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A Note on Fuzzy Upper and Lower Weakly Semi - Precontinuous Multifunction

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Abstract: ***This paper is devoted to the introduction and study of fuzzy lower and upper weakly semi-precontinuous multifunction between a general topological space (X,τ) and a fuzzy topological space (y,σ) using the lower inverse f and upper f^+ several equivalent conditions of a lower and upper weakly Semi-precontinuous multifunctions are obtained.***

Keywords: Fuzzy Continuity, Fuzzy Semicontinuous, Fuzzy semipre-continuous, Fuzzy multifunctions

1. Introduction

Through the paper by(x,τ) or simply by X we will mean a topological space in a classical sense and (y,σ) or simply by Y will stand for a fuzzy topological space (fts, for short) as defined by chang [2], Fuzzy sets in to Y will be denoted λ, μ, ϑ & etc. And interior and closure fuzzy sets λ in an fts Y will be denoted by int λ and cl(λ) and respectively a fuzzy set λ of fts Y is called fuzzy α- open set (fuzzy α- closed) if A ≤ int cl(λ)). Then α- closure and α-interior of λ are defined as follows:

αcl_= D { μ / μ is fuzzy α closed and 1 ≤ μ }

When a fuzzy set 1 is quasi-coincident (q-coincident,for short) with a fuzzy set λ in (y,σ) we shell write lq,m if λ and μ are not quasi-coincident denoted by (λqμ).The words 'neighbourhood ' and fuzzy topological space 'will be abbreviated as 'nbd' and 'fts' respectively.

On Fuzzy Weakly Semi-precontinuous Multifunctions

Definition (1.1.1) Let (X,τ) be a topological space in the classical sens and (y,σ) be a fuzzy topological space f:x!y is called fuzzy multifunction for each x 2 X.,f(x) is a fuzzy set in y.

Definition (1.1.2) For a fuzzy multifunction f :x → y the f(A) of fuzzy set A in Y are defined as follows:
f(A) = f (x) ε X : F(x)qA,
f(A) = f (x) ε X : F(x) ≤ A

Theorem (1.1.1) For a fuzzy multifunction f : x→ y we have f(1-A)= x- f(A) for any fuzzy set A in Y.

Definition (1.13) For a fuzzy multifunction f: x → y is called:

(a) Fuzzy lower and precontinuous at a point x_0 ε X if for every fuzzy open set V in Y will x_0 ε F^-(V), there exist a semi preopen nbd U of x_0 in X such that U Ċ F(v).
(b) Fuzzy upper and precontinuous at a point x_0ε X if for every fuzzy open set V in Y will x_0 ε F^+(V),there exist a semi preopen nbd U of x_0 in X such that U F(v).
(c) Fuzzy lower semii- precontinuous on X iff it is respectively so that each x_0 ε X.
(d) Fuzzy semi-pre continuous on X,iff it is fuzzy lower semi-precontinuous and fuzzy upper semi precontinuous mapping.

Definition (1.14) For a fuzzy multifunction f : x → y is called :

(a) Fuzzy lower weakly semi-continuous at a point x_0 ε X. Iff for every fuzzy open set V in Y will x_0 εf(v), there exists a semi open nbd U of x_0 in X such that U ε f_0(SCIV).
(b) Fuzzy lower weakly semi-continuous at a point x_0 ε X iff for every fuzzy open set V in Y will x_0 ε f(v), there exists a semi open nbd U of x_0 in X such that U ε f0(SCIV).
(c) Fuzzy lower weakly semi-continuous on X iff it is respectively so each x_0 ε X.
(d) Fuzzy weakly semi- continuous on X iff it is fuzzy lower weakly semi-continuous and fuzzy upper weakly semi-continuous mapping.

2. Lower and Upper Weakly Semi-Precontinuous Fuzzy Multifunctions

Definition (2.1) For a fuzzy multifunction f : x → y is called :

(a) Fuzzy lower weakly semi-precontinuous at a point x0 ε X iff for every fuzzy open set V in Y will x_0 ε .F(v).There exist a semi preopen nbd of U of x_0 in X such that UĊF(SCLV).
(b) Fuzzy lower weakly semi-precontinuous at a point x_0 ε X iff for every fuzzy open set V in Y will x_0ε F(v). There exist a semi preopen nbd of U of x_0 in X such that UĊF(SCLV).
(c) Fuzzy lower weakly semi-precontinuous on X iff it is respectively so at each x_0 ε X.
(d) Fuzzy weakly semi-precontinuous on X iff it is fuzzy lower weakly semi-precontinuous and fuzzy upper weakly semi-precontinuous mapping.

