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Totally $\psi\alpha g$-Continuous functions in Topological Spaces

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Abstract: ***The aim of this paper is to define a new class of functions namely totally $\psi\alpha g$-continuous functions and slightly $\psi\alpha g$-continuous functions and study their properties. Additionally, we relate and compare these functions with some other functions in topological spaces.***

Keywords: Totally $\psi\alpha g$-continuous functions and Slightly $\psi\alpha g$-continuous functions

1. Introduction

Continuity is an important concept in mathematics and many forms of continuous functions have been introduced over the years. Dr. V. Kokilavani and P. R. Kavitha [10] defined $\psi\alpha g$-continuous functions. R. C. Jain [2] introduced the concept of totally continuous functions and slightly for topological spaces. In this paper, we define totally $\psi\alpha g$-continuous functions and Slightly $\psi\alpha g$-continuous functions and basic properties of these functions are investigated and obtained.

2. Prelimieries

Throughout this paper (X,τ), (Y,σ) and (Z,η) or X,Y,Z represent non-empty topological spaces on which no separation axioms are assumed unless otherwise mentioned. For a subset A of a space (X,τ), $cl(A)$ and $int(A)$ denote the closure and the interior of A respectively. The power set of X is denoted by $P(X)$. If A is $\psi\alpha g$-open and $\psi\alpha g$-closed, then it is said to be $\psi\alpha g$-clopen.

Definition: 2.1 A subset A of a topological space X is said to be a $\psi\alpha g$-open[10] if $\psi cl(A) \subseteq U$ whenever $A \subseteq U$ and U is αg-open.

Definition: 2.2 A function $f:(X,\tau) \to (Y,\sigma)$ is called totally continuous [2] if $f^{-1}(V)$ is clopen set in X for each open set V of Y.

Definition: 2.3 A function $f:(X,\tau) \to (Y,\sigma)$ is called a $\psi\alpha g$-continuous [10] if $f^{-1}(V)$ is $\psi\alpha g$-open set of (X,τ) for every open set V of (Y,σ).

Definition: 2.4 A function $f:(X,\tau) \to (Y,\sigma)$ is called slightly continuous [2] if the inverse image of every clopen set in Y is open in X.

Definition: 2.5 A function $f:(X,\tau) \to (Y,\sigma)$ is called a contra continuous [1] if $f^{-1}(V)$ is closed in (X,τ) for every open set V in (Y,σ).

Definition: 2.6 A topological space X is said to be connected [9] if X cannot be expressed as the union off two disjoint non empty open sets in X.

Definition: 2.7 A topological space X is said to be $\psi\alpha g$-connected if X cannot be expressed as a disjoint union of two non empty $\psi\alpha g$-open sets.

Definition: 2.8 A map $f:(X,\tau) \to (Y,\sigma)$ is said to be pre $\psi\alpha g$-open if the image of every $\psi\alpha g$-open set of X is $\psi\alpha g$-open in Y.

Definition: 2.9 A map $f:(X,\tau) \to (Y,\sigma)$ is said to be perfectly $\psi\alpha g$-continuous if the inverse image of every $\psi\alpha g$-open in (Y,σ) is both open and closed in (X,τ).

Definition: 2.10 A function $f:(X,\tau) \to (Y,\sigma)$ is called strongly $\psi\alpha g$-continuous if the inverse image of every $\psi\alpha g$-open in (Y,σ) is open in (X,τ).

Definition: 2.11 A space (X,τ) is called a locally indiscrete space [3] if every open set of X is closed in X.

Definition: 2.12 [10] Every open set is $\psi\alpha g$-open and every closed set is $\psi\alpha g$-closed.

3. Totally ψαg-Continuous Fucntions

Definition: 3.1 A function $(X,\tau) \to (Y,\sigma)$ is called totally $\psi\alpha g$-continuous functions if the inverse image of every open set of (Y,σ) is both $\psi\alpha g$-open and $\psi\alpha g$-closed subset of (X,τ).

