

# On Quasi-Normal and Quasi-Regular Spaces in Hereditary Generalized Topological Spaces

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**Abstract:** The concept of generalized open sets in generalized topologies was investigated by Csaszar [2]. In this paper we introduce spaces namely Quasi  $\mu$ - $\mathcal{H}$ -regular spaces, Almost  $\mu$ - $\mathcal{H}$ -Normal spaces and Quasi ultra  $\mu$ - $\mathcal{H}$ -Normal spaces with a fixed set of parameters and obtain some properties in the light of these notion. We also introduce Quasi  $\mu_\beta$ - $\mathcal{H}$ -regular spaces and Quasi  $\mu_\beta$ - $\mathcal{H}$ -normal spaces and investigate some properties of these new notions by using some basic properties of  $(\mu, \lambda)$ -continuity in generalized topological spaces introduced by M.Rajamani, V.Inthumathi and R.Ramesh [5]. Moreover we obtain relations between Quasi  $\mu$ - $\mathcal{H}$  normal spaces and Almost  $\mu$ - $\mathcal{H}$ -normal spaces with respect to  $(\mu, \lambda)$ -continuity and  $(\mu, \lambda)$ -open map.

**Keywords:** Quasi  $\mu$ - $\mathcal{H}$ -normal spaces, Quasi  $\mu$ - $\mathcal{H}$ -regular spaces, Quasi ultra  $\mu$ - $\mathcal{H}$ -Normal space, Almost  $\mu$ - $\mathcal{H}$ -Normal spaces,  $(\mu, \lambda)$ -continuity and  $(\mu, \lambda)$ -open,  $(\mu, \lambda)$   $\beta$ -irresolute,  $(\mu, \lambda)$   $\beta$ -continuous,  $(\mu, \lambda)$  R-irresolute,  $(\mu, \lambda)$  R-pre-closed.

## 1. Introduction and Preliminaries

The idea of generalized topology and hereditary class was introduced and studied by Csaszar[2]. A subfamily  $\mu$  of  $P(X)$  is called a generalized topology if  $\phi \in \mu$  and union of elements of  $\mu$  belongs to  $\mu$ . The space  $X$  together with the generalized topology  $\mu$  is said to be generalized topological space and denoted by  $(X, \mu)$ .  $i_\mu(A)$  and  $c_\mu(A)$  denotes the interior and closure of  $A$  in  $(X, \mu)$ . The elements of  $\mu$  are called open and is denoted by  $\mu$ -open. The complement of  $\mu$ -open is  $\mu$ -closed. We say a hereditary class  $\mathcal{H}$  on  $(X, \mu)$  is a non-empty collection of subset of  $X$  such that  $A \subseteq B, B \in \mathcal{H}$  implies

$A \in \mathcal{H}$ . With respect to the generalized topology  $\mu$  and a hereditary class  $\mathcal{H}$ , for a subset  $A$  of  $X$  we define  $A_{\mu\alpha}^*(\mathcal{H})$  or simply  $A_\mu^* = \{x \in X : M \cap A \notin \mathcal{H} \text{ for every } M \in \mu \text{ such that } x \in M\}$ . The closure  $c_\mu^*(A) = A \cup A_\mu^*(\mathcal{H})$ . The space  $(X, \mu)$  with the hereditary class  $\mathcal{H}$  is called hereditary generalized topological space and denoted by  $(X, \mu, \mathcal{H})$ . A subset  $A$  of  $(X, \mu)$  is  $\mu\alpha$ -open [2] (resp.  $\mu$ -semi open [2],  $\mu$ -pre open [2],  $\mu\beta$ -open[2]), if  $A \subseteq i_\mu(c_\mu(i_\mu(A)))$  (resp.  $A \subseteq c_\mu(i_\mu(A))$ ,  $A \subseteq i_\mu(c_\mu(A))$ ,  $A \subseteq c_\mu(i_\mu(c_\mu(A)))$ ). We denote the family of all  $\mu\alpha$ -open sets,  $\mu$ -semi open sets,  $\mu$ -pre open sets and  $\mu\beta$ -open sets by  $\alpha(\mu)$ ,  $\sigma(\mu)$ ,  $\pi(\mu)$  and  $\beta(\mu)$  respectively. On generalized topology,  $\mu \subseteq \alpha(\mu) \subseteq \pi(\mu) \subseteq \beta(\mu)$ .

In GTS  $c_{\mu\alpha}(A)$  and  $c_{\mu\beta}(A)$  denotes  $\alpha$ -closure of  $A$  and  $\beta$ -closure of  $A$  in  $(X, \mu)$  respectively. A subset  $A$  of  $(X, \mu)$  is

said to be  $\mu$ -regular open[4] if  $A = i_\mu(c_\mu(A))$  and the complement is  $\mu$ -regular closed. The finite union of  $\mu$ -regular open sets is called  $\mu\pi$ -open sets and its complement is  $\mu\pi$ -closed set. A set  $A$  is said to be  $\mu g$ -closed [2] if,  $c_\mu(A) \subseteq A$  whenever  $A \subseteq U$  and  $U$  is  $\mu$ -open and its complement is  $\mu g$ -open.

