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Intuitionistic Fuzzy Almost Continuity and Weakly Continuity

M. Shrivastava[1], Jyoti Gupta[2]

[1]Professor, Department of Mathematics & Computer Science, Rani Durgavati University, Jabalpur, India

[2]Department of Mathematics & Computer Science, Rani Durgavati University, Jabalpur, India

Abstract: ***In the present paper, we introduce the concepts of intuitionistic fuzzy almost continuous and intuitionistic fuzzy weakly continuous mappings in the intuitionistic fuzzy topological spaces and also show their properties. We will also introduce and investigate intuitionistic fuzzy semi regular spaces and intuitionistic fuzzy regular spaces.***

Keywords: Intuitionistic fuzzy topological space, intuitionistic fuzzy continuous mapping, fuzzy regular open (closed) sets

1. Introduction

The notion of intuitionistic fuzzy sets was introduced by Atanassov [1] in 1983 as a generalization of fuzzy sets introduced by Zadeh [5]. Coker [4], generalized the concept of fuzzy topological spaces given by Chang [3] and studied intuitionistic fuzzy continuity, intuitionistic fuzzy compactness and intuitionistic fuzzy connectedness in intuitionistic fuzzy topological spaces.

Azad [2] introduced the concepts of fuzzy almost continuity and fuzzy weakly continuity in fuzzy topological spaces. In the present paper we introduce and study intuitionistic fuzzy almost continuous mappings and intuitionistic fuzzy weakly continuous mappings and also generalize the concepts of fuzzy semi regular space and fuzzy regular space in intuitionistic fuzzy topological spaces.

2. Preliminaries

Let X be a nonempty universal set. An intuitionistic fuzzy set (IF-set) A in X is an object defined as

$A = \{< x, \mu_A(x), \nu_A(x) > : \mu_A(x) + \nu_A(x) \leq 1, x \in X\}$

where the functions $\mu_A : X \to [0,1]$ and $\nu_A : X \to [0,1]$ are called membership function and non-membership function respectively. Also the values $\mu_A(x)$ and $\nu_A(x)$ are called the degree of membership and degree of non-membership respectively of an element $x \in X$ in the IF-set A.

The intuitionistic fuzzy sets $0 = \{< x, 0, 1 > : x \in X\}$ and $1 = \{< x, 1, 0 > : x \in X\}$ are respectively called the null set and the whole set of X. For the sake of simplicity we use the notation $A = \{< x, \mu_A, \nu_A >\}$ instead of $A = \{< x, \mu_A(x), \nu_A(x) > : x \in X\}$.

Let (X, τ) be an intuitionistic fuzzy topological space and $A = \{< x, \mu_A, \nu_A >\}$ be an IF-set in X. Then IF-interior and IF-closure of A are defined as

$Int(A) = \cup\{G : G \text{ is an } IF - open \text{ set in } X \text{ and } G \subseteq A\}$

$Cl(A) = \cap\{K : K \text{ is an } IF - closed \text{ set in } X \text{ and } A \subseteq K\}$

3. IF-Almost Continuity

We recall that if (X, τ) and (Y, σ) are two IF-topological spaces, then a mapping $f : (X, \tau) \to (Y, \sigma)$ is said to be an IF-continuous map if the pre-image of each IF-open set in Y is an IF-open set in X.

Further an IF-set A in an IF-topological space (X, τ) is called an IF-regular open set of X if $Int\ Cl\ (A) = A$ and IF-set A is called an IF-regular closed set in X if $Cl\ Int\ (A) = A$. Also we know that

(a) Every IF-regular open set is an IF-open set.
(b) Every IF-regular closed set is an IF-closed set.
(c) The closure of an IF-open set is an IF-regular closed set.
(d) The interior of an IF-closed set is an IF-regular open set.

Now we define IF-almost continuous maps from one IF-topological space to another.

Definition 3.1: Let (X, τ) and (Y, σ) be two IF-topological spaces. A mapping $f : X \to Y$ is said to be an IF-almost continuous map if $f^{-1}(A)$ is an IF-open set in X, for each IF-regular open set A of Y.