Example: 3.2 Let $X = Y = \{a,b,c\}$, $\tau = \{X,\phi,\{a\},\{b\},\{a,b\}\}$, $\sigma = \{Y,\phi,\{c\},\{a,c\},\{b,c\}\}$, $\psi\alpha gO(X,\tau) = \{X,\phi,\{b,c\},\{a,c\},\{a,b\},\{a\},\{b\}\}$ and $\psi\alpha gC(X,\tau) = \{X,\phi,\{a\},\{b\},\{c\},\{b,c\},\{a,c\}\}$. Let $g:(X,\tau) \to (Y,\sigma)$ be defined by $g(a) = b$, $g(b) = a$, $g(c) = c$. Since $g^{-1}(b) = a$, $g^{-1}(c) = c$ and $g^{-1}(b,c) = \{a,c\}$ is both $\psi\alpha g$-open and $\psi\alpha g$-closed in X. Therefore g is totally $\psi\alpha g$-continuous.

Theorem: 3.3 Every totally $\psi\alpha g$-continuous functions is $\psi\alpha g$-continuous.

Proof: Let V be an open set of (Y,σ). Since f is totally $\psi\alpha g$-continuous functions, $f^{-1}(V)$ is both $\psi\alpha g$-open and $\psi\alpha g$-closed in (X,τ). Therefore f is $\psi\alpha g$-continuous.

Remark: 3.4 The converse of the above theorem need not be true.

Example: 3.5 Let $X=Y=\{a,b,c\}$, $\tau=\{X,\phi,\{b\},\{a,b\}\}$, $\sigma=\{Y,\phi,\{a\},\{a,c\}\}$. Let $g:(X,\tau)\to(Y,\sigma)$ be defined by $g(a)=b$, $g(b)=a$, $g(c)=c$. $\psi\alpha gO(X,\tau)=\{X,\phi,\{b\},\{a,b\},\{b,c\}\}$ and $\psi\alpha gC(X,\tau)=\{X,\phi,\{a\},\{c\},\{a,c\}\}$. Clearly g is not totally $\psi\alpha g$-continuous. Since $g^{-1}(\{a\})=\{b\}$ is $\psi\alpha g$-open in X but not $\psi\alpha g$-closed. However g is $\psi\alpha g$-continuous.

Theorem: 3.6 Every totally continuous function is totally $\psi\alpha g$-continuous.

Proof: Let V be an open set of (Y,σ). Since f is totally continuous functions, $f^{-1}(V)$ is both open and closed in (X,τ). Since every open set is $\psi\alpha g$-open and every closed set is $\psi\alpha g$-closed. $f^{-1}(V)$ is both $\psi\alpha g$-open and $\psi\alpha g$-closed in (X,τ). Therefore f is totally $\psi\alpha g$-continuous.

Remark: 3.7 The converse of the above theorem need not be true.

Example: 3.8 Let $X=Y=\{a,b,c,d\}$, $\tau=\{X,\phi,\{a,b\}\}$, $\tau^c=\{Y,\phi,\{c,d\}\}$, $\sigma=\{Y,\phi,\{a\},\{b\},\{a,b\},\{b,c\},\{a,b,c\}\}$ Let $g:(X,\tau)\to(Y,\sigma)$ be defined by $g(a)=a$, $g(b)=b$, $g(c)=c$, $g(d)=d$. $\psi\alpha gO(X,\tau)=P(X)=\psi\alpha gC(X,\tau)$. Clearly g is not totally $\psi\alpha g$-continuous but $g^{-1}(\{a,b\})=\{a,b\}$ is open in X but not closed in X. Therefore g is not totally continuous.

Theorem: 3.9 Every perfectly $\psi\alpha g$-continuous map is totally $\psi\alpha g$-continuous.

Proof: Let $f:(X,\tau)\to(Y,\sigma)$ be a perfectly $\psi\alpha g$-continuous map. Let V be an open set of (Y,σ). Then V is $\psi\alpha g$-open in (Y,σ). Since f is perfectly $\psi\alpha g$-continuous, $f^{-1}(V)$ is both open and closed in (X,τ), implies $f^{-1}(V)$ is both $\psi\alpha g$-open and $\psi\alpha g$-closed in (X,τ). Therefore, f is totally $\psi\alpha g$-continuous.

