# On Fuzzy Weakly $\alpha$ -continuous Multifunction

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**Abstract:** This paper is devoted to the introduction and study of fuzzy lower weakly  $\alpha$ -continuous multifunction between a general topological space  $(X,\tau)$  and a fuzzy topological space  $(y,\sigma)$  using the lower inverse f and upper  $f^{\dagger}$  severl equivalent conditions of a lower and upper weakly  $\alpha$ -continuous multifunctions are obtained.

Keywords: Topological space, Fuzzy topological space, fuzzy set, continuity

#### 1. Introduction

Through the paper by(  $x,\tau$ ) or simply by X we will mean a topological space in a classical sense and  $(y,\sigma)$  or simply by Y will stand for a fuzzy topological space (fts, for short) as defined by chang [2], Fuzzy sets in to Y will be denoted  $\lambda$ ,  $\mu$ ,  $\vartheta$  & etc. And interior and closure fuzzy sets  $\lambda$  in an fts Y will be denoted by int  $\lambda$  and cl( $\lambda$ ) and respectively a fuzzy set  $\lambda$  of fts Y is called fuzzy  $\alpha$ - open set (fuzzy  $\alpha$ - closed) if  $A \leq int$  cl( $\lambda$ )). Then  $\alpha$ - closure and  $\alpha$ -interior of  $\lambda$  are defined as follows:

 $\alpha cl_= D \{ \mu / \mu \text{ is fuzzy } \alpha \text{ closed and } 1 \leq \mu \}$ 

When a fuzzy set 1 is quasi-coincident (q-coincident,for short) with a fuzzy set  $\lambda$  in  $(y,\sigma)$  we shell write lq,m if  $\lambda$  and  $\mu$  are not quasi-coincident denoted by  $(\lambda q \mu)$ . The words 'neighbourhood ' and fuzzy topological space 'will be abbreviated as 'nbd' and 'fts' respectively.

## 2. On Fuzzy Weakly α-continuous Multifunctions

**Definition (1.1.1)** Let  $(X,\tau)$  be a topological space in the classical sens and  $(y,\sigma)$  be a fuzzy topological space f:x!y is called fuzzy multifunction for each x 2 X., f(x) is a fuzzy set in y.

**Definition (1.1.2)** For a fuzzy multifunction  $f : x \to y$  the f(A) of fuzzy set A in Y are defined as follows:  $f(A) = f(x) \varepsilon X : F(x)qA$ ,  $f(A) = f(x) \varepsilon X : F(x) \le A$ 

**Theorem (1.1.1)** For a fuzzy multifunction  $f : x \rightarrow y$  we have f(1-A) = x - f(A) for any fuzzy set A in Y.

**Definition (1.13)** For a fuzzy multifunction  $f: x \rightarrow y$  is called:

- (a) Fuzzy lower and precontinuous at a point  $x_0 \in X$  if for every fuzzy open set V in Y will  $x_0 \in F^-(V)$ , there exist a semi preopen nbd U of  $x_0$  in X such that U C F(v).
- (b) Fuzzy upper and precontinuous at a point  $x_0 \epsilon X$  if for every fuzzy open set V in Y will  $x_0 \epsilon F^+(V)$ , there exist a semi preopen nbd U of  $x_0$  in X such that U F(v).
- (c) Fuzzy lower semii- precontinuous on X iff it is respectively so that each  $x_0 \in X$ .
- (d) Fuzzy semi-pre continuous on X,iff it is fuzzy lower semi-precontinuous and fuzzy upper semi precontinuous mapping.

**Definition (1.14)** For a fuzzy multifunction  $f: x \rightarrow y$  is called :

- (a) Fuzzy lower weakly semi-continuous at a point  $x_0 \in X$ . Iff for every fuzzy open set V in Y will  $x_0 \in f(v)$ , there exists a semi open nbd U of  $x_0$  in X such that U  $\epsilon f_0(SCIV)$ .
- (b) Fuzzy lower weakly semi-continuous at a point x<sub>0</sub> ε X iff for every fuzzy open set V in Y will x<sub>0</sub> ε f(v), there exists a semi open nbd U of x<sub>0</sub> in X such that U ε f0(SCIV).
- (c) Fuzzy lower weakly semi-continuous on X iff it is respectively so each  $x_0 \in X$ .
- (d) Fuzzy weakly semi- continuous on X iff it is fuzzy lower weakly semi-continuous and fuzzy upper weakly semi-continuous mapping.

#### **3.** Lower and Upper Weakly Semi-Precontinuous Fuzzy Multifunctions

**Definition (2.1)** For a fuzzy multifunction  $f : x \rightarrow y$  is called :

- (a) Fuzzy lower weakly semi-precontinuous at a point x0  $\varepsilon$  X iff for every fuzzy open set V in Y will  $x_0 \varepsilon$ . F(v). There exist a semi preopen nbd of U of  $x_0$  in X such that UĊF(SCLV).
- (b) Fuzzy lower weakly semi-precontinuous at a point  $x_0 \in X$  iff for every fuzzy open set V in Y will  $x_0 \in F(v)$ . There exist a semi preopen nbd of U of  $x_0$  in X such that UCF(SCLV).
- (c) Fuzzy lower weakly semi-precontinuous on X iff it is respectively so at each  $x_0 \in X$ .
- (d) Fuzzy weakly semi-precontinuous on X iff it is fuzzy lower weakly semi-precontinuous and fuzzy upper weakly semi-precontinuous mapping.