Example 3.1: Let $f : (X, \tau) \to (Y, \sigma)$ be a mapping defined as $f(x) = a$and $f(y) = b$, where $X = \{x, y\}$ and $Y = \{a, b\}$. Let A, B, C, D, E be IF-sets defined as

$$A = \{< x, .6, .3 >, < y, .4, .5 >\}$$
$$B = \{< x, .2, .7 >, < y, .4, .5 >\}$$
$$C = \{< a, .6, .3 >, < b, .4, .5 >\}$$
$$D = \{< a, .2, .7 >, < b, .4, .5 >\}$$
$$E = \{< a, .2, .8 >, < b, .3, .7 >\}$$

Consider $\tau = \{0, A, B, 1\}$ and $\sigma = \{0, C, D, E, 1\}$ as IF-topologies on X and Y respectively. Then we see that IF-sets $0, C, D$ and 1 are the only IF-regular open sets in Y. Also $f^{-1}(0) \equiv 0$, $f^{-1}(C) \equiv A$, $f^{-1}(D) \equiv B$ and $f^{-1}(1) \equiv 1$ are IF-open sets in X. Hence f is an IF-almost continuous map.

Theorem 3.1: Let $f : (X, \tau) \to (Y, \sigma)$ be a mapping, then following statements are equivalent

(a) f is an IF-almost continuous mapping.
(b) $f^{-1}(B)$ is an IF-closed set for each IF-regular closed set B of Y.
(c) $f^{-1}(A) \leq Int\, f^{-1}(Int\, Cl\, A)$, for each IF-open set A of Y.
(d) $Cl\, f^{-1}(Cl\, Int\, B) \leq f^{-1}(B)$, for each IF-closed set B of Y.

Proof: (i) (a)$\Leftrightarrow$(b) It can be proved easily using relation $(f^{-1}(B^c))^c = f^{-1}(B)$, for any IF-regular closed set B of Y.
(ii) (a) $\Longrightarrow$(c) Suppose f is an IF-almost continuous map and suppose A is any IF-open set in Y. Then clearly $A \leq Int\, Cl\, A$.
Hence $f^{-1}(A) \leq f^{-1}(Int\, Cl\, A)$ (3.1)
Since $Int\, Cl(A)$ is an IF-regular open set in Y, $f^{-1}(Int\, Cl\, A)$ is an IF-open set in X, so that

$$f^{-1}(Int\, Cl\, A) = Int\, f^{-1}(Int\, Cl\, A)$$

which clearly leads to the result in view of (3.1).
(iii) (c) $\Longrightarrow$(a) It can be proved easily.
(iv) (b) $\Longrightarrow$(d) Let B be an IF-closed set in Y, then clearly $Cl\, Int\, B \leq B$. Therefore

$$f^{-1}(Cl\, Int\, B) \leq f^{-1}(B) \quad (3.2)$$

Since $Cl\, Int\, B$ is an IF-regular closed set, $f^{-1}(Cl\, Int\, B)$ is an IF-closed set in X. Using (3.2), we have
$Cl\, f^{-1}(Cl\, Int\, B) = f^{-1}(Cl\, Int\, B) \leq f^{-1}(B)$
Hence $Cl\, f^{-1}(Cl\, Int\, B) \leq f^{-1}(B)$
(vi) (d) $\Longrightarrow$(b) It can be proved similarly.

Theorem 3.2: If $f:(X,\tau) \rightarrow (Y,\sigma)$ is an IF-continuous map, then it is an IF-almost continuous map.

Proof: Let $f:(X,\tau) \rightarrow (Y,\sigma)$ be an IF-continuous map. Suppose A is an IF-regular open set in Y, then by (a), A is an IF-open set in Y. Since f is an IF-continuous map, therefore $f^{-1}(A)$ is an IF-open set in X. Hence $f: X \rightarrow Y$ is an IF-almost continuous map.

Remark 3.1: The converse of Theorem 3.2 may not be true in general. This can be shown through following example.

Example 3.2: Considering Example 3.1, we see that the map f is an IF-almost continuous map and the pre-images of IF-open sets $0, 1, C$ and D are IF-open sets in X, but $f^{-1}(E) \notin \tau$, whereas $E \in \sigma$. Hence f is not an IF-continuous map.