**Definition 1.1:** For a subset  $A$  of hereditary generalized topological space  $(X, \mu, \mathcal{H})$

- $A_{\mu\alpha}^*(\mathcal{H}) = \{x \in X : M \cap A \notin \mathcal{H} \text{ for every } M \in \alpha(\mu) \text{ such that } x \in M\}$  [2].
- $A_{\mu\beta}^*(\mathcal{H}) = \{x \in X : M \cap A \notin \mathcal{H} \text{ for every } M \in \beta(\mu) \text{ such that } x \in M\}$  [2].

**Definition 1.2:** A subset  $A$  of a hereditary generalized topological space  $(X, \mu, \mathcal{H})$  is said to be  $\mu^*$ -closed [3] if  $A_\mu^*(\mathcal{H}) \subseteq A$  and  $\mu_\beta^*$ -closed [3] if  $A_{\mu\beta}^*(\mathcal{H}) \subseteq A$ . Then  $c_{\mu\beta}^*(A) = A \cup A_{\mu\beta}^*(\mathcal{H})$ .

**Definition 1.3:** Let  $(X, \mu)$  and  $(Y, \lambda)$  be generalized topologies. A function  $f: (X, \mu) \rightarrow (Y, \lambda)$  is said to be

- $(\mu, \lambda)$ -continuous if for every closed set  $V$  in  $(Y, \lambda)$  (denoted by  $\lambda$ -closed set),  $f^{-1}(V)$  is  $\mu$ -closed,
- $(\mu, \lambda)$ -open if for every  $\mu$ -open set  $U$ ,  $f(U)$  is open in  $(Y, \lambda)$  (denoted by  $\lambda$ -open).

## 2. Quasi $\mu\mathcal{H}$ -regular spaces, Almost $\mu\mathcal{H}$ -Normal spaces and Quasi ultra $\mu\mathcal{H}$ -Normal space

**Definition 2.1:** A hereditary generalized topological space  $(X, \mu, \mathcal{H})$  is said to be Quasi  $\mu\mathcal{H}$ -regular space if for every  $\mu\pi$ -closed set  $A$  and a point  $x \notin A$ , there exist  $\mu$ -open sets  $U$  and  $V$  such that  $A - U \in \mathcal{H}$ ,  $x \in V$  and  $U \cap V \in \mathcal{H}$ .

**Definition 2.2:** Let  $(X, \mu, \mathcal{H})$  be a hereditary generalized topological spaces. A space  $(X, \mu, \mathcal{H})$  is said to be quasi  $\mu\mathcal{H}$ -normal if for every pair of  $\mu\pi$ -closed sets  $A$  and  $B$  of  $X$ , there exist  $\mu$ -open sets  $U$  and  $V$  such that  $A - U \in \mathcal{H}$ ,  $B - V \in \mathcal{H}$ , and  $U \cap V \in \mathcal{H}$ .

**Theorem 2.3:** Let  $(X, \mu, \mathcal{H})$  be a Hereditary generalized topological space.

Then the followings are equivalent:

- $X$  is a quasi  $\mu\mathcal{H}$ -regular space.
- for each point  $x \in X$  and for each  $\mu\pi$ -open neighbourhood  $F$  of  $x$ , there exists a  $\mu$ -open set  $V$  of  $X$  such that  $c_\mu^*(V) - F \in \mathcal{H}$ .
- For each point  $x \in X$  and for each  $\mu\pi$ -closed set  $A$  not containing, there exists a  $\mu$ -open set  $V$  of  $X$  such that  $c_\mu^*(V) \cap A \in \mathcal{H}$ .

**Proof:**

**(a)  $\Rightarrow$  (b)** Let  $F$  be  $\mu\pi$ -open neighbourhood of  $x$ . Then there exist a  $\mu\pi$ -open subset  $G$  of  $X$  such that  $x \in G \subseteq F$ . Since  $G^c$  is  $\mu\pi$ -closed and  $x \in G$  by hypothesis, there exist disjoint  $\mu$ -open sets  $U$  and  $V$  such that  $G^c - U \in \mathcal{H}$ ,  $x \in V$  and  $U \cap V \in \mathcal{H}$  and so  $V - U^c \in \mathcal{H}$ . Since  $U^c$  is  $\mu$ -closed,  $c_\mu^*(V) - U^c \in \mathcal{H}$  implies  $U^c - G \in \mathcal{H}$ . Hence  $c_\mu^*(V) - F \in \mathcal{H}$ .