Theorem (2.3) Let f :x → y is fuzzy lower weakly semi-precontinuous iff every fuzzy open set V in F(V)_SP int F(SCIV) (respectively,F(v) Ċ SPint F(SCIV).

Theorem (2.4) If f : x → y is lower weakly semi-precontinuous, for every fuzzy pre- semiopen set V in F(v) Ċ SPint F(SCIV).

Theorem (2.5) Iff : x → y is fuzzy upper weakly semi-precontinuous for every fuzzy pre-semiopen set V in Y , F(V) Ċ SPint F(SCIV).

Theorem(2.6) If f : x → y is fuzzy lower weakly semi-precontinuous on X ,Then SPCL f(V)Ċ F(SCLV)for any fuzzy open set V in Y.

Theorem (2.7) If f: x → y is fuzzy upper weakly semi-precontinuous on X, Then SPCL f(V)Ċ F(SCLV) for any fuzzy open set V in Y.

Theorem (2.8) Let f : x → y is fuzzy multifunction then following are equivalent :

(a) F is lower weakly semi-precontinuous.
(b) For any fuzzy open set V in Y,F(V)Ċ SP int F(SCLV) int F(SCLV) and F(V) Ċ F(SCLV).

Theorem (2.9) Let f : x → y is fuzzy lower weakly semi-precontinuous if for each fuzzy open V,F(SCLV) is fuzzy semi pre-open set.

Definition(2.2) For a fuzzy multifunction f :x → y the F^-(A) and F^+(A) of a fuzzy set
λ in Y are defined as follows:
$F^-(\lambda)$ = f (x) ε X : F(x)qλ
$F^+(\lambda)$ = f (x) ε X: F(x)λ

Definition (2.3) A fuzzy multifunction f: (x,τ) → (y,σ) is said to be :

(a) Fuzzy upper continuous at a point x of X for any fuzzy open set λ of Y such that F(x) ≤ λ there exist u ε τ containing x such thatF(u) ≤ λ for all u ε U.
(b) Fuzzy upper continuous at a point x of X for any fuzzy open set λ of Y such that F(x)qλ, there exist u ε τ containing x such that F(u)qλ for all u ε U.
(c) For upper (resp.fuzzy lower) continuous if it is fuzzy upper (resp.lower) continuous at every point of X.

Definition (2.4): A fuzzy multifunction f :x → y is called:

(a) A fuzzy lower weakly α-continuous at a point x ε X if for every fuzzy open set λ in Y with x ε X such that F^-(λ).There exist a nbd u of x_0 in X such that U Ċ F^- ((Cl(λ)
(b) A fuzzy upper weakly α-continuous at a point x ε X if for every fuzzy open set λ in Y with x ε X such that F+(λ).There exist a nbd u of x_0 in X such that U Ċ F+ ((Cl(λ))

3. Lower and Upper Weakly α-Continuous Fuzzy Multifunctions

Definition (3.1): For a fuzzy multifunction f :x → y is called:

(a) Fuzzy lower weakly α-continuous at a point x ε X if for every fuzzy open set λ in Y with x_0ε $F^-(\lambda)$.There exist a α-nbd u of x_0 in X such that UĊF^-(α(cl(λ)).
(b) Fuzzy lower weakly α-continuous at a point x ε X if for every fuzzy openset λ in Y with x_0 ε $F^+(\lambda)$. There exist a α-nbd u of x_0 in X such that U Ċ F^+(α (cl(λ)).
(c) Fuzzy lower weakly α-continuous on X if it is respectively so as each x_0 ε X.
(d) Fuzzy weakly α-continuous on X if it is fuzzy lower weakly α- continuous and fuzzy upper weakly continuous.

Theorem (3.1) Let f : x → y is fuzzy multifunction,Then following are equivalent:

(a) F is fuzzy weakly α-continuous.
(b) For any fuzzy open set in Y,$F^-(\lambda)$Ċ α int F^-(α cl(λ)).
Proof: Evident.

Theorem (3.2): Let f : x → y is fuzzy lower weakly α-continuous if for each fuzzy open set λ,F^-(cl(λ)) is fuzzy α-open set.

Proof: Straight forward.

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