Remark: 3.10 The converse of the above theorem is need not be true.

Example: 3.11 Let $X=Y=\{a,b,c,d\}$, $\tau=\{X,\phi,\{a,b\}\}$, $\tau^c=\{Y,\phi,\{c,d\}\}$, $\sigma=\{Y,\phi,\{a\},\{b\},\{a,b\},\{b,c\},\{a,b,c\}\}$ Let $g:(X,\tau)\to(Y,\sigma)$ be defined by $g(a)=a$, $g(b)=b$, $g(c)=c, g(d)=d, \psi\alpha gO(X,\tau)=P(X)=\psi\alpha gC(X,\tau)$. $\psi\alpha gO(Y,\sigma)=\{Y,\phi,\{a\},\{b\},\{c\},\{a,b\},\{a,c\},\{b,c\},\{a,b,c\},\{a,b,d\}\}$. Clearly g is totally $\psi\alpha g$-continuous but $g^{-1}(\{a,b\})=\{a,b\}$ is open in X but not closed in X. Therefore g is not perfectly continuous.

Remark: 3.12 The concept of totally $\psi\alpha g$-continuous and strongly $\psi\alpha g$-continuous are independent of each other.

Example: 3.13 Let $X=Y=\{a,b,c,d\}$, $\tau=\{X,\phi,\{a,b\}\}$, $\tau^c=\{Y,\phi,\{c,d\}\}$, $\sigma=\{Y,\phi,\{a\},\{b\},\{a,b\},\{b,c\},\{a,b,c\}\}$ Let $g:(X,\tau)\to(Y,\sigma)$ be defined by $g(a)=a$, $g(b)=b$, $g(c)=c, g(d)=d, \psi\alpha gO(X,\tau)=P(X)=\psi\alpha gC(X,\tau)$. $\psi\alpha gO(Y,\sigma)=\{Y,\phi,\{a\},\{b\},\{c\},\{a,b\},\{a,c\},\{b,c\},\{a,b,c\},\{a,b,d\}\}$. Clearly g is totally $\psi\alpha g$-continuous but $g^{-1}(\{b\})=\{b\}$ is not open in X. Therefore g is not strongly $\psi\alpha g$-continuous.

Example: 3.14 Let $X=Y=\{a,b,c\}$, $\tau=\{X,\phi,\{a\},\{a,c\}\{a,b\}\}$, $\sigma=\{Y,\phi,\{a\},\{b\},\{a,b\}\}$ Let $g:(X,\tau)\to(Y,\sigma)$ be defined by $g(a)=a=g(b)$, $g(c)=c$. $\psi\alpha gO(X,\tau)=\{X,\phi,\{a\},\{a,c\}\{a,b\}$, $\psi\alpha gC(X,\tau)=\{X,\phi,\{b\},\{c\},\{b,c\}\}$ and $\psi\alpha gO(Y,\sigma)=\{Y,\phi,\{a\},\{b\},\{a,b\},\{a,c\}\}$. Clearly g is Strongly $\psi\alpha g$-continuous but $g^{-1}(\{a\})=\{a,b\}$ is $\psi\alpha g$-open in X but not $\psi\alpha g$-closed. Therefore g is not totally $\psi\alpha g$-continuous.

Theorem: 3.15 If $f:X\to Y$ is a totally $\psi\alpha g$-continuous map and X is $\psi\alpha g$-connected, then Y is an indiscrete space.

Proof: Suppose that Y is not an indiscrete space. Let A be a non-empty open subset of Y. Since, f is totally $\psi\alpha g$-continuous map, then $f^{-1}(A)$ is a non-empty $\psi\alpha g$-clopen subset of X. Then $X=f^{-1}(A)\cup(f^{-1}(A))^C$. Thus, X is a union of two non-empty disjoint $\psi\alpha g$-open sets which is contradiction to the fact that X is $\psi\alpha g$-connected. Therefore, Y must be an indiscrete space.