**(b)  $\Rightarrow$  (a):** Let  $F^c$  be any  $\mu\pi$ -closed set and  $x \notin F^c$ . Then  $x \in F$  and  $F$  is  $\mu\pi$ -open neighbourhood of  $x$ . By hypothesis, there exist a  $\mu$ -open set  $V$  of  $x$  such that  $x \in V$  and  $c_\mu^*(V) - F \in \mathcal{H}$ , which implies  $F^c - c_\mu^*(V) \in \mathcal{H}$ . Then  $(c_\mu^*(V))^c$  is  $\mu$ -open set containing  $F^c$  and  $V \cap (c_\mu^*(V))^c \in \mathcal{H}$ . Therefore is  $X$  quasi  $\mu\mathcal{H}$ -regular space.

**(b)  $\Rightarrow$  (c):** Let  $x \in X$  and  $A$  be  $\mu\pi$ -closed set such that  $x \notin A$ . Since  $A^c$  is  $\mu\pi$ -open neighbourhood of  $x$  and by hypothesis, there exist a  $\mu$ -open set  $V$  of  $X$  such that

$$c_\mu^*(V) - A^c \in \mathcal{H} \text{ and } c_\mu^*(V) \cap A \in \mathcal{H}.$$

**(c)  $\Rightarrow$  (a):** Let  $x \in X$  and  $A$  be a  $\mu\pi$ -closed set such that  $x \notin A$ . By hypothesis, there exists  $\mu$ -open set  $U$  such that  $c_\mu^*(V) \cap A \in \mathcal{H}$ . Let  $V = X - c_\mu^*(U)$ . Since  $V$  is  $\mu$ -open set and  $U \cap V \in \mathcal{H}$ ,  $X$  is quasi  $\mu\mathcal{H}$ -regular space.

**Definition 2.4:** Let  $(X, \mu)$  and  $(Y, \lambda)$  be generalized topologies. A function  $f: (X, \mu) \rightarrow (Y, \lambda)$  is said to be

- Completely  $(\mu, \lambda)$ -irresolute if for every  $\pi$ -closed set  $V$  in  $(Y, \lambda)$  (denoted by  $\lambda\pi$ -closed set),  $f^{-1}(V)$  is  $\pi$ -closed in  $(X, \mu)$  (denoted by  $\mu\pi$ -closed).
- Completely  $(\mu, \lambda)$ -continuous if for every  $\lambda$ -closed set  $V$ ,  $f^{-1}(V)$  is  $\mu\pi$ -closed.
- Almost  $(\mu, \lambda)$ -open if for every  $\pi$ -open set  $V$  in  $(X, \mu)$  (denoted by  $\mu\pi$ -open set),  $f(V)$  is  $\pi$ -open in  $(Y, \lambda)$  (denoted by  $\lambda\pi$ -open).
- Almost  $(\mu, \lambda)$ -closed if for every  $\mu\pi$ -closed set  $F$ ,  $f(F)$  is  $\lambda\pi$ -closed.
- Perfectly  $(\mu, \lambda)$ -continuous if for every open set  $F$  in  $(Y, \lambda)$  (denoted by  $\lambda$ -open set),  $f^{-1}(F)$  is  $\mu$ -open and  $\mu$ -closed.
- $(\mu, \lambda)$ -R-irresolute if for every regular-closed set  $V$  in  $(Y, \lambda)$  (denoted by  $\lambda$ -regular-closed),  $f^{-1}(V)$  is  $\mu$ -regular-closed.
- $(\mu, \lambda)$   $\beta$ -irresolute if for every  $\beta$ -closed set  $V$  in  $(Y, \lambda)$  (denoted by  $\lambda\beta$ -closed set),  $f^{-1}(V)$  is  $\beta$ -closed in  $(X, \mu)$  (denoted by  $\mu\beta$ -closed set).
- $(\mu, \lambda)$   $\beta$ -continuous if for every  $\lambda$ -closed set  $V$ ,  $f^{-1}(V)$  is  $\mu\beta$ -closed in  $(X, \mu)$ .
- $(\mu, \lambda)$ -R-pre-closed if for every  $\mu$ -regular-closed set  $U$ ,  $f(U)$  is  $\lambda$ -regular-closed.

**Lemma 2.5** [3]: If  $\mathcal{H} \neq \phi$  is a hereditary class on  $(X, \mu)$  and  $f: (X, \mu) \rightarrow (Y, \lambda)$  is a function, then  $f(\mathcal{H}) = \{f(H) : H \in \mathcal{H}\}$  is a hereditary class on  $(Y, \lambda)$ .

**Theorem 2.6:** Let  $(X, \mu, \mathcal{H})$  be a hereditary generalized topological space and  $(Y, \lambda)$  be a generalized topology. A function  $f: (X, \mu) \rightarrow (Y, \lambda)$  is bijective, completely  $(\mu, \lambda)$ -irresolute and  $(\mu, \lambda)$ -open. If  $X$  is quasi  $\mu\mathcal{H}$ -regular space, then  $Y$  is quasi  $\lambda$ - $f(\mathcal{H})$ -regular space.

