Then we have $\begin{cases} \frac{1}{2}p_0 = b \\ \frac{1}{2}p_1 = a+3b \\ \frac{1}{2}p_2 = 3a+3b \\ \frac{1}{2}p_3 = 3a+b \\ \frac{1}{2}p_4 = a \end{cases}$, from (2),we can solution

$$\begin{cases} a = \frac{1}{8} - \frac{\sqrt{2}}{8}\cos\alpha\cos\beta - \frac{C}{4}\cos\eta \\ \quad +i\left(-\frac{\sqrt{2}}{8}\sin\alpha\cos\gamma - \frac{C}{4}\sin\eta\right) \\ b = \frac{1}{8} - \frac{\sqrt{2}}{8}\cos\alpha\cos\beta + \frac{C}{4}\cos\eta \\ \quad +i\left(-\frac{\sqrt{2}}{8}\sin\alpha\cos\gamma + \frac{C}{4}\sin\eta\right) \end{cases}$$

Where $\alpha, \beta, \lambda, \eta, C$ satisfies

$$\begin{cases} \frac{\sqrt{2}}{2}\cos\alpha\sin\beta = \frac{1}{2} - \sqrt{2}\cos\alpha\cos\beta + C\cos\eta \\ \frac{\sqrt{2}}{2}\sin\alpha\sin\gamma = -\sqrt{2}\sin\alpha\cos\gamma + C\sin\eta \\ C = \frac{1}{2}\sqrt{\left(1-\sqrt{2}\cos\alpha\cos\beta\right)^2 + 2\cos^2\gamma\sin^2\alpha} \end{cases}$$

Theorem 5 Let $P_5(\xi) = \frac{1}{2}\sum_{k=0}^{4} p_k e^{-ik\xi}$, $\{p_k = x_k + iy_k\}_{k=0}^{4}$ is orthogonal complex values scale filter with M=4, and $P_5(\xi) = e^{-i4\xi}\overline{P_4}(-\xi)$, then

$$\begin{cases} C\cos\eta = 0 \\ \sin\alpha\cos\gamma = 0 \\ \cos\alpha\sin\beta = 0 \\ C = \frac{1}{2}\sqrt{\left(1-\sqrt{2}\cos\alpha\cos\beta\right)^2 + 2\cos^2\gamma\cos^2\alpha} \end{cases}.$$

Proof: By $P_5(\xi) = e^{-i4\xi}\overline{P_5}(-\xi)$ and $P_5(\xi) = \frac{1}{2}\sum_{k=0}^{4} p_k e^{-ik\xi}$,then we have

$$p_0 + p_1e^{-i\xi} + p_2e^{-i2\xi} + p_3e^{-i3\xi} + p_4e^{-i4\xi}$$
$$= e^{-i4\xi}\left(\overline{p_0} + \overline{p_1}e^{i\xi} + \overline{p_2}e^{i2\xi} + \overline{p_3}e^{i3\xi} + \overline{p_4}e^{i4\xi}\right)$$
$$= \overline{p_0}e^{-i4\xi} + \overline{p_1}e^{-i3\xi} + \overline{p_2}e^{-i2\xi} + \overline{p_3}e^{-i\xi} + \overline{p_4}$$

so $\begin{cases} p_0 = \overline{p_4} \\ p_1 = \overline{p_3} \\ p_2 = \overline{p_2} \end{cases}$ or $\begin{cases} x_0 = x_4 \\ y_0 = -y_4 \\ x_1 = x_3 \\ y_1 = -y_3 \\ y_2 = -y_2 \end{cases}$

From (2), we can solution

$$\begin{cases} C\cos\eta = 0 \\ \sin\alpha\cos\gamma = 0 \\ \cos\alpha\sin\beta = 0 \\ C = \frac{1}{2}\sqrt{\left(1-\sqrt{2}\cos\alpha\cos\beta\right)^2 + 2\cos^2\gamma\sin^2\alpha} \end{cases}$$

Example 6 Let $\alpha = \frac{\pi}{2}, \beta = 0, \gamma = \frac{\pi}{2}, \eta = \frac{\pi}{2}$, then

$$P_5(\xi) = \frac{1}{8}\left[(1+i) + \left(2+2\sqrt{2}\right)ie^{-i\xi} + 2e^{-i2\xi} + \left(2-2\sqrt{2}\right)ie^{-i3\xi} + (1-i)e^{-i4\xi}\right]$$

3. Conclusion

At present, the symmetric orthogonal compactly supported complex wavelets were widely applied to image processing and statistical model, based on orthogonal wavelet necessary condition. In this paper, we present a construction method for parameterizing orthogonal complex wavelets. By this method, we can construct some complex wavelets with high sum rules or conjugate symmetric, at the same time, some examples are given. In addition, with the increase of positive integer M, the number of parameters is increased.

References

[1] Tang Xiaochu. Wavelet analysis and its application [M]. Chongqing: Chongqing publishing house, 2006.

[2] FERNANDES Felix C A，IVAN W Selesnick，Van RUTGER L C Spaendonck． Complex wavelet transforms with allpass filters［J. Signal Processing，2003，83(8) : 1 689-1 706

[3] Lawton W. Applications of complex valued wavelet transforms to subband decomposition[J]. IEEE Transaction on Signal Process, 1993,44(12):3566-3568

[4] Lina J M. Complex daubechies wavelets[J]. Applied and Computational Harmonic Analysis,1995,2(3):219-229

[5] Lai M J, Roach D W. Parameterization of univariate orthogonal wavelets with short support [C] // Approximation theory X: Wavelets, Splines, and Applications, Nashville, Tenn, USA, Vanderbilt University Press, 2002:369-384

[6] FENG Yan, YUAN De-hui, YANG Shou-zhi, Parameterizations of Orthogonal Complex Wavelets with Short Support, CHINESE JOURNAL OF ENGINEERING MATHEMATICS, 2013,5(17):773-780

Author Profile

Qingming Zhang is reading the M.S. degrees in College of Science from Tianjin University of Technology and Education .