Theorem: 3.16 Let $f:X\to Y$ and $g:Y\to Z$ be functions. Then $g\circ f:X\to Z$.
(i)If f is $\psi\alpha g$-irresolute and g is totally $\psi\alpha g$-continuous then $g\circ f$ is totally $\psi\alpha g$-continuous.
(ii) If f is totally $\psi\alpha g$-continuous g is continuous then $g\circ f$ is totally $\psi\alpha g$-continuous.

Proof: (i) Let V be an open set in Z. Since, g is totally $\psi\alpha g$-continuous map, $g^{-1}(V)$ is $\psi\alpha g$-clopen in Y. Since f is $\psi\alpha g$-irresolute, $f^{-1}(g^{-1}(V))$ is $\psi\alpha g$-open and $\psi\alpha g$-closed in X. Since $(g\circ f)^{-1}(V)=f^{-1}(g^{-1}(V))$. Therefore, $g\circ f$ is totally $\psi\alpha g$-continuous.

(ii) Let V be an open set in Z. Since, g is continuous, $g^{-1}(V)$ is open in Y. Since, f is totally $\psi\alpha g$-continuous, $f^{-1}(g^{-1}(V))$ is $\psi\alpha g$-clopen in X. Hence, $g\circ f$ is totally $\psi\alpha g$-continuous.

4. Slightly ψαg-Continuous Fucntions

Definition: 4.1 A function $f:(X,\tau)\to(Y,\sigma)$ is called slightly $\psi\alpha g$-continuous at a point $x\in X$ if for each clopen subset V of Y containing $f(x)$, there exists a $\psi\alpha g$-open subset U in X containing x such that $f(U)\subseteq V$. The function f is said to be slightly $\psi\alpha g$-continuous if f is slightly $\psi\alpha g$-continuous at each of its points.

Definition: 4.2 A function $f:(X,\tau)\to(Y,\sigma)$ is said to be slightly $\psi\alpha g$-continuous if the inverse image of every clopen set in Y is $\psi\alpha g$-open in X.

Example: 4.3 Let $X = Y = \{a,b,c,d\}$, $\tau = \{X,\phi,\{a,b\},\{a,b,c\}\}$, $\sigma = \{Y,\phi,\{a\},\{b,c,d\}\}$ and $\psi\alpha gO(X,\tau) = \{X,\phi,\{a\},\{b\},\{c\},\{a,b\},\{a,c\},\{a,d\},\{b,c\},\{b,d\},\{a,b,c\},\{a,b,d\},\{a,c,d\},\{b,c,d\}\}$. Let $g:(X,\tau) \to (Y,\sigma)$ be defined by $g(a) = b$, $g(b) = a$ $g(c) = d$, $g(d) = c$. Clearly g is Slightly $\psi\alpha g$-continuous.

Proposition: 4.4 The definition 4.1 and 4.2 are equivalent.

Proof: Suppose the definition 4.1 holds. Let V be a clopen set in Y and $x \in f^{-1}(V)$. Then $f(x) \in V$ and thus there exists a $\psi\alpha g$-open set U_x such that $x \in U_x \subseteq f^{-1}(V)$ and $f^{-1}(V) = \bigcup_{x\in f^{-1}(V)} U_x$. Since, arbitrary union of $\psi\alpha g$-open set is $\psi\alpha g$-open. $f^{-1}(V)$ is $\psi\alpha g$-open in X and therefore, f is slightly $\psi\alpha g$-continuous.

Suppose, the definition 4.2 holds. Let $f(x) \in V$ where V is a clopen set in Y. Since, f is slightly $\psi\alpha g$-continuous, $x \in f^{-1}(V)$ where $f^{-1}(V)$ is $\psi\alpha g$-open in X. Let $U = f-1V$. Then U is $\psi\alpha g$-open in X, $x\in X$ and $f(U)\subseteq V$.

Theorem: 4.5 For a function $f:(X,\tau) \to (Y,\sigma)$, the following statements are equivalent.
(i) f is slightly $\psi\alpha g$-continuous.
(ii) The inverse image of every clopen set V of Y is $\psi\alpha g$-open in X.
(iii) The inverse image of every clopen set V of Y is $\psi\alpha g$-closed in X.
(iv) The inverse image of every clopen set V of Y is $\psi\alpha g$-clopen in X.