**Proof:** Let  $y \in Y$  and  $A$  be any  $\lambda\pi$ -closed set. Since  $f$  is completely  $(\mu, \lambda)$ -irresolute,  $f^{-1}(A)$  is  $\mu\pi$ -closed subset of  $X$ . Since  $f$  is a bijection,  $f(x) = y$ , then  $y \neq f^{-1}(x)$  for every  $x \in X$ . Since  $(X, \mu, \mathcal{H})$  is quasi  $\mu\mathcal{H}$ -regular space, there exists  $\mu$ -open sets  $U$  and  $V$  such that  $x \in U$ ,

$f^{-1}(A) - V \in \mathcal{H}$  and  $U \cap V \in \mathcal{H}$ . Since  $f$  is  $(\mu, \lambda)$ -open,  $f(U)$  and  $f(V)$  are  $\mu$ -open sets in  $Y$ . Also  $y \in f(U)$ , and  $A - f(V) \in f(\mathcal{H})$  and  $f(U) \cap f(V) = f(U \cap V) \in f(\mathcal{H})$ . Hence by using lemma(2.5)  $Y$  is quasi  $\lambda$ - $f(\mathcal{H})$ -regular space.

**Lemma 2.7 [3]:** If  $\mathcal{H} \neq \phi$  is a hereditary class on  $(Y, \lambda)$  and  $f: (X, \mu) \rightarrow (Y, \lambda)$ , then  $f^{-1}(\mathcal{H}) = \{f^{-1}(H): H \in \mathcal{H}\}$  is a hereditary class on  $(X, \mu)$ .

**Theorem 2.8:** Let  $(X, \mu)$  be a generalized topological space and  $(Y, \lambda, \mathcal{H})$  be hereditary generalized topology. A function  $f: (X, \mu) \rightarrow (Y, \lambda)$  is injective, Almost  $(\mu, \lambda)$ -closed and  $(\mu, \lambda)$ -continuous. If  $Y$  is quasi  $\lambda$ - $\mathcal{H}$ -regular space, then  $X$  is quasi  $\mu$ - $f^{-1}(\mathcal{H})$ -regular space.

**Proof:** Let  $x \in X$  and  $A$  be any  $\mu\pi$ -closed subset of  $X$ . Since  $f$  is Almost  $(\mu, \lambda)$ -closed,  $f(A)$  is  $\lambda\pi$ -closed subset of  $Y$ . Since  $(Y, \lambda, \mathcal{H})$  is quasi  $\lambda$ - $\mathcal{H}$ -regular space, there exists  $\lambda$ -open sets  $U$  and  $V$  such that  $f(x) \in U$ ,  $f(A) - V \in \mathcal{H}$  and  $U \cap V \in \mathcal{H}$ . Since  $f$  is  $(\mu, \lambda)$ -continuous and injective,  $f^{-1}(U)$  and  $f^{-1}(V)$  are  $\mu$ -open sets in  $X$ , such that  $x \in f^{-1}(U)$ ,  $A - f^{-1}(V) \in f^{-1}(\mathcal{H})$  and  $f^{-1}(U) \cap f^{-1}(V) = f^{-1}(U \cap V) \in f^{-1}(\mathcal{H})$ . Hence by using lemma (2.7)  $X$  is quasi  $\mu$ - $f^{-1}(\mathcal{H})$ -regular space.

**Lemma 2.9 [3]:** If  $\mathcal{H} \neq \phi$  is a hereditary class on  $(Y, \lambda)$  and  $Y$  is a subset of  $X$ . Then  $\mathcal{H}_Y = \{Y \cap H: H \in \mathcal{H}\}$  is a hereditary class on  $Y$ .

**Theorem 2.10:** Let  $(X, \mu, \mathcal{H})$  be a generalized topological space. If  $X$  is quasi  $\mu$ - $\mathcal{H}$ -regular space and  $Y \subset X$  is  $\mu\pi$ -closed set, then  $Y$  is quasi  $\mu$ - $\mathcal{H}_Y$ -regular space.

**Proof:** Let  $y \in Y$  and  $A$  be  $\mu\pi$ -closed subset of  $Y$  and  $y \notin A$ . Since  $Y$  is  $\mu\pi$ -closed set and  $Y \subset X$ ,  $A$  is  $\mu\pi$ -closed subset of  $X$ . Since  $X$  is quasi  $\mu$ - $\mathcal{H}$ -regular space, there exist  $\mu$ -open sets  $U$  and  $V$  such that  $A - U \in \mathcal{H}$ ,  $x \in V$  and  $U \cap V \in \mathcal{H}$ . If  $A - U = H \in \mathcal{H}$ , then  $A \subset (U \cup H)$ . Since  $A \subset F$ ,  $A \subset (F \cap (U \cup H))$  and so  $A \subset (F \cap U) \cup (F \cap H)$ . Therefore,  $A - (F \cap U) \subset (F \cap H) \in \mathcal{H}_Y$ ,  $y \in (V \cap F)$ . Hence  $(F \cap U)$  and  $(F \cap V)$  are  $\mu$ -open sets in  $Y$  such that  $A - (F \cap U) \in \mathcal{H}_Y$ ,  $y \in V$  and

$(F \cap U) \cap (F \cap V) \in \mathcal{H}_Y$ . Hence  $Y$  is quasi  $\mu$ - $\mathcal{H}_Y$ -regular space.

