Synchronization of Quadratically Damped Mathieu – Duffing Based Chaotic Circuits with Applications to Secure Communications

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Abstract: This paper presents an effective secure chaotic synchronized communication using quadratically damped Mathieu – Duffing system. The circuit simulation results are used to visualize and illustrate the effectiveness of quadratically damped Mathieu – Duffing chaotic system in synchronization and application of secure communication. The synchronization between two chaotic oscillators, serving as a transmitter-receiver scheme using one way coupling is studied.

Keywords: Chaotic circuits, Mathieu - Duffing oscillator, Coupling, Synchronization, Secure communication

1. Introduction

In recent years chaos theory has attracted the attention of many researchers, scientists and engineers in both the academic area and industry. Most of the electronic/ electrical devices are nonlinear in nature. Computer is one of the most important nonlinear devices being used in daily life. The computer networks are very insecure medium for transmission of data. The demand in the various applications of internet has rapidly increased its traffic and hence requires various measures for secure communications. Because of this the private and secure communication is very much significant for the transmission of large amounts of information. The public key cryptosystems using software techniques are the popular solutions for secure communications. The use of hardware complexity to hide or mask the message on chaotic carrier is also an alternative method of increasing security in communications [1][2][3].

Due to strong dependence on initial conditions, two identical chaotic circuits show different waveforms even if they start from almost identical initial conditions. The signals generated by chaotic systems have statistical properties similar to randomness in spite of being deterministic. Chaotic systems have been used for secure communication and different encryption systems based on chaos have been proposed in order to hide the information signal: additive masking [4], chaos shift keying [5], two channels transmission [6], parametric modulation [7] and inclusion encryption [8]. In chaos-based communication system, the procedure adopted by the transmitter to generate the chaotic signal is deterministic and signal that allows an authorized receiver to replicate, or synchronize, the chaotic signal and then to recover the message by subtracting the chaotic carrier [1][2][3]. The use of synchronized chaotic systems for communication usually relies on the robustness of the synchronization within the transmitter and receiver pair [1][2][3][9]. If the communication channel is imperfect and /or there is internal noise at the electronic circuitry the distorted signal at the receiver input might cause synchronization mismatch between the transmitter-receiver

pair [10][11]. The chaotic shift keying in existing frequencyhop spread spectrum systems is used for secure communication [12].

The work presented in this paper proposes a synchronization of quadratically damped Mathieu - Duffing oscillator circuit for secure communication. The effectiveness of the proposed work is illustrated through PSpice circuit simulation. In the next section we discuss the electronic circuit designed and its simulation results for Mathieu – Duffing oscillator circuit. In Section 3, we discuss the electronic circuit designed and its simulation results for synchronization of the Mathieu - Duffing Oscillator Circuit. In Section 4 we discuss the application for Secure Communication Systems and Section 5 concludes the paper.

2. Mathieu – Duffing Oscillator Circuit

Here we consider the Mathieu-Duffing oscillator described by Singh et al. [13] as the following equation of motion:

$$\ddot{x} + \alpha \dot{x} \dot{x}^{p-1} - \omega_o^2 (\delta + \varepsilon \cos \omega t) x + \beta x^3 = 0$$
(1)

where, $\dot{x} = \frac{dx}{dt}$ represents the derivative with respect to time t, α is the damping coefficient, ε and ω are respectively, the amplitude and frequency of the parametric excitation, δ and β are, respectively, the Mathieu parameter and a stiffness constant. We have considered a nonlinear damping term i.e. proportional to the pth power of velocity. Here modulus of velocity is taken to consider the sign of velocity. We have used $\omega_{\varepsilon}^2 = 1$, $\beta = 1$, $\omega = 1$, $\delta = 1$, $\alpha = 0.1$, p=2 and $\varepsilon = 0.42$.

2.1 Description of the Circuit

We have constructed an analog circuit equivalent to Eq. (1) using conventional operational amplifiers and four quadrant multipliers, which is shown in Figure 1.



Figure 1: Schematic of quadratically damped Mathieu – Duffing oscillator circuit

The circuit is constructed using conventional operational amplifiers and five quadrant multipliers. Here A1 and A2 represent two integrators; A3 and A4 are summing amplifiers, A5 is inverter; A6 is an absolute rectifier; and M1, M2, M3 and M4 are four quadrant multipliers. The operational amplifiers used in A1, A2, A3, A4, A5 and A6 are op-amp μ A741C and the four multipliers M1, M2, M3 and M4 are Analog Devices multiplier AD633 with the inbuilt gain of 0.1. Analog Devices multiplier AD633 is the low cost chip with 2% error at Full Scale.