Proof: (i)⇒(ii): Follows from the proposition 4.4.
(ii)⇒(iii): Let V be a clopen set in Y which implies V^C is clopen in Y. By (ii), $f^{-1}(V^C) = (f^{-1}(V))^C$ is $\psi\alpha g$-open in X. Therefore, $f^{-1}(V)$ is $\psi\alpha g$-closed in X.
(iii)⇒(iv): By (ii) and (iii), $f^{-1}(V)$ is $\psi\alpha g$-clopen in X.
(iv)⇒(i): Let V be a clopen set in Y containing $f(x)$, by (iv), $f^{-1}(V)$ is $\psi\alpha g$-clopen in X. Take $U = f^{-1}(V)$, then $f(U) \subseteq V$. Hence, f is slightly $\psi\alpha g$-continuous.

Theorem: 4.6 Every slightly continuous function is slightly $\psi\alpha g$-continuous.

Proof: Let $f:(X,\tau) \to (Y,\sigma)$ be a slightly continuous function. Let V be a clopen set in Y. Then, $f^{-1}(V)$ is open in X. Since, every open set is $\psi\alpha g$-open. Hence, f is slightly $\psi\alpha g$-continuous.

Remark: 4.7 The converse of the above theorem need not be true as can be seen from the following example.

Example: 4.8 Let $X = Y = \{a,b,c,d\}$, $\tau = \{X,\phi,\{a,b\},\{a,b,c\}\}$, $\sigma = \{Y,\phi,\{a\},\{b,c,d\}\}$ and $\psi\alpha gO(X,\tau) = X,\phi,\{a\},\{b\},\{c\},\{a,b\},\{a,c\},\{a,d\},\{b,c\},\{b,d\},\{a,b,c\},\{a,b,d\},\{a,c,d\},\{b,c,d\}\}$. Let $g:(X,\tau) \to (Y,\sigma)$ be defined by $g(a) = b$, $g(b) = a$ $g(c) = d$, $g(d) = c$. Clearly g is Slightly $\psi\alpha g$-continuous but not slightly continuous. Since $g^{-1}(\{a\}) = \{b\}$ where $\{a\}$ is clopen in Y but $\{b\}$ is not open in X.

Theorem: 4.9 Every $\psi\alpha g$-continuous function is slightly $\psi\alpha g$-continuous.

Proof: Let $f:(X,\tau) \to (Y,\sigma)$ be a $\psi\alpha g$-continuous function. Let V be a clopen set in Y. Then, $f^{-1}(V)$ is $\psi\alpha g$-open and $\psi\alpha g$-closed in X. Hence, f is slightly $\psi\alpha g$-continuous.

Remark: 4.10 The converse of the above theorem need not be true as can be seen from the following example.

Example: 4.11 Let $X = \{a,b,c\}$, $Y = \{a,b\}$, $\tau = \{X,\phi,\{a\},\{b\},\{a,b\}\}$, $\sigma = \{Y,\phi,\{a\}\}$ and $\psi\alpha gO(X,\tau) = \{X,\phi,\{a\},\{b\},\{a,b\},\{a,c\}\}$. Let $g:(X,\tau) \to (Y,\sigma)$ be defined by $g(a) = b$, $g(b) = g(c) = a$. The function g is Slightly $\psi\alpha g$-continuous but not $\psi\alpha g$-continuous. Since $g^{-1}(\{a\}) = \{b,c\}$ is not $\psi\alpha g$-open in X.

Theorem: 4.12 Every contra $\psi\alpha g$-continuous function is slightly $\psi\alpha g$-continuous.

Proof: Let $f:(X,\tau) \to (Y,\sigma)$ be a contra $\psi\alpha g$-continuous function. Let V be a clopen set in Y. Then, $f^{-1}(V)$ is $\psi\alpha g$-open in X. Hence, f is slightly $\psi\alpha g$-continuous.

Remark: 4.13 The converse of the above theorem need not be true as can be seen from the following example.