**Definition 2.11:** A generalized topological space  $(X, \mu)$  with the hereditary class  $\mathcal{H}$  is said to be Almost  $\mu$ - $\mathcal{H}$ -normal space if for every pair of disjoint  $\mu$ -regular closed sets  $A$  and  $B$  there exist  $\mu$ -open sets  $F$  and  $G$  such that  $A - F \in \mathcal{H}$ ,  $A - G \in \mathcal{H}$  and  $F \cap G \in \mathcal{H}$ .

**Theorem 2.12:** Let  $(X, \mu, \mathcal{H})$  be a hereditary generalized topological space and  $(Y, \lambda)$  be a generalized topological space. Also let  $f: (X, \mu) \rightarrow (Y, \lambda)$  is  $(\mu, \lambda)$  R-pre-closed and  $(\mu, \lambda)$ -continuous injective function. If  $Y$  is Almost  $\lambda$ - $\mathcal{H}$ -normal space, then  $X$  is Almost  $\mu$ - $f^{-1}(\mathcal{H})$ -normal space.

**Proof:** Let  $A$  and  $B$  be disjoint  $\mu$ -regular-closed subsets of  $X$ . Since  $f$  is  $(\mu, \lambda)$  R-pre-closed,  $f(A)$  and  $f(B)$  are disjoint  $\lambda$ -regular-closed. Since  $Y$  is Almost  $\lambda$ - $\mathcal{H}$ -normal space, there exist  $\lambda$ -open sets  $U$  and  $V$  in  $Y$  such that  $f(A) - U \in \mathcal{H}$ ,  $f(B) - V \in \mathcal{H}$ ,  $U \cap V \in \mathcal{H}$ . Then  $f^{-1}(f(A)) - f^{-1}(U) \in f^{-1}(\mathcal{H})$  and  $f^{-1}(f(B)) - f^{-1}(V) \in f^{-1}(\mathcal{H})$  which implies  $A - f^{-1}(U) \in f^{-1}(\mathcal{H})$ ,  $B - f^{-1}(V) \in f^{-1}(\mathcal{H})$  and  $f^{-1}(U) \cap f^{-1}(V) \in f^{-1}(\mathcal{H})$ . Since  $f$  is  $(\mu, \lambda)$ -continuous,  $f^{-1}(U)$  and  $f^{-1}(V)$  are  $\mu$ -open subsets of  $X$ . Hence  $(X, \mu)$  is Almost  $\mu$ - $f^{-1}(\mathcal{H})$ -normal space.

**Definition 2.13:** A generalized topological space  $(X, \mu)$  with the hereditary class  $\mathcal{H}$  is said to be quasi ultra  $\mu$ - $\mathcal{H}$  normal space if for every pair of disjoint  $\mu\pi$ -closed sets  $A$  and  $B$  there exist  $\mu$ -open sets  $F$  and  $G$  such that  $A - F \in \mathcal{H}$ ,  $A - G \in \mathcal{H}$  and  $F \cap G \in \mathcal{H}$ .

**Theorem 2.14:** Let  $(X, \mu, \mathcal{H})$  be a hereditary generalized topological space and  $(Y, \lambda)$  be a generalized topological space. Also let  $f: (X, \mu) \rightarrow (Y, \lambda)$  is Almost  $(\mu, \lambda)\pi$ -closed and perfectly  $(\mu, \lambda)$ -continuous injective function. If  $Y$  is Quasi  $\lambda$ - $\mathcal{H}$ -normal space, then  $X$  is Quasi ultra  $\mu$ - $f^{-1}(\mathcal{H})$ -normal space.

**Proof:** Let  $F$  and  $G$  be disjoint  $\mu\pi$ -closed subsets of  $X$ . Since  $f$  is Almost  $(\mu, \lambda)\pi$ -closed,  $f(F)$  and  $f(G)$  are disjoint  $\lambda\pi$ -closed subsets of  $Y$ . Since  $Y$  is quasi  $\lambda$ - $\mathcal{H}$ -normal space, there exist  $\lambda$ -open sets  $U$  and  $V$  in  $Y$  such that  $f(F) - U \in \mathcal{H}$ ,  $f(G) - V \in \mathcal{H}$  and  $U \cap V \in \mathcal{H}$ . Then  $F - f^{-1}(U) \in f^{-1}(\mathcal{H})$ ,  $G - f^{-1}(V) \in f^{-1}(\mathcal{H})$  and



$f^{-1}(U) \cap f^{-1}(V) \in f^{-1}(\mathcal{H})$ . Since  $f$  is perfectly  $(\mu, \lambda)$ -continuous,  $f^{-1}(U)$  and  $f^{-1}(V)$  are  $\mu$ -open and  $\mu$ -closed subsets of  $(X, \mu)$ . Hence  $X$  is quasi ultra  $\mu$ - $f^{-1}(\mathcal{H})$ -normal space.