At junction A i.e. at the output of summing amplifier, we obtain the following equation

$$\ddot{x} = -\alpha \dot{x} |\dot{x}| + \omega_o^2 (\delta + \varepsilon \cos \omega t) x - \beta z$$

Where, $|\omega_0|^2| = \frac{(0.1)R9}{R10}$, $\alpha = \frac{R9}{R7}$, $and\beta = \frac{(0.01)R9}{R9}$ The values of various resistors and capacitors used in the circuit are as given below: - $R1 = 100k\Omega$, $R2 = 100k\Omega$, $R3 = 1k\Omega$, $R4 = 1k\Omega$, $R5 = 1k\Omega$, $R6 = 10k\Omega$, $R3 = 100\Omega$, $R9 = 10k\Omega$, $R10 = 1k\Omega$, $R11 = 1k\Omega$, $R12 = 1k\Omega$, $R13 = 1k\Omega$, and $C1 = C2 = 0.01\mu$ F

Here $\omega_0^2 = 1$, $\beta \approx 1$, $\delta = 1$ and $\omega = 1$. $\omega = 1$ is obtained at external frequency $f \approx 160$ Hz since here $\omega = 2\pi * f * (R1 * C1)$ or $2\pi * f * (R2 * C2)$.

2.2 Results and Discussion

By using $\omega_o^2 = 1$, $\beta = 1$, $\omega = 1$ and $\delta = 1$ with various combinations of α and ε we can obtain chaotic signals of different nature. Here for our study we are using $\alpha = 0.1$ and $\varepsilon = 0.42V$ to generate the chaotic signal. The chaotic signal obtained at \vec{x} vs time and the phase plot \vec{x} vs x of PSpice simulation result obtained from the circuit is shown in Figure 2(a) and Figure 2(b) respectively.





Figure 2: Signal **x** and phase behavior (**x** vs **x**) of coupled quadratically damped Mathieu – Duffing oscillators (i.e., behaviour of Eq. (1)).

3. Synchronization of the Mathieu - Duffing Oscillator Circuit

Synchronization between chaotic systems has received considerable attention due to the considerable progress in communication applications. The communication scheme consists of two parts: a drive system, which is called the transmitter and a response system, which is called receiver. To achieve synchronization, the output of transmitter is sent to receiver. The two basic coupling methods for synchronization are one-way coupling and both-way coupling. In this paper we have used one-way coupling method for synchronization of Mathieu - Duffing system.

3.1 Description of the Circuit

In Figure 3, we have used the same chaotic circuit of Mathieu – Duffing oscillator shown in Figure 1 with same values of all the parameters as mentioned in Section 2.1 and Section 2.2 for both transmitter and a response system.



Figure 3: Synchronization of quadratically damped Mathieu – Duffing oscillator circuit.

Here we have used a buffer circuit (amplifier A7) made up of an Op-amp μ A741C for a one way coupling between transmitter and a response system.

3.2 Results and Discussion

3.2.1Before synchronization i.e. without coupling

The chaotic signal obtained at \vec{x} vs time and the phase plot \vec{x} vs \vec{x} of PSpice simulation result obtained from the Transmitter system is same as shown in Figure 2(a) and Figure 2(b) respectively. Whereas at Response system, obtained signal is chaotic but of different nature as compared to the chaotic signal generated at Transmitter system as shown in Figure 4(a) and Figure 4(b) respectively for \vec{x} vs

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time and the phase plot \vec{x} vs \vec{x} . The drive and response system chaotic signals obtained at \vec{x} vs time of transmitter system and receiver system is shown in Figure 5. The unsynchronized phase plot between \vec{x} signal of transmitter and \vec{x} signal of receiver is shown in Figure 6. From Figure 5 and Figure 6, we can observe that the obtained chaotic signals at both the systems i.e drive and response are unsynchronized due to the fact that chaotic circuits are strongly dependent on initial conditions.



Figure 4: Chaotic signal x vs time and phase plot x vs x at the response system before synchronization obtained from the circuit shown in Figure 3.



Figure 5: Drive and response system chaotic signals before synchronization obtained from the circuit shown in Figure 3.



Figure 6: Phase plot (\vec{x} at drive system vs \vec{x} at response system) of unsynchronized case obtained from the circuit shown in Figure 3.

