A Note on Fuzzy Upper and Lower Weakly Semi-Precontinuous Multifunction

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Abstract: This paper is devoted to the introduction and study of fuzzy lower and upper weakly semi-precontinuous multifunction between a general topological space \((X,\tau)\) and a fuzzy topological space \((y,\sigma)\) using the lower inverse \(f\) and upper \(f^*\) several equivalent conditions of a lower and upper weakly Semi-precontinuous multifunctions are obtained.

Keywords: Fuzzy Continuity, Fuzzy Semicontinuous, Fuzzy semipre-continuous, Fuzzy multifunctions

1. Introduction

Through the paper by \((x,\tau)\) or simply by \(X\) we will mean a topological space in a classical sense and \((y,\sigma)\) or simply by \(Y\) will stand for a fuzzy topological space (fts, for short) as defined by Chang [2]. Fuzzy sets in to \(Y\) will be denoted \(\lambda, \mu, \vartheta\) etc. And interior and closure fuzzy sets \(\lambda\) in an fts \(Y\) will be denoted by \(\text{int} \lambda\) and \(\text{cl}(\lambda)\) and respectively a fuzzy set \(\lambda\) of fts \(Y\) is called fuzzy \(\alpha\)- open set (fuzzy \(\alpha\)- closed) if \(A \leq \text{int} \text{cl}(\lambda)\). Then \(\alpha\)- closure and \(\alpha\)-interior of \(\lambda\) are defined as follows:

\[
\text{acl}_\alpha = D \{ \mu / \mu \text{ is fuzzy } \alpha \text{ closed and } I \leq \mu \}
\]

When a fuzzy set \(1\) is quasi-coincident \((q\text{-coincident, for short})\) with a fuzzy set \(\lambda\) in \((y,\sigma)\) we shell write \(lq,m \lambda\) if \(k\) and \(\lambda\) are not quasi-coincident denoted by \(\lambda(q\mu)\). The words 'neighbourhood ' and fuzzy topological space ' will be abbreviated as 'nbd' and 'fts' respectively.

On Fuzzy Weakly Semi-precontinuous Multifunctions

Definition (1.1.1) Let \((X,\tau)\) be a topological space in the classical sens and \((y,\sigma)\) be a fuzzy topological space \(f: x \rightarrow y\) is called fuzzy multifunction for each \(x \in X, f(x)\) is a fuzzy set in \(y\).

Definition (1.1.2) For a fuzzy multifunction \(f: x \rightarrow y\) the \(f(A)\) of fuzzy set \(A\) in \(Y\) are defined as follows:

\[
f(A) = f(x) \in X : F(x)qA, \quad f(A) = f(x) \in X : F(x) \leq A
\]

Theorem (1.1.1) For a fuzzy multifunction \(f : x \rightarrow y\) we have \(f(1-A) = x - f(A)\) for any fuzzy set \(A\) in \(Y\).

Definition (1.13) For a fuzzy multifunction \(f: x \rightarrow y\) is called:

(a) Fuzzy lower and precontinuous at a point \(x_0 \epsilon X\) if for every fuzzy open set \(V\) in \(Y\) will \(x_0 \epsilon F(V)\), there exist a semi preopen nbd \(U\) of \(x_0\) in \(X\) such that \(U \epsilon F(V)\).

(b) Fuzzy upper and precontinuous at a point \(x_0 \epsilon X\) if for every fuzzy open set \(V\) in \(Y\) will \(x_0 \epsilon F^*(V)\), there exist a semi preopen nbd \(U\) of \(x_0\) in \(X\) such that \(U \epsilon F(V)\).

(c) Fuzzy lower semi-precontinuous on \(X\) iff it is respectively so that each \(x_0 \epsilon X\).

(d) Fuzzy semi-pre continuous on \(X\) iff it is fuzzy lower semi-precontinuous and fuzzy upper semi precontinuous mapping.