4. IF-Weakly Continuity

Definition 4.1: A mapping $f:(X,\tau) \rightarrow (Y,\sigma)$ from an IF-topological space (X,τ) to another IF-topological space (Y,σ) is called an IF-weakly continuous map if for each IF-open set A of Y

$$f^{-1}(A) \leq Int\, f^{-1}(Cl\, A) \quad (4.1)$$

Example 4.1: Considering Example 3.1, we observe that $0, C, D, E, 1$ are IF-open sets in Y and also $f^{-1}(C) \equiv A$, $f^{-1}(D) \equiv B$, $f^{-1}(E) \equiv \{< x, .2, .8 >, < y, .3, .7 >\}$. By easy calculations, we see that

$$f^{-1}(0) \equiv 0 \leq Int\, f^{-1}(Cl\, 0) = 0$$
$$f^{-1}(1) \equiv 1 \leq Int\, f^{-1}(Cl\, 1) = 1$$
$$f^{-1}(C) \equiv A \leq Int\, f^{-1}(Cl\, C)$$
$$f^{-1}(D) \equiv B \leq Int\, f^{-1}(Cl\, D)$$
$$f^{-1}(E) \leq Int\, f^{-1}(Cl\, E)$$

Thus each of the IF-open sets of Y fulfills the required condition (4.1) for IF-weakly continuity. Hence f is an IF-weakly continuous map.
Theorem 4.1: An IF-continuous map from an IF-topological space (X,τ) to another IF-topological space (Y,σ) is an IF-weakly continuous map.
Proof: Let $f:(X,\tau) \rightarrow (Y,\sigma)$ be an IF-continuous map and let A be an IF-open set in Y, so that $f^{-1}(A)$ is an IF-open set in X. We clearly have $A \leq Cl\, A$. Hence $f^{-1}(A) \leq f^{-1}(Cl\, A)$. Since $f^{-1}(A)$ is an IF-open set in X, we have $f^{-1}(A) = Int\,(f^{-1}(A))$. Hence $f^{-1}(A) = Int(f^{-1}(A)) \leq Int\,(f^{-1}(Cl\, A))$. It implies $f^{-1}(A) \leq Int\,(f^{-1}(Cl\, A))$. Thus f is an IF-weakly continuous map.

Remark 4.1: The converse of Theorem 4.1 may not be true in general. This can be shown by the following example:

Example 4.2: In view of Example 3.1 and 4.1, we observe that f is an IF-weakly continuous map, but it is not an IF-continuous map because $f^{-1}(E) \notin \tau$, whereas $E \in \sigma$.

Theorem 4.2: An IF-almost continuous map is also an IF-weakly continuous map.

Proof: Let $f:(X,\tau) \rightarrow (Y,\sigma)$ be an IF-almost continuous map and A be an IF-open set in Y, then from Theorem 3.1(c), we have
$f^{-1}(A) \leq Int\, f^{-1}(Int\, Cl\, A)$ (4.2)

Further we know that $Int\, Cl\, A \leq Cl\, A$. Therefore $f^{-1}(Int\, Cl\, A) \leq f^{-1}(Cl\, A)$. Hence
$Int\, f^{-1}(Int\, Cl\, A) \leq Int\, f^{-1}(Cl\, A)$ (4.3)
In view of (4.2) and (4.3), we get that
$f^{-1}(A) \leq Int\, f^{-1}(Cl\, A)$
which shows that f is an IF-weakly continuous map.

Remark 4.2: From Theorem 3.2, Theorem 4.1 and Theorem 4.2, it is clear that the relationship of IF-continuous, IF-almost continuous and IF-weakly continuous mappings may be presented by figure 4.1

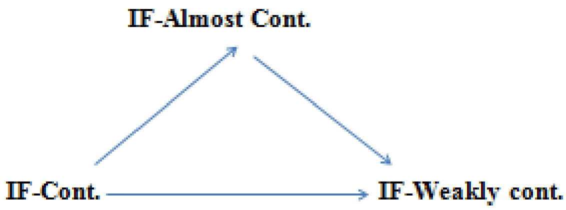

Figure 4.1

5. IF-Semi Regular Space and IF-Regular Space

Definition 5.1 IF-Semi Regular Space: An IF-topological space (X,τ) is said to be an IF-semi regular space iff the collection of all IF-regular open sets of X forms a base for IF-topology τ.

Example 5.1: Let $X = \{a,b\}$ and A,B,C,D be IF-sets on X defined as follows :
A = $\{< a,.4,.5 >, < b,.6,.2 >\}$
B = $\{< a,.5,.4 >, < b,.2,.6 >\}$
C = $\{< a,.5,.4 >, < b,.6, .2 >\}$
D = $\{< a,.4,.5 >, < b,.2,.6 >\}$

Let $\tau = \{0,A,B,C,D,1\}$ be an IF-topology on X. We see that IF-open sets $0,A,B,C,D,1$ are the IF-regular open sets. Further we observe that
$0 = 0 \cup 0$, A = A ∪ 0,
B = 0 ∪ B, C =$0 \cup A \cup B \cup C$,
D =$0 \cup D$, 1 = $0 \cup A \cup B \cup C \cup D \cup 1$

Hence each of these IF-open sets of τ is a union of (some of) these six IF-regular open sets. Thus the collection $\{0,A,B,C,D,1\}$ forms a base for IF-topology τ. Therefore IF-topological space (X,τ) is an IF-semi regular space.