**Remark 2.15:** Let  $(X, \mu, \mathcal{H})$  be a hereditary generalized topological space and  $(Y, \lambda)$  be a generalized topological space. Also let  $f: (X, \mu) \rightarrow (Y, \lambda)$  is  $(\mu, \lambda)$   $\pi$ -pre-closed and  $(\mu, \lambda)$ -continuous injective function. If  $Y$  is Quasi  $\lambda$ - $\mathcal{H}$ -normal space, then  $X$  is Almost  $\mu$ - $f^{-1}(\mathcal{H})$ -normal space.

**Proof:** Let  $F$  and  $G$  be disjoint  $\mu$ -regular-closed subsets of  $X$  and hence  $\mu\pi$ -open. Since  $f$  is Almost  $(\mu, \lambda)\pi$ -closed,  $f(F)$  and  $f(G)$  are disjoint  $\lambda\pi$ -closed. Since  $Y$  is quasi  $\lambda$ - $\mathcal{H}$ -normal space, there exist  $\lambda$ -open sets  $U$  and  $V$  in  $Y$  such that  $f(F) - U \in \mathcal{H}$ ,  $f(G) - V \in \mathcal{H}$  and  $U \cap V \in \mathcal{H}$ . Then  

$$F - f^{-1}(U) \in f^{-1}(\mathcal{H}),$$

$$G - f^{-1}(V) \in f^{-1}(\mathcal{H})$$
 and  

$$f^{-1}(U) \cap f^{-1}(V) \in f^{-1}(\mathcal{H}).$$
 Since  $f$  is  $(\mu, \lambda)$ -continuous,  $f^{-1}(U)$  and  $f^{-1}(V)$  are  $\mu$ -open subsets of  $(X, \mu)$ . Hence  $X$  is Almost  $\mu$ - $f^{-1}(\mathcal{H})$ -normal space.

**Theorem 2.16:** Let  $(X, \mu, \mathcal{H})$  be a hereditary generalized topological space and  $(Y, \lambda)$  be a generalized topological space. Also let  $f: (X, \mu) \rightarrow (Y, \lambda)$  be a bijection  $(\mu, \lambda)$ - $R$ -irresolute and  $(\mu, \lambda)$ -open function. If  $X$  is Almost  $\mu$ - $\mathcal{H}$ -normal space, then  $Y$  is Almost  $\lambda$ - $f(\mathcal{H})$ -normal space.

**Proof:** Proof is similar to the proof of (2.6).

### 3. Quasi $\mu_\beta$ - $\mathcal{H}$ -regular spaces and Quasi $\mu_\beta$ - $\mathcal{H}$ -normal spaces

**Definition 3.1:** A hereditary generalized topological space  $(X, \mu, \mathcal{H})$  is said to be Quasi  $\mu_\beta$ - $\mathcal{H}$ -regular space if for every  $\mu\pi$ -closed set  $A$  and  $x \notin A$ , there exist  $\mu\beta$ -open sets  $U$  and  $V$  such that  $A - U \in \mathcal{H}$ ,  $x \in V$  and  $U \cap V \in \mathcal{H}$ .

**Definition 3.2:** Let  $(X, \mu, \mathcal{H})$  be a hereditary generalized topological spaces. A space  $(X, \mu, \mathcal{H})$  is said to be quasi  $\mu_\beta$ - $\mathcal{H}$ -normal if for every pair of  $\mu\pi$ -closed sets  $A$  and  $B$  of  $X$ , there exist  $\mu\beta$ -open sets  $U$  and  $V$  such that  $A - U \in \mathcal{H}$ ,  $B - V \in \mathcal{H}$ , and  $U \cap V \in \mathcal{H}$ .

**Theorem 3.3:** Let  $(X, \mu, \mathcal{H})$  be a hereditary generalized topological spaces. Then the followings are equivalent:

(a)  $X$  is a quasi  $\mu_\beta$ - $\mathcal{H}$ -normal space.

(b) for every  $\mu\pi$ -closed set  $F$  and  $\mu\pi$ -open set  $G$  containing  $F$ , there exists a  $\mu\beta$ -open set  $V$  such that  $F - U \in \mathcal{H}$  and  $c_{\mu_\beta}^*(V) - G \in \mathcal{H}$ .

