists \( k \in (0, 1) \) such that for all \( x, y \in X \) and \( t > 0 \),
\[
\frac{M(Px, Qy, t)}{M(x, y, t)} \geq \frac{M(Sx, Ty, t)}{M(x, y, t)}
\]
And
\[
\frac{M(Px, Qy, t)}{M(x, y, t)} \leq \frac{M(Sx, Ty, t)}{M(x, y, t)}
\]
Then \( P, Q, S \) and \( T \) have a unique common fixed point in \( X \).

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References