3.2.2After synchronization i.e with coupling

The chaotic signal obtained at \vec{x} vs time and the phase plot \vec{x} vs \vec{x} obtained from the Transmitter system is shown in Figure 7(a) and Figure 7(b) respectively. For the Response system, obtained chaotic signal at \vec{x} vs time and the phase plot \vec{x} vs \vec{x} is shown in Figure 8(a) and Figure 8(b) respectively. The drive and response system chaotic signals obtained at \vec{x} vs time of transmitter system and receiver system is shown in

Figure 9. The synchronized phase plot between \vec{x} signal of transmitter and \vec{x} signal of receiver is shown in Figure 10. From Figure 9 and Figure 10, we can observe that due to perfect coupling between two identical chaotic circuits through one-way coupling method, both the system i.e Transmitter and Response system are well synchronized with each other and obtained the same signal.



Figure 7: Chaotic signal **x** vs time and phase plot **x** vs **x** at the drive system after synchronization obtained from the circuit shown in Figure 3.



Figure 8: Chaotic signal **x** vs time and phase plot **x** vs **x** at the response system after synchronization obtained from the circuit shown in Figure 3.



Figure 9: Drive and response system chaotic signals after synchronization obtained from the circuit shown in Figure 3.

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Figure 10: Phase plot (\vec{x} at drive system vs \vec{x} at response system) of synchronized case obtained from the circuit shown in Figure 3.

4. Application for Secure Communication Systems

To design of secure communication system, it is necessary to make sure the parameters of transmitter and receiver are identical for implementing the chaotic masking communication [14][15][16][17][18][19]. In this masking signal, a low level message signal is added to the synchronizing driving chaotic signal to generate a perfect driving signal at receiver.

The information signal is added to the generated chaotic signal and the resultant signal is transmitter to the receiver end. The chaotic signal is subtracted from the received signal at receiver end to retrieve the transmitted signal. Figure 11 shows the circuit schematic for implementing the Mathieu - Duffing Chaotic Masking Communication.



Figure 11: Schematic of quadratically damped Mathieu – Duffing chaotic masking communication circuit

Here we have designed the chaotic masking communication system by using adder circuit (adder amplifier A9) that masks the information signal by mixing it with the chaotic signal generated by the transmitter end and the subtractor circuit (subtractor amplifier A11) that receives the transmitted signal after subtracting the synchronized chaotic signal, in addition to the synchronization network as shown in Figure 3. All the resistors used in additional circuit i.e R14, R15, R16, R17, R18 and R19 are of $1K\Omega$.

Figure 12 shows PSpice simulation results of this Chaotic Masking Circuit. The information signal (square wave) Inf_Signal, chaotic masking transmitted signal Mix_Signal and the retrieved signal Ret_Signal are shown in Figure 12(a), Figure 12(b) and Figure 12(c) respectively.



Figure 12: Simulation results of masking communication circuit: (a) Information signal (square wave) Inf_Signal; (b) chaotic masking transmitted signal Mix_Signal; and (c) retrieved signal Ret_Signal

From Figure 12(a) and Figure 12(b) we can observe that the transmitted signal is well retrieved at the receiver end. Figure 13 shows an excellent agreement between the transmitted signal and the retrieved signal, without any error.



Figure 13: Overlapping of information signal (Square wave of 0.5Vpp) Inf_Signal and retrieved signal Ret_Signal obtained from the circuit shown in Figure 11.

The effectiveness of designed circuit is also illustrated by transmitting another type of information signal (sinusoidal signal) as shown in Figure 14(a), Figure 14(b) and Figure 14(c) respectively for the information signal Inf_Signal, chaotic masking transmitted signal Mix_Signal and the retrieved signal Ret_Signal.





Figure 14: Simulation results of masking communication circuit: (a) Information signal (sinusoidal wave) Inf_Signal; (b) chaotic masking transmitted signal Mix_Signal; and (c) retrieved signal Ret_Signal.

From Figure 14(a) and Figure 14(c) we can observe that the transmitted signal is well retrieved at the receiver end. Figure 15 shows an excellent agreement between the transmitted signal and the retrieved signal, without any error.



Figure 15: Overlapping of information signal (Sinusoidal wave of 1 Vpp) Inf_Signal and retrieved signal Ret_Signal obtained from the circuit shown in Figure 11.