(c) For each pair of disjoint  $\mu\pi$ -closed sets  $A$  and  $B$ , there exists an  $\mu\beta$ -open set  $U$  such that  $A - U \in \mathcal{H}$  and  $c_{\mu_\beta}^*(U) \cap B \in \mathcal{H}$ .

**Proof:**

(a)  $\Rightarrow$  (b) Let  $F$  be  $\mu\pi$ -closed set and  $G$  be a  $\mu\pi$ -open subset of  $X$ . Since  $X - G$  is  $\mu\pi$ -closed and  $F \subset G$ ,  $F \cap (X - G) = \phi$ . Since  $X$  is quasi  $\mu_\beta$ - $\mathcal{H}$ -normal space, there exist disjoint  $\mu\beta$ -open sets  $U$  and  $V$  such that  $F - U \in \mathcal{H}$  and  $(X - G) - V \in \mathcal{H}$ . Then  

$$c_{\mu_\beta}^*(V) \subset (X - U)$$
 and  

$$(X - G) \cap c_{\mu_\beta}^*(V) \subset (X - G) \cap (X - U).$$
 Hence  

$$c_{\mu_\beta}^*(V) - G \in \mathcal{H}.$$

(b)  $\Rightarrow$  (c) It is obvious.

(c)  $\Rightarrow$  (a) Let  $A$  and  $B$  be  $\mu\pi$ -closed sets. By (c) there exists a  $\mu\beta$ -open  $U$  such that  $A - U \in \mathcal{H}$  and  $c_{\mu_\beta}^*(U) \cap B \in \mathcal{H}$ . Let  $V = X - c_{\mu_\beta}^*(U)$ . Since  $V$  is  $\mu\beta$ -open and  

$$V = X - c_{\mu_\beta}^*(U), U \cap V \in \mathcal{H}$$
 Hence  $X$  is quasi  $\mu_\beta$ - $\mathcal{H}$ -normal space.

**Theorem 3.4:** Let  $(X, \mu, \mathcal{H})$  be a hereditary generalized topological space and  $(Y, \lambda)$  be a generalized topological space. Also let  $f: (X, \mu) \rightarrow (Y, \lambda)$  is a bijective function, completely  $(\mu, \lambda)$ -continuous and  $(\mu, \lambda)$   $\beta$ -open. If  $X$  is quasi  $\mu_\beta$ - $\mathcal{H}$ -normal space, then  $Y$  is quasi  $\lambda_\beta$ - $f(\mathcal{H})$ -normal space.

**Proof:** Let  $A$  and  $B$  be  $\lambda\pi$ -closed subsets of  $Y$ . Since  $f$  is completely  $(\mu, \lambda)$ -continuous,  $f^{-1}(A)$  and  $f^{-1}(B)$  are  $\mu\pi$ -closed subsets of  $X$ . Since  $X$  is quasi  $\mu_\beta$ - $\mathcal{H}$ -normal space, there exist  $\mu\beta$ -open sets  $U$  and  $V$  in  $X$  such that  $f^{-1}(A) - U \in \mathcal{H}$ ,  $f^{-1}(B) - V \in \mathcal{H}$  and  $U \cap V \in \mathcal{H}$ . Since  $f$  is bijective,  $f(f^{-1}(A)) - f(U) \in f(\mathcal{H})$ ,  $f(f^{-1}(B)) - f(V) \in f(\mathcal{H})$  and  $f(U) \cap f(V) \in f(\mathcal{H})$  and hence  $A - f(U) \in f(\mathcal{H})$ ,  $B - f(V) \in f(\mathcal{H})$ . Since  $f$  is  $(\mu, \lambda)$   $\beta$ -open,  $f(U)$  and  $f(V)$  are  $\mu\beta$ -open sets in  $Y$ . Hence it follows that  $Y$  is quasi  $\lambda_\beta$ - $f(\mathcal{H})$ -normal space.

**Theorem 3.5:** Let  $(X, \mu, \mathcal{H})$  be a hereditary generalized topological space and  $(Y, \lambda)$  be a generalized topological space. Also let  $f: (X, \mu) \rightarrow (Y, \lambda)$  is an injective function, and Almost  $(\mu, \lambda)$ -closed and  $(\mu, \lambda)\beta$ -irresolute. If  $Y$  is quasi  $\lambda_\beta$ - $\mathcal{H}$ -normal space, then  $X$  is quasi  $\mu_\beta$ - $f^{-1}(\mathcal{H})$ -normal space.