Definition (1.14) For a fuzzy multifunction \(f : x \rightarrow y\) is called:

(a) Fuzzy lower weakly semi-continuous at a point \(x_0 \epsilon X\) iff for every fuzzy open set \(V\) in \(Y\) will \(x_0 \epsilon f(V)\), there exist a semi open nbd \(U\) of \(x_0\) in \(X\) such that \(U \epsilon f(V)\).

(b) Fuzzy lower weakly semi-continuous at a point \(x_0 \epsilon X\) iff for every fuzzy open set \(V\) in \(Y\) will \(x_0 \epsilon f(V)\), there exist a semi open nbd \(U\) of \(x_0\) in \(X\) such that \(U \epsilon f(V)\).

(c) Fuzzy lower weakly semi-continuous on \(X\) iff it is respectively so at each \(x_0 \epsilon X\).

(d) Fuzzy weakly semi-pre-continuous on \(X\) iff it is fuzzy lower weakly semi-precontinuous and fuzzy upper weakly semi-continuous mapping.

Theorem (2.3) Let \(f : x \rightarrow y\) is fuzzy lower weakly semi-precontinuous iff every fuzzy open set \(V\) in \(F(V)_{SP}\) int \(F(SCIV)\) (respectively, \(F(V)_{SP}\) int \(F(SCIV)\)).
Theorem (2.4) If f : x → y is lower weakly semi-precontinuous, for every fuzzy pre-semiopen set V in F(V) C SP int F(SCIV).

Theorem (2.5) If f : x → y is fuzzy upper weakly semi-precontinuous for every fuzzy pre-semiopen set V in Y, F(V) C SP int F(SCIV).

Theorem (2.6) If f : x → y is fuzzy lower weakly semi-precontinuous on X, then SP CL f(V) for any fuzzy open set V in Y.

Theorem (2.7) If f : x → y is fuzzy upper weakly semi-precontinuous on X, then SP CL f(V) C F(SCLV) for any fuzzy open set V in Y.

Theorem (2.8) Let f : x → y is fuzzy multifunction then following are equivalent:

(a) F is lower weakly semi-precontinuous.
(b) For any fuzzy open set V in Y, F(SCLV) for any fuzzy open set V in Y.

Theorem (2.9) Let f : x → y is fuzzy lower weakly semi-precontinuous if for each fuzzy open set V in X, if F(SCLV) is fuzzy open set.

Definition (2.2): For a fuzzy multifunction f : x → y the fuzzy (A) and F*(A) of a fuzzy set λ in Y are defined as follows:

F(λ) = f(x) ∈ X : F(x) ≤ λ.
F+(λ) = f(x) ∈ X : F(x) > λ.

Definition (2.3) A fuzzy multifunction f : (x, τ) → (y, σ) is said to be:

(a) Fuzzy upper continuous at a point x of X for any fuzzy open set λ of Y such that F(x) ≤ τ containing x such that F(u) ≤ λ, for all u ∈ U.
(b) Fuzzy upper continuous at a point x of X for any fuzzy open set λ of Y such that F(u) > λ, containing x such that F(u) > λ, for all u ∈ U.
(c) For upper (resp. fuzzy lower) continuous if it is fuzzy upper (resp. lower) continuous at every point of X.

Definition (2.4) A fuzzy multifunction f : x → y is called:

(a) Fuzzy lower weakly α-continuous at a point x ∈ X if for every fuzzy open set λ of Y with x ∈ x such that F(λ) → (Cl(λ)). There exist a nbd u of x in X such that U C F(α Cl(λ)).
(b) Fuzzy upper weakly α-continuous at a point x ∈ X if for every fuzzy open set λ of Y with x ∈ x such that F+(λ) C (Cl(λ)).

3. Lower and Upper Weakly α-Continuous Fuzzy Multifunctions

Definition (3.1): For a fuzzy multifunction f : x → y is called:

(a) Fuzzy lower weakly α-continuous at a point x ∈ X if for every fuzzy open set λ of Y with x ∈ x such that F(λ). There exist a α-nbd u of x in X such that U C F(α Cl(λ)).

References