Example: 4.14 Let $X = Y = \{a,b,c\}$, $\tau = \{X,\phi,\{a\},\{b\},\{a,b\}\}$, $\sigma = \{Y,\phi,\{a\},\{c\},\{a,b\},\{a,c\}\}$ and $\sigma^c = \{Y,\phi,\{b\},\{c\},\{a,b\},\{b,c\}\}$ and $\psi\alpha gO(X,\tau) = \{X,\phi,\{a\},\{b\},\{a,b\},\{a,c\}\}$ and Let $g:(X,\tau) \to (Y,\sigma)$ be defined by $g(a) = a$, $g(b) = c$, $g(c) = b$. The function g is Slightly $\psi\alpha g$-continuous but not contra $\psi\alpha g$-continuous. Since $g^{-1}(\{b\}) = \{c\}$ is not $\psi\alpha g$-open in X.

Remark: 4.15 Composition of two slightly $\psi\alpha g$-continuous need not be slightly $\psi\alpha g$-continuous as it can be seen from the following example.

Example: 4.16 Let $X = Y = \{a,b,c,d\}$, $Z = \{a,b,c\}$ and the topologies are $\tau = \{X,\phi,\{a\},\{a,b\},\{a,b,c\}\}$, $\sigma = \{Y,\phi,\{a\},\{b,d\}\}$ and $\eta = \{Z,\phi,\{b\},\{a,c\}\}$. Define $f:(X,\tau) \to (Y,\sigma)$ by $f(a) = b$, $f(b) = a$, $f(c) = c$, $f(d) = d$.
$\psi\alpha gO(X,\tau) = \left\{\begin{matrix} X,\phi,\{a\},\{b\},\{c\},\{a,b\},\{a,c\},\{a,d\},\{b,c\},\{a,b,c\}, \\ \{a,b,d\},\{a,c,d \end{matrix}\right\}$.
Clearly f is Slightly $\psi\alpha g$-continuous. Define $g:(Y,\sigma) \to (Z,\eta)$ by $g(a) = a$, $g(b) = b = g(c)$, $g(d) = c$. $\psi\alpha gO(Y,\sigma) = P(Y)$. Clearly, g is Slightly $\psi\alpha g$-continuous. But $g \circ f:(X,\tau) \to (Z,\eta)$ is not Slightly $\psi\alpha g$-continuous. Since $(g \circ f)^{-1}(\{a,c\}) = f^{-1}(g^{-1}(\{a,c\})) = f^{-1}(\{a,d\}) = \{b,d\}$ is not $\psi\alpha g$-open in (X,τ).

Theorem: 4.17 Let $f:X \to Y$ and $g:Y \to Z$ be functions. Then the following properties hold:
(i) If f is $\psi\alpha g$-irresolute and g is slightly $\psi\alpha g$-continuous then $(g \circ f)$ is slightly $\psi\alpha g$-continuous.
(ii) If f is $\psi\alpha g$-irresolute and g is $\psi\alpha g$-continuous then $(g \circ f)$ is slightly $\psi\alpha g$-continuous.

(iii) If f is $\psi\alpha g$-irresolute and g is slightly continuous then $(g \circ f)$ is slightly $\psi\alpha g$-continuous.
(iv) If f is $\psi\alpha g$-continuous and g is slightly continuous then $(g \circ f)$ is slightly $\psi\alpha g$-continuous.
(v) If f is strongly $\psi\alpha g$-continuous and g is slightly $\psi\alpha g$-continuous then $(g \circ f)$ is slightly continuous.
(vi) If f is slightly $\psi\alpha g$-continuous and perfectly $\psi\alpha g$-continuous then $(g \circ f)$ is $\psi\alpha g$-irresolute.
(vii) If f is slightly $\psi\alpha g$-continuous and g is contra continuous then $(g \circ f)$ is slightly $\psi\alpha g$-continuous.
(viii) If f is $\psi\alpha g$-irresolute and g is contra $\psi\alpha g$-continuous then $(g \circ f)$ is slightly $\psi\alpha g$-continuous.