Definition 5.2 IF-Regular Space: An IF-topological space (X,τ) is called an IF-regular space iff each IF-open set A of X is a union of a collection $\{A_\alpha\}_{\alpha\in J}$ of IF-open sets of X such that
$Cl\ A_\alpha \ \leq A$ for each $\alpha \in J$ (5.1)
where J is an arbitrary index set.

Example 5.2: Let $X = \{a,b\}$ and $A,\ B,\ C,\ D$ be IF-sets on X defined as
$A = \{< a,.6,.3 >, < b,.4,.5 >\}$
$B = \{< a,.3,.6 >, < b,.5,.4 >\}$
$C = \{< a,.6,.3 >, < b,.5,.4 >\}$
$D = \{< a,.3,.6 >, < b,.4,.5 >\}$

Let $\tau = \{0,A,B,C,D,1\}$ be an IF-topology on X. We see that
$Cl\ A = B^c$, $Cl\ B = A^c$,
$Cl\ C = D^c$, $Cl\ D = C^c$

Hence we observe that
$0 = 0 \cup 0$ and $Cl(0) \leq 0$
$A = A \cup 0$, and $Cl(0),\ Cl(A) \leq A$
$B = 0 \cup B$, and $Cl(0),\ Cl(B) \leq B$
$C = 0 \cup A \cup B \cup C$, and
$Cl(0),\quad Cl(A),\quad Cl(B),\quad Cl(C) \leq C$
$D = 0 \cup D$, and $Cl(0),\ Cl(D) \leq D$
$1 = 0 \cup A \cup B \cup C \cup D \cup 1$ and
$Cl(0),\ Cl(A),\ Cl(B),\ Cl(C),\ Cl(D),\ Cl(1) \leq 1$

Therefore each of the IF-open sets in X is the union of these six sets and satisfies the required condition (5.1), for IF-regularity of the topological space. Hence IF-topological space (X,τ) is an IF-regular space.

Theorem 5.1: An IF-regular space is also an IF-semi regular space.

Proof: Let(X,τ) be an IF-regular space and A be any IF-open set in X. Suppose A is the union of A_α, $\alpha \in \Lambda$, where each A_α is an IF-open set in X such that $Cl\ A_\alpha \leq A$.
Thus $A = \cup_\alpha A_\alpha$, $\quad \alpha \in \Lambda$ (5.2)
Now for each $\alpha \in \Lambda$, we have $A_\alpha \leq Cl\ A_\alpha \leq A$.
Hence $Int\ A_\alpha \equiv A_\alpha \leq Int\ (Cl\ A_\alpha) \leq Int\ A \equiv A$
This implies that $A_\alpha \leq Int(Cl\ A_\alpha) \leq A$.
Hence $\cup_{\alpha\in\Lambda} A_\alpha \leq \cup_{\alpha\in\Lambda} Int(Cl\ A_\alpha) \leq A$ (5.3)
Therefore in view of (5.2) & (5.3), we have
$A = \cup_{\alpha\in\Lambda} A_\alpha = \cup_{\alpha\in\Lambda} Int(Cl\ A_\alpha) \leq A$
Thus $A = \cup_{\alpha\in\Lambda} Int(Cl\ A_\alpha)$. Here each of the set $Int(Cl\ A_\alpha)$, $\alpha \in \Lambda$ is an IF-regular open set. Thus the collection $\{A_\alpha\}_{\alpha\in\Lambda}$, where $A_\alpha = Int\ (Cl\ A_\alpha)$ forms a base for X. Hence X is an IF-semi regular space.

Theorem 5.2: Let $f : (X,\tau) \to (Y,\sigma)$ be a mapping from an IF-topological space X to an IF-semi regular topological space Y. Then f is an IF-almost continuous map iff f is an IF-continuous map.