**Theorem (2.3)** Let  $f :x \rightarrow y$  is fuzzy lower weakly semiprecontinuous iff every fuzzy open set V in F(V)\_SP int F(SCIV) (respectively,F(v) Ċ SPint F(SCIV).

**Theorem (2.4 )** If  $f : x \rightarrow y$  is lower weakly semiprecontinuous, for every fuzzy pre- semiopen set V in F(v) C SPint F(SCIV). **Theorem (2.5)** Iff :  $x \rightarrow y$  is fuzzy upper weakly semiprecontinuous for every fuzzy pre- semiopen set V in Y , F(V) Ċ SPint F(SCIV).

**Theorem(2.6)** If  $f: x \to y$  is fuzzy lower weakly semiprecontinuous on X ,Then SPCL  $f(V)\dot{C} F(SCLV)$  for any fuzzy open set V in Y.

**Theorem (2.7)** If f:  $x \rightarrow y$  is fuzzy upper weakly semiprecontinuous on X,Then SPCL f(V)Ċ F(SCLV) for any fuzzy open set V in Y.

**Theorem (2.8)** Let  $f : x \to y$  is fuzzy multifunction then following are equivalent :

(a) F is lower weakly semi-precontinuous.

(b) For any fuzzy open set V in Y,F(V ) $\dot{C}$  SP int F(SCLV) int F(SCLV) and F(V )  $\dot{C}$  F(SCLV).

**Theorem (2.9)** Let  $f : x \to y$  is fuzzy lower weakly semiprecontinuous if for each fuzzy open V,F(SCLV) is fuzzy semi pre-open set.

**Definition (2.2)** For a fuzzy multifunction  $f:x\to y$  the  $F^{\text{-}}(A)$  and  $F^{\text{+}}(A)$  of a fuzzy set

 $\lambda$  in Y are defined as follows:

 $F(\lambda) = f(x) \varepsilon X : F(x)q\lambda$ 

 $F^{+}(\lambda) = f(x) \varepsilon X: F(x)\lambda$ 

**Definition (2.3)** A fuzzy multifunction f: (  $x,\tau$  )  $\rightarrow$  (  $y,\sigma$  ) is said to be :

- (a) Fuzzy upper continuous at a point x of X for any fuzzy open set  $\lambda$  of Y such that  $F(x) \leq \lambda$  there exist u  $\varepsilon \tau$  containing x such that  $F(u) \leq \lambda$  for all u  $\varepsilon$  U.
- (b) Fuzzy upper continuous at a point x of X for any fuzzy open set  $\lambda$  of Y such that  $F(x)q\lambda$ , there exist u  $\epsilon \tau$  containing x such that  $F(u)q\lambda$  for all u  $\epsilon$  U.
- (c) (c)For upper (resp.fuzzy lower) continuous if it is fuzzy upper (resp.lower) continuous at every point of X.

**Definition (2.4):** A fuzzy multifunction  $f: x \rightarrow y$  is called:

- (a) A fuzzy lower weakly α-continuous at a point x ε X if for every fuzzy open set λ in Y with x ε X such that F (λ).There exist a nbd u of x<sub>0</sub> in X such that U Ċ F ((Cl(λ)
- (b) A fuzzy upper weakly α-continuous at a point x ε X if for every fuzzy open set λ in Y with x ε X such that F+(λ).There exist a nbd u of x<sub>0</sub> in X such that U Ċ F+ ((Cl(λ))

### 4. Lower And Upper Weakly α-Continuous Fuzzy Multifunctions :

**Definition (3.1):** For a fuzzy multifunction  $f : x \rightarrow y$  is called:

- (a) Fuzzy lower weakly α-continuous at a point x ε X if for every fuzzy open set λ in Y with x <sub>0</sub>ε F<sup>-</sup>(λ). There exist a α-nbd u of x<sub>0</sub> in X such that UĊF<sup>-</sup>(α(cl(λ)).
- (b) Fuzzy lower weakly  $\alpha$ -continuous at a point x  $\epsilon$  X if for every fuzzy openset  $\lambda$  in Y with  $x_0 \epsilon F^+(\lambda)$ . There exist a  $\alpha$ -nbd u of  $x_0$  in X such that U Ċ F<sup>+</sup>( $\alpha$  (cl( $\lambda$ )).
- (c) Fuzzy lower weakly  $\alpha$ -continuous on X if it is respectively so as each  $x_0 \in X$ .

(d) Fuzzy weakly α-continuous on X if it is fuzzy lower weakly α- continuous and fuzzy upper weakly continuous.

**Theorem (3.1)** Let  $f: x \rightarrow y$  is fuzzy multifunction, Then following are equivalent:

(a) F is fuzzy weakly  $\alpha$ -continuous.

(b) For any fuzzy open set in Y,  $F(\lambda)\dot{C} \alpha$  int  $F(\alpha cl(\lambda))$ .

Proof: Evident.

**Theorem (3.2):** Let  $f : x \to y$  is fuzzy lower weakly  $\alpha$ continuous if for each fuzzy open set  $\lambda$ ,  $F'(cl(\lambda))$  is fuzzy  $\alpha$ open set.

**Proof:** Straight forward.

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