Proof: (i) Let V be a clopen set in Z. Since g is slightly $\psi\alpha g$-continuous, $g^{-1}(V)$ is $\psi\alpha g$-open in Y. Since f is $\psi\alpha g$-irresolute, $f^{-1}(g^{-1}(V))$ is $\psi\alpha g$-open in X. Since $(g \circ f)^{-1}(V) = f^{-1}(g^{-1}(V))$, $g \circ f$ is slightly $\psi\alpha g$-continuous.
(ii) Let V be a clopen set in Z. Since g is $\psi\alpha g$-continuous, $g^{-1}(V)$ is $\psi\alpha g$-open in Y. Since f is $\psi\alpha g$-irresolute, $f^{-1}(g^{-1}(V))$ is $\psi\alpha g$-open in X. Hence $g \circ f$ is slightly $\psi\alpha g$-continuous.
(iii) Let V be a clopen set in Z. Since g is slightly continuous, $g^{-1}(V)$ is open in Y. Since f is $\psi\alpha g$-irresolute, $f^{-1}(g^{-1}(V))$ is $\psi\alpha g$-open in X. Hence $g \circ f$ is slightly $\psi\alpha g$-continuous.
(iv) Let V be a clopen set in Z. Since g is slightly continuous, $g^{-1}(V)$ is open in Y. Since f is $\psi\alpha g$-continuous, $f^{-1}(g^{-1}(V))$ is $\psi\alpha g$-open in X. Hence $g \circ f$ is slightly $\psi\alpha g$-continuous.
(v) Let V be a clopen set in Z. Since g is slightly $\psi\alpha g$-continuous, $g^{-1}(V)$ is $\psi\alpha g$-open in Y. Since f is strongly $\psi\alpha g$-continuous, $f^{-1}(g^{-1}(V))$ is open in X. Therefore $g \circ f$ is slightly continuous.
(vi) Let V be a $\psi\alpha g$-open set in Z. Since g is perfectly $\psi\alpha g$-continuous, $g^{-1}(V)$ is open and closed in Y. Since f is slightly $\psi\alpha g$-continuous, $f^{-1}(g^{-1}(V))$ is $\psi\alpha g$-open in X. Hence $g \circ f$ is $\psi\alpha g$-irresolute.
(vii) Let V be a clopen set in Z. Since g is contra continuous, $g^{-1}(V)$ is open and closed in Y. Since f is slightly $\psi\alpha g$-continuous, $f^{-1}(g^{-1}(V))$ is $\psi\alpha g$-open in X. Hence$g \circ f$ is slightly $\psi\alpha g$-continuous.
(viii) Let V be a clopen set in Z. Since g is contra $\psi\alpha g$-continuous, $g^{-1}(V)$ is $\psi\alpha g$-open and $\psi\alpha g$-closed in Y. Since f is $\psi\alpha g$-irresolute, $f^{-1}(g^{-1}(V))$ is $\psi\alpha g$-open and $\psi\alpha g$-closed in X. Hence $g \circ f$ is slightly $\psi\alpha g$-continuous.

Theorem: 4.18 If the function $f: (X, \tau) \to (Y, \sigma)$ is slightly $\psi\alpha g$-continuous and (X, τ) is $\psi_{\alpha g}T_{1/2}$ space, then f is slightly continuous.

Proof: Let V be a clopen set in Z. Since g is slightly $\psi\alpha g$-continuous, $f^{-1}(V)$ is $\psi\alpha g$-open in X. Since X is $\psi_{\alpha g}T_{1/2}$ space, $f^{-1}(V)$ is open in X. Hence f is slightly continuous.

Theorem: 4.19 Let $f: (X, \tau) \to (Y, \sigma)$ and $g: (Y, \sigma) \to (Z, \eta)$ be functions. If f is surjective and pre $\psi\alpha g$-open and $(g \circ f): (X, \tau) \to (Z, \eta)$ is slightly $\psi\alpha g$-continuous, then g is slightly $\psi\alpha g$-continuous.

