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An Advanced Method for Finding Optimal Solution of Assignment Problem

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Abstract: ***In this paper, an advanced method named NS-AVSNM MAP (NS-AVSNM Method for Assignment Problem) is proposed to find an optimal solution of an assignment problem. Here we directly obtain optimal solution of assignment problem. The optimality of the problem is checked by illustrating numerical examples. The results obtained in the examples are compared with the results yielded with Hungarian method. This method is one of the efficient, simple, accurate method for obtaining an optimal solution of assignment problem.***

Keywords: Assignment problem, balanced and unbalanced assignment problem, optimal solution, Hungarian method, NS-AVSNM MAP.

1. Introduction

The assignment problem is a special case of linear programming problem. It is applicable in assigning vehicles to routes, machines to jobs, products to factories, school buses to various routes, aircrafts to particular trips, networking computers etc., In real life, it can also be used to determine marriage partners, friends etc.,

Assignment problem can be either minimization or maximization problem. The problem is to maximize the revenue from the sales or can be minimizing the assignment cost by deploying the vehicles to various routes.

Previously, a considerable number of methods have been so far presented for assignment problems among which Hungarian method is the best among them. The main idea of assignment problem is to assign „n" number of tasks „m" number of workers at a minimum cost or maximum profit. If m=n then assignment problem is said to be balanced. If m≠n then the problem is called unbalanced assignment problem.

Previously, assignment problems are solved using Hungarian method[1]. Later, A.Ahmed etal.,[2], AnjuKhandelwal[3], Jameer G.Kotwal[4] proposed innovative methods for solving assignment problems. Recently, A.Thiruppathi etal.,[5] introduced an innovative method for finding optimal solution to balanced assignment problems.This method is confined to balanced assignment problems. Also, Ghadle P. Kirtiwantetal[6] has given a new approach to solve balanced and unbalanced assignment problems. Previously, Hungarian method is the only method for finding maximal solution of unbalanced assignment problems. All the other methods are not suitable for finding maximal and optimal solution of unbalanced assignment problems. In this paper we presented an advanced, simple, efficient and time consuming method when compared with the Hungarian and other methods named as NS-AVSNM MAP so as to find an optimal solution of unbalanced assignment problems .The optimal solution of assignment problems is checked by numerical examples. Also, this method is suitable to find maximalsolution of an unbalanced assignment problem.

2. Formulation of Assignment Problem

Let $x_{ij} = \begin{cases} 1, & if\ i^{th}\ job\ is\ assigned\ to\ j^{th}\ person \\ 0, & if\ i^{th}\ job\ is\ not\ assigned\ to\ j^{th}\ person \end{cases}$

Determine x_{ij} ; i,j=1,2,...,m so as to optimize

$$Z = \sum_{i=1}^{m}\sum_{j=1}^{m} C_{ij}x_{ij} \quad \text{such that}$$

$$\sum_{i=1}^{m} x_{ij} = 1, for\ j = 1,2,..., m$$

$$\sum_{j=1}^{m} x_{ij} = 1, for\ i = 1,2,..., m$$

and $x_{ij} = 0\ or\ 1$.

Here C_{ij} is the cost or effectiveness of assigning ith job to jth facility. A cost matrix or effective matrix C_{ij} represents cost of assigning ith job to jth facility.

3. Algorithm for NS-AVSNM MAP

This algorithm is suitable for both balanced and unbalanced assignment problems. Steps for NS-AVSNM MAP method is as follows.

Step1: (a) If the assignment problem is minimization problem then go to step2.

(b)If the assignment problem is maximization problem then subtract maximum element from all the elements of the cost matrix. Then go to step 2.

Step 2: Find the minimum in each column and subtract the same from the respective column.

Step 3: If there is a tie in minimum of a column then find difference between first and second smallest elements in each column.

Step 4: Identify the column for which the difference is maximum.
Step 5: Assign cell (i, j) with minimum/maximum according to the given assignment problem.

Step 6: Delete i^{th} row and j^{th} column.

Step 7: Continue steps 3 to 6 until all the assignment process gets completed.

3.1 Numerical Examples

Example 3.1.1: Consider the following assignment problem. Assign 4 jobs to 5 persons with minimum cost.

$$\begin{array}{c} \\ 1 \\ 2 \\ 3 \\ 4 \end{array}\begin{array}{c} \begin{array}{ccccc} A & B & C & D & E \end{array} \\ \begin{bmatrix} 5 & 7 & 11 & 6 & 5 \\ 8 & 5 & 5 & 6 & 5 \\ 6 & 7 & 10 & 7 & 3 \\ 10 & 4 & 8 & 2 & 4 \end{bmatrix} \end{array}$$

Solution: Optimal assignment by Hungarian method is (1,A) (2,B) (3,E) (4,D)
Associated cost is 5+5+3+2=15.
Optimal assignment by NS-AVSNM MAP is (1,A) (2,C) (3,E) (4,D)
Associated cost is 5+5+3+2=15.

Example 3.1. 2:
Consider the following assignment problem. Assign 4 jobs to 10 persons with minimum cost.

$$\begin{array}{c} \\ A \\ B \\ C \\ D \\ E \\ F \\ G \\ H \\ I \\ J \end{array}\begin{array}{c} \begin{array}{cccc} 1 & 2 & 3 & 4 \end{array} \\ \begin{bmatrix} 11 & 8 & 9 & 8 \\ 4 & 5 & 29 & 33 \\ 10 & 5 & 29 & 33 \\ 1 & 18 & 25 & 31 \\ 23 & 22 & 33 & 30 \\ 3 & 9 & 13 & 19 \\ 6 & 8 & 27 & 32 \\ 32 & 30 & 39 & 38 \\ 36 & 35 & 31 & 21 \\ 15 & 11 & 10 & 28 \end{bmatrix} \end{array}$$

Solution: Optimal assignment by Hungarian method: (A,2) (B,1) (J,3) (I,4)
Associated cost = 8+4+10+21=43
Optimal assignment by NS-AVSNM MAP: (A,4) (B,2) (J,3) (D,1)
Associated cost=8+5+10+1=24

Example 3.1.3:

Consider the following assignment problem. Assign 4 jobs to 3 machines so that the profit is maximum.

$$\begin{array}{c} \\ A \\ B \\ C \\ D \end{array}\begin{bmatrix} 11 & 8 & 8 \\ 4 & 33 & 5 \\ 10 & 33 & 5 \\ 1 & 25 & 10 \end{bmatrix}$$

Solution: Given problem is of maximization, we subtract maximum element 33 from all the elements of the matrix. Now the cost matrix is

$$\begin{array}{c} \\ A \\ B \\ C \\ D \end{array}\begin{array}{c} \begin{array}{ccc} 1 & 2 & 3 \end{array} \\ \begin{bmatrix} 22 & 25 & 25 \\ 29 & 0 & 28 \\ 23 & 0 & 28 \\ 32 & 8 & 23 \end{bmatrix} \end{array}$$

Optimal assignment by Hungarian method: (A,1) (B,2) (D,3)
Associated cost =11+33+10=54
Optimal assignment by NS-AVSNM MAP: (A,1) (B,2) (D,3)
Associated cost =11+33+10=54

4. Conclusion

In this paper, we introduced a new method which is simple, efficient when compared with Hungarian method. In examples 1,2 and 3 the optimal solution yielded is same for both Hungarian and NS-AVSNM MAP methods. When compared with Hungarian method, NS-AVSNM MAP is simple, time consuming, optimal and efficient.

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