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On Quasi-Normal and Quasi-Regular Spaces in Hereditary Generalized Topological Spaces

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Abstract: ***The concept of generalized open sets in generalized topologies was investigated by Csaszar [2]. In this paper we introduce spaces namely Quasi μ-$\mathcal{H}$ -regular spaces, Almost μ-$\mathcal{H}$ -Normal spaces and Quasi ultra μ- $\mathcal{H}$-Normal spaces with a fixed set of parameters and obtain some properties in the light of these notion. We also introduce Quasi μ_β- $\mathcal{H}$-regular spaces and Quasi μ_β- $\mathcal{H}$ -normal spaces and investigate some properties of these new notions by using some basic properties of (μ, λ)-continuity in generalized topological spaces introduced by M.Rajamani, V.Inthumathi and R.Ramesh [5]. Moreover we obtain relations between Quasi μ-$\mathcal{H}$ normal spaces and Almost μ-$\mathcal{H}$ -normal spaces with respect to (μ, λ)-continuity and (μ, λ)-open map.***

Keywords: Quasi μ-$\mathcal{H}$ -normal spaces, Quasi μ-$\mathcal{H}$ -regular spaces, Quasi ultra μ- $\mathcal{H}$-Normal space, Almost μ-$\mathcal{H}$ -Normal spaces, (μ, λ)-continuity and (μ, λ)-open, (μ, λ) β-irresolute, (μ, λ) β-continuous, (μ, λ) R-irresolute, (μ, λ) R-pre-closed.

1. Introduction and Preliminaries

The idea of generalized topology and hereditary class was introduced and studied by Csaszar[2]. A subfamily μ of P(X) is called a generalized topology if $\phi \in \mu$ and union of elements of μ belongs to μ. The space X together with the generalized topology μ is said to be generalized topological space and denoted by (X, μ). $i_\mu(A)$ and $c_\mu(A)$ denotes the interior and closure of A in (X, μ). The elements of μ are called open and is denoted by μ-open. The complement of μ-open is μ-closed. We say a hereditary class $\mathcal{H}$ on (X, μ) is a non-empty collection of subset of X such that $A \subseteq B$, $B \in \mathcal{H}$ implies

$A \in \mathcal{H}$. With respect to the generalized topology μ and a hereditary class $\mathcal{H}$, for a subset A of X we define A_μ^* $(\mathcal{H})$ or simply $A_\mu^* = \{x \in X : M \cap A \notin \mathcal{H}$ for every $M \in \mu$ such that $x \in M\}$. The closure $c_\mu^*(A) = A \cup A_\mu^*(\mathcal{H})$. The space (X, μ) with the hereditary class $\mathcal{H}$ is called hereditary generalized topological space and denoted by $(X, \mu, \mathcal{H})$. A subset A of (X, μ) is μα-open [2] (resp. μ-semi open [2], μ-pre open [2], μβ-open[2]), if $A \subseteq i_\mu(c_\mu(i_\mu(A)))$ (resp. $A \subseteq c_\mu(i_\mu(A))$, $A \subseteq i_\mu\left(c_\mu(A)\right)$, $A \subseteq c_\mu(i_\mu(c_\mu(A)))$). We denote the family of all μ α -open sets, μ-semi open sets, μ -pre open sets and μ β-open sets by α (μ), σ(μ), π(μ) and β (μ) respectively. On generalized topology, $\mu \subseteq \alpha(\mu) \subseteq \pi(\mu) \subseteq \beta(\mu)$.

In GTS $c_{\mu_\alpha}(A)$ and $c_{\mu_\beta}(A)$ denotes α-closure of A and β-closure of A in (X, μ) respectively. A subset A of (X, μ) is said to be μ- regular open[4] if $A = i_\mu(c_\mu(A))$ and the complement is μ -regular closed. The finite union of μ -regular open sets is called μπ -open sets and its complement is μπ -closed set. A set A is said to be μg-closed [2] if, $c_\mu(A) \subseteq A$ whenever $A \subseteq U$ and U is μ -open and its complement is μ g-open.

Definition 1.1: For a subset A of hereditary generalized topological space $(X, \mu, \mathcal{H})$

i) $A_{\mu_\alpha}^*(\mathcal{H}) = \{x \in X : M \cap A \notin \mathcal{H}$ for every $M \in \alpha(\mu)$ such that $x \in M\}$ [2].

ii) $A_{\mu_\beta}^*(\mathcal{H}) = \{x \in X : M \cap A \notin \mathcal{H}$ for every $M \in \beta(\mu)$ such that $x \in M\}$ [2].

Definition 1.2: A subset A of a hereditary generalized topological space $(X, \mu, \mathcal{H})$ is said to be μ^*-closed [3] if $A_\mu^*(\mathcal{H}) \subseteq A$ and μ_β^*-closed [3] if $A_{\mu_\beta}^*(\mathcal{H}) \subseteq A$. Then $c_{\mu_\beta}^*(A) = A \cup A_{\mu_\beta}^*(\mathcal{H})$.

Definition 1.3: Let (X, μ) and (Y, λ) be generalized topologies. A function $f: (X, \mu) \to (Y, \lambda)$ is said to be

i) (μ, λ)-continuous if for every closed set V in (Y, λ) (denoted by λ-closed set), $f^{-1}(V)$ is μ-closed,

ii) (μ, λ)-open if for every μ-open set U, $f(U)$ is open in (Y, λ) (denoted by λ-open).