**Proof:** Let  $A$  and  $B$  be  $\mu\pi$ -closed subsets of  $X$ . Since  $f$  is Almost  $(\mu, \lambda)$ -closed,  $f(A)$  and  $f(B)$  are disjoint  $\lambda\pi$ -closed. Since  $Y$  is quasi  $\lambda_\beta$ - $\mathcal{H}$ -normal space, there exist  $\mu\beta$ -open sets  $U$  and  $V$  in  $Y$  such that  $f(A) - U \in \mathcal{H}$  and  $f(B) - V \in \mathcal{H}$  and  $U \cap V \in \mathcal{H}$ . Then  
 $f^{-1}(f(A)) - f^{-1}(U) \in f^{-1}(\mathcal{H})$  and  
 $f^{-1}(f(B)) - f^{-1}(V) \in f^{-1}(\mathcal{H})$  and  
 $f^{-1}(U \cap V) = f^{-1}(U) \cap f^{-1}(V) \in f^{-1}(\mathcal{H})$  which implies  
 $A - f^{-1}(U) \in f^{-1}(\mathcal{H})$  and  
 $B - f^{-1}(V) \in f^{-1}(\mathcal{H})$ . Since  $f$  is  $(\mu, \lambda)\beta$ -irresolute,  $f^{-1}(U)$  and  $f^{-1}(V)$  are  $\mu\beta$ -open subsets of  $X$ . Hence it follows that  $X$  is quasi  $\mu_\beta$ - $f^{-1}(\mathcal{H})$ -normal space.

**Theorem 3.6:** Let  $(X, \mu, \mathcal{H})$  be a generalized topological space. If  $X$  is quasi  $\mu_\beta$ - $\mathcal{H}$ -normal space, and  $Y \subset X$  is  $\mu\pi$ -closed set, then  $Y$  is quasi  $\mu_\beta$ - $\mathcal{H}_Y$ -normal space.

**Proof:** Let  $A$  and  $B$  be disjoint  $\mu\pi$ -closed subsets of  $Y$ . Since  $Y$  is  $\mu\pi$ -closed set and  $Y \subset X$ ,  $A$  and  $B$  are  $\mu\pi$ -closed subsets of  $X$ . Since  $X$  is quasi  $\mu_\beta$ - $\mathcal{H}$ -normal space, there exist  $\mu\beta$ -open sets  $U$  and  $V$  such that  $A - U \in \mathcal{H}$ ,  $B - V \in \mathcal{H}$  and  $U \cap V \in \mathcal{H}$ . If  $A - U = H \in \mathcal{H}$ ,  $B - V = G \in \mathcal{H}$ , then  $A \subset (U \cup H)$  and  $B \subset (V \cup G)$ . Since  $A \subset Y$ ,  $A \subset Y \cap (U \cup H)$  and so  $A \subset (Y \cap U) \cup (Y \cap H)$ . Therefore  
 $A - (Y \cap U) \subset (Y \cap H) \in \mathcal{H}_Y$ . Similarly  
 $B - (Y \cap V) \subset (Y \cap G) \in \mathcal{H}_Y$ . Hence  $Y \cap U$  and  $Y \cap V$  are  $\mu\beta$ -open sets in  $Y$  such that  
 $A - (Y \cap U) \in \mathcal{H}_Y$  and  $B - (Y \cap V) \in \mathcal{H}_Y$ . Hence  $Y$  is quasi  $\mu_\beta$ - $\mathcal{H}_Y$ -normal space.

#### 4. Conclusion

In this paper we introduced new classes of spaces namely quasi  $\mu$ - $\mathcal{H}$ -regular space, quasi  $\mu_\beta$ - $\mathcal{H}$ -regular space, quasi  $\mu_\beta$ - $\mathcal{H}$ -normal space, Almost  $\mu$ - $\mathcal{H}$ -Normal spaces, Quasi ultra  $\mu$ - $\mathcal{H}$ -Normal space in hereditary generalized topological spaces and derived some properties by using

some basic properties of  $(\mu, \lambda)$ -continuity in generalized topological spaces.

#### References

- [1] Bishwambhar Roy, On a type of generalised open sets, Applied general topology, Universidad Politecnica de Valencia. Volume 12, no. 2, pp.163-173, **2011**.
- [2] A.Csaszar, generalized open sets in generalized topologies., Acta Mathematica Hungaria. 106, pp.53-66, **2005**.
- [3] A.Csaszar, Modifications in Generalized topologies via hereditary., Acta Mathematica Hungaria. 115, pp. 29-36, **2007**.
- [4] R.Jamunarani and P. Jeyanthi, Regular sets in generalized topological spaces., Acta Mathematica Hungaria, 135, pp. 342-349, **2012**.
- [5] K.Karuppaiy, A note on RH-open sets in GTS with hereditary classes., International Journal of Mathematical Archive-5(1), pp:312-316, **2014**.
- [6] M.Rajamani, V.Inthumathi and R.Ramesh, A decomposition of  $(\mu, \lambda)$ -continuity in generalized topological spaces, Jordan J.Math.Stat.6(1), pp.15-27, **2013**.
- [7] J.Tong, Expansion of open sets and decomposition of continuous mappings, Rend. Circ.Mat.Polermo 43(2), 303-308, **1994**.