5. Conclusion

In this paper we have demonstrated in simulations that two identical chaotic systems can be synchronized and used for the secure communication. The unknown information is encrypted in the transmitter and is restructured in the receiver end. The effectiveness of the system is illustrated by transmitting two different types of signal. The retrieved signal at the receiver end is obtained without any error that makes the strong recommendation of proposed system for the secure communication.

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References

- Carroll, T. L., Pecora, L. M. [1993] "Synchronizing nonautonomous chaotic circuits", IEEE Transactions on Circuits and Systems II,40 (10), 646-650.
- [2] Pecora, L. M., Carroll, T. L. [1990] "Synchronization in chaotic systems", Physical Review Letter, 64 (8), 821-823.
- [3] Murali, M., Lakshmanan, M. [1994] "Drive-response scenario of chaos synchronization in identical nonlinear systems", Physical Review, E49 (5), 4882-4885.
- [4] Cuomo, K. M., Oppenheim, A. V., Strogatz, S. H. [1993] "Synchronization of Lorenz-based chaotic circuits with applications to communications", IEEE Transactions on Circuits and Systems II, 40, 626-633.
- [5] L'Hernault, M., Barbot, J. P., Ouslimani, A. [2008] "Feasibility of analog realization of a sliding mode observer: application to data transmission", IEEE Trans. Circuits Syst. I, 55, 614-624.
- [6] Jiang, Z. P. [2002] "A note on chaotic secure communication system" IEEE Trans. Circuits Syst. I: Fundamental Theory and Applications, 49, 92-96.
- [7] Kemih, K., Halimi, M., Ghanes, M. [2011] "An Application of Chaotic Chua's System for Secure Chaotic Communication Based on Sliding Mode observer", AIP Conf. Proc., 1400, 344-347.
- [8] Lian, K. Y., Liu, P. [2000] "Synchronization with message embedded for generalized lorenz chaotic circuits and its error analysis", IEEE Trans. Circuits Syst. I: Fundamental Theory and Applications, 47 (9), 1418-1424.
- [9] Kolumban, G., Kennedy, M. P., Chua, L. O. [1997] "The role of synchronization in digital communications using chaos- Part I: Fundamentals of digital communications", IEEE Transactions on Circuits and Systems I, 44 (10), 927-936.
- [10] Yang, X., Wu, T. X., Jaggard, D. L. [2002] "Synchronization recovery of chaotic wave through an imperfect channel", IEEE Antennas & Wireless Propagation Letter 1, 154-156.
- [11] Carroll, T. L. [2001] "Noise-resistant chaotic synchronization", Physical Review, E64 (015201), 1-4.
- [12] Raja Kumar, R., Sampath, A., Indumathi, P. [2011] "Improving security of communication systems using chaos", Indian Journal of Science and Technology, 4(7), 773-778.
- [13] Singh, U. R., Purohit, G., Patidar, V. [2016] "An Analogue Circuit to Study the Forced and Quadratically Damped Mathieu-Duffing Oscillator", Progress in Nonlinear Dynamics and Chaos, 4(1), 1-6.
- [14] Feng, J. C., Tse, C. K. [2007] "Reconstruction of Chaotic Signals with Applications to Chaos-Based Communications", Tsinghua University Press and World Scientific Publishing Co. Pte. Ltd.,
- [15] Pehlivan, I., Uyaroglu, Y. [2007] "Rikitake Attractor and It's Synchronization Application for Secure Communication Systems", Journal of Applied Sciences, 7(2), 232-236.
- [16] Lee, T. H., Park, J. H. [2010] "Generalized functional projective synchronization of Chen-Lee chaotic systems

and its circuit implementation", International Journal of the Physical Sciences, 5(7), 1183-1190.

- [17] Pehlivan, I., Uyaroglu, Y., Yogun, M. [2010] "Chaotic oscillator design and realizations of the Rucklidge attractor and its synchronization and masking simulations", Scientific Research and Essays, 5(16), 2210-2219.
- [18] Mamat, M., Salleh, Z., Mada, Sanjaya W. S., Muhammad, Noor N. M., Ahmad, M. F. [2012] "Numerical simulation of unidirectional chaotic synchronization of nonautonomous chaotic circuit and its application for secure communication" Adv. Studies Theor. Phys, 6(10), 497-509.
- [19] Mada, Sanjaya W. S., Halimatussadiyah, Maulana, D. S. [2011] "Bidirectional chaotic synchronization of nonautonomous chaotic circuit and its application for secure communication", World Academy of Science, Engineering, and Technology, 80.

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