Proof: Let $f : (X,\tau) \to (Y,\sigma)$ be a mapping from an IF-topological space X to an IF-semi regular space Y. Let f be an IF-continuous map, then by Theorem 3.2, we know that f is also an IF-almost continuous map. Now to prove the theorem, it is sufficient to show that if (Y,σ) is an IF-semi regular space and f is an IF-almost continuous map, then f is an IF-continuous map. Let A be an IF-open set in Y, we show that $f^{-1}(A)$ is an IF-open set in X. Since A is a union of a collection of IF-regular open sets $\{A_\alpha\}_{\alpha\in\Lambda}$ in Y, where Λ is an arbitrary index set. Thus
$A = \cup A_\alpha, \alpha \in \Lambda$ (5.4)
Now if f is an IF-almost continuous mapping, then pre-image of each IF-regular open set A_α is an IF-open set in X.
Therefore,
$f^{-1}(A) = f^{-1}(\cup A_\alpha) = \cup f^{-1}(A_\alpha)$
$= \cup\ Int\ f^{-1}(A_\alpha)$
$\leq Int\ f^{-1}(\cup A_\alpha)$
$= Int\ f^{-1}(A)$
Hence $f^{-1}(A) = Int\ f^{-1}(A)$
which shows that $f^{-1}(A)$ is an IF-open set in X. Hence f is an IF-continuous map.

Theorem 5.3: Let $f : (X,\tau) \to (Y,\sigma)$ be a mapping from an IF-topological space X to an IF-regular topological space Y. Then f is an IF-weakly continuous map iff f is an IF-continuous map.

roof: Let $f:(X,\tau)\to(Y,\sigma)$ be a mapping from an IF-topological space X to an IF-regular space Y. In view of Theorem 4.1, we know that if f is an IF-continuous map, then it is an IF-weakly continuous map. Therefore to prove the theorem, it is sufficient to show that if f is an IF-weakly continuous, then it is an IF-continuous map, where Y is an IF-regular space. Let A be any IF-open set in Y. Since Y is an IF-regular space, then

$A=\cup A_\alpha,\ A_\alpha\in\sigma$ and $Cl\,A_\alpha\le A$, for each $\alpha\in\wedge$ (5.5)

Now if f is an IF-weakly continuous mapping and $A_\alpha\in\sigma$, we have $f^{-1}(A_\alpha)\le\cup\,Int f^{-1}(Cl\,A_\alpha),\quad \alpha\in\wedge$ (5.6)

$f^{-1}(A)=f^{-1}(\cup A_\alpha)=\cup f^{-1}(A_\alpha)$

$\le\cup\,Int f^{-1}(Cl\,A_\alpha)$, by (5.6)

$\le\cup\,Int\ f^{-1}(A)$ by (5.5)

$=Int\ f^{-1}(A)$

Hence $f^{-1}(A)\ =Int\ f^{-1}(A)$

which shows that $f^{-1}(A)$ is an IF open set in X. Therefore f is an IF continuous map.

6. Conclusion

In this paper, we have introduced the IF-almost continuity and IF-weakly continuity in IF-topological spaces. We have also studied the IF-semi regular space and IF-regular space and significant results are obtained.

These results may proof to be the pathway for the study of IF-almost continuous and IF-weakly continuous maps in any IF-topological space or in an IF-semi regular or IF-regular topological space.

References

[1] Atanassov K., Intuitionistic fuzzy sets, ITKR's Session Sofia, VII (1983), (in Bulgaria).
[2] Azad K.K., On fuzzy semi continuity, fuzzy almost continuity and fuzzy weakly continuity, J. Math. Anal. Appl. **82** (1981), 14-32.
[3] Chang C.L., Fuzzy topological spaces, J. Math. Anal. Appl. **24** (1968), 182-190.
[4] Coker D., An introduction to intuitionistic fuzzy topological space, Fuzzy Sets and System, **88** (1997), 79-84.
[5] Zadeh L.A., Fuzzy sets, Information and Control, **8** (1965), 338-353.

Author Profile

Prof. Mrs. M. Shrivastava, holds M.Sc., Ph.D., D.Sc. degrees. She has been awarded National scholarships, UGC Teacher Fellowship and National Radha Krishnan Award for her research. She is a member of many National Academic Societies. She has a number of research papers on her credit. She has worked on many minor and major projects of UGC, DST, MPCST etc. She has been Program Coordinator of prestigeous SAP Program of UGC awarded to the Department and has organized many national and international conferences/seminars in the Department. Presently, she is working as Professor in Department of Mathematics and Computer Science, Rani Durgavati University, Jabalpur (M.P.), India.

Ms. Jyoti Gupta, has achieved M.Sc. and M.Phil. degrees from Rani Durgavati University, Jabalpur (M.P.). Presently, she is pursuing research for Ph.D. in field of Fuzzy Mathematics Theory.