Proof: Let V be a clopen set in (Z, η). Since $(g \circ f): (X, \tau) \to (Z, \eta)$ is slightly $\psi\alpha g$-continuous, $f^{-1}(g^{-1}(V))$ is $\psi\alpha g$-open in X. Since, f is surjective and pre $\psi\alpha g$-open $f(f^{-1}(g^{-1}(V))) = g^{-1}(V)$ is $\psi\alpha g$-open. Hence g is slightly $\psi\alpha g$-continuous.

Theorem: 4.20 Let $f: (X, \tau) \to (Y, \sigma)$ and $g: (Y, \sigma) \to (Z, \eta)$ be functions. If f is surjective and pre $\psi\alpha g$-open and $\psi\alpha g$-irresolute, then $(g \circ f): (X, \tau) \to (Z, \eta)$ is slightly $\psi\alpha g$-continuous if and only if g is slightly $\psi\alpha g$-continuous.

Proof: Let V be a clopen set in (Z, η). Since $(g \circ f): (X, \tau) \to (Z, \eta)$ is slightly $\psi\alpha g$-continuous, $f^{-1}(g^{-1}(V))$ is $\psi\alpha g$-open in X. Since, f is surjective and pre $\psi\alpha g$-open $f(f^{-1}(g^{-1}(V))) = g^{-1}(V)$ is $\psi\alpha g$-open in Y. Hence g is slightly $\psi\alpha g$-continuous.

Conversely, let g is slightly $\psi\alpha g$-continuous. Let V be a clopen set in (Z, η), then g is $\psi\alpha g$-open in Y. Since, f is $\psi\alpha g$-irresolute, $f^{-1}(g^{-1}(V))$ is $\psi\alpha g$-open in X. $(g \circ f): (X, \tau) \to (Z, \eta)$ is slightly $\psi\alpha g$-continuous.

Theorem: 4.21 If f is slightly $\psi\alpha g$-continuous from a $\psi\alpha g$-connected space (X, τ) onto a space (Y, σ) then Y is not a discrete space.

Proof: Suppose that Y is a discrete space. Let V be a proper non empty open subset of Y. Since, f is slightly $\psi\alpha g$-continuous, $f^{-1}(V)$ is a proper non empty $\psi\alpha g$-clopen subset of X which is a contradiction to the fact that X is $\psi\alpha g$-connected.

Theorem: 4.22 If $f: (X, \tau) \to (Y, \sigma)$ is a slightly $\psi\alpha g$-continuous surjection and X is $\psi\alpha g$-connected, then Y is connected.

Proof: Suppose Y is not connected, then there exists non empty disjoint open sets U and V such that $Y = U \cup V$. Therefore, U and V are clopen sets in Y. Since, f is slightly $\psi\alpha g$-continuous, $f^{-1}(U)$ and $f^{-1}(V)$ are non empty disjoint $\psi\alpha g$-open in X and $X = f^{-1}(U) \cup f^{-1}(V)$. This shows that X is not $\psi\alpha g$-connected. This is a contradiction and hence, Y is connected.

Theorem: 4.23 If $f: (X, \tau) \to (Y, \sigma)$ is a slightly $\psi\alpha g$-continuous and (Y, σ) is locally indiscrete space then f is $\psi\alpha g$-continuous.

Proof: Let V be an open subset of Y. Since, (Y, σ) is a locally indiscrete space, V is closed in Y. Since, f is slightly $\psi\alpha g$-continuous, $f^{-1}(V)$ is $\psi\alpha g$-open in X. Hence, f is $\psi\alpha g$-continuous.

Theorem: 4.24 If $f: (X, \tau) \to (Y, \sigma)$ is a slightly $\psi\alpha g$-continuous and A is an open subset of X then the restriction $f|_A: (A, \tau_A) \to (Y, \sigma)$ is slightly $\psi\alpha g$-continuous.

Proof: Let V be an clopen subset of Y. Then $(f|_A)^{-1}(V) = f^{-1}(V) \cap A$. Since, $f^{-1}(V)$ is $\psi\alpha g$-open and A is open, $(f|_A)^{-1}(V)$ is $\psi\alpha g$-open in the relative topology of A. Hence, $f|_A: (A, \tau_A) \to (Y, \sigma)$ is slightly $\psi\alpha g$-continuous.

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