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2. Quasi μ-$\mathcal{H}$ -regular spaces, Almost μ-$\mathcal{H}$ -Normal spaces and Quasi ultra μ- $\mathcal{H}$-Normal space

Definition 2.1: A hereditary generalized topological space $(X, \mu, \mathcal{H})$ is said to be Quasi μ-$\mathcal{H}$-regular space if for every μπ -closed set A and a point $x \notin A$, there exist μ-open sets U and V such that $A - U \in \mathcal{H}$, $x \in V$ and $U \cap V \in \mathcal{H}$.

Definition 2.2: Let $(X, \mu, \mathcal{H})$ be a hereditary generalized topological spaces. A space $(X, \mu, \mathcal{H})$ is said to be quasi μ-$\mathcal{H}$- normal if for every pair of μ π -closed sets A and B of X, there exist μ-open sets U and V such that $A - U \in \mathcal{H}$, $B - V \in \mathcal{H}$, and $U \cap V \in \mathcal{H}$.

Theorem 2.3: Let $(X, \mu, \mathcal{H})$ be a Hereditary generalized topological space.

Then the followings are equivalent:
(a) X is a quasi μ-$\mathcal{H}$ -regular space.
(b) for each point $x \in X$ and for each μπ -open neighbourhood F of x, there exists a μ -open set V of X such that $c_\mu{}^*(V) - F \in \mathcal{H}$.
(c) For each point $x \in X$ and for each μ π -closed set A not containing, there exists a μ -open set V of X such that $c_\mu{}^*(V) \cap A \in \mathcal{H}$.

Proof:
$(a) \Rightarrow (b)$ Let F be μπ-open neighbourhood of x. Then there exist a μ π -open subset G of X such that $x \in G \subseteq F$. Since G^c is μπ -closed and $x \in G$ by hypothesis, there exist disjoint μ-open sets U and V such that $G^c - U \in \mathcal{H}$, $x \in V$ and $U \cap V \in \mathcal{H}$ and so $V - U^c \in \mathcal{H}$. Since U^c is μ-closed, $c_\mu{}^*(V) - U^c \in \mathcal{H}$ implies $U^c - G \in \mathcal{H}$. Hence $c_\mu{}^*(V) - F \in \mathcal{H}$.
$(b) \Rightarrow (a)$: Let F^c be any μπ -closed set and $x \notin F^c$. Then $x \in F$ and F is μπ-open neighbourhood of x. By hypothesis, there exist a μ-open set V of x such that $x \in V$ and $c_\mu{}^*(V) - F \in \mathcal{H}$, which implies $F^c - c_\mu{}^*(V) \in \mathcal{H}$. Then $(c_\mu{}^*(V))^c$ is μ-open set containing $F^c$and $V \cap (c_\mu{}^*(V))^c \in \mathcal{H}$. Therefore is X quasi μ-$\mathcal{H}$-regular space.
$(b) \Rightarrow (c)$: Let $x \in X$ and A be μπ-closed set such that $x \notin A$. Since A^c is μπ -open neighbourhood of x and by hypothesis, there exist a μ-open set V of X such that $c_\mu{}^*(V) - A^c \in \mathcal{H}$ and $c_\mu{}^*(V) \cap A \in \mathcal{H}$.
$(c) \Rightarrow (a)$: Let $x \in X$ and A be a μπ-closed set such that $x \notin A$. By hypothesis, there exists μ-open set U such that $c_\mu{}^*(V) \cap A \in \mathcal{H}$. Let $V = X - c_\mu{}^*(U)$. Since V is μ -open set and $U \cap V \in \mathcal{H}$, X is quasi μ-$\mathcal{H}$ -regular space.

Definition 2.4: Let (X, μ) and (Y, λ) be generalized topologies. A function $f: (X, \mu) \rightarrow (Y, \lambda)$ is said to be
i) Completely (μ, λ)-irresolute if for every π-closed set V in (Y, λ) (denoted by λπ- closed set), $f^{-1}(V)$ is π -closed in (X, μ) (denoted by μπ-closed).
ii) Completely (μ, λ)-continuous if for every λ-closed set V, $f^{-1}(V)$ is μπ-closed.
iii) Almost (μ, λ)-open if for every π -open set V in (X, μ) (denoted by μπ-open set) , $f(V)$ is π -open in (Y, λ) (denoted by λπ-open).
iv) Almost (μ, λ)-closed if for every μπ -closed set F, $f(F)$ is λπ -closed.
v) Perfectly (μ, λ)-continuous if for every open set F in (Y, λ) (denoted by λ-open set), $f^{-1}(F)$ is μ-open and μ-closed.
vi) (μ, λ)-R-irresolute if for every regular-closed set V in (Y, λ) (denoted by λ-regular-closed), $f^{-1}(V)$ is μ-regular closed.
vii) (μ, λ) β-irresolute if for every β-closed set V in (Y, λ) (denoted by λβ-closed set), $f^{-1}(V)$ is β-closed in (X, μ) (denoted by μ β-closed set).
viii) (μ, λ) β-continuous if for every λ-closed set V, $f^{-1}(V)$ is μβ-closed in (X, μ).
ix) (μ, λ)R-pre-closed if for every μ-regular-closed set U, $f(U)$ is λ-regular-closed.

Lemma 2.5 [3]: If $\mathcal{H} \neq \phi$ is a hereditary class on (X, μ) and $f: (X, \mu) \rightarrow (Y, \lambda)$ is a function, then $f(\mathcal{H}) = \{f(H): H \in \mathcal{H}\}$ is a hereditary class on (Y, λ).

Theorem 2.6: Let $(X, \mu, \mathcal{H})$ be a hereditary generalized topological space and (Y, λ) be a generalized topology. A function $f: (X, \mu) \rightarrow (Y, \lambda)$ is bijective, completely (μ, λ)-irresolute and (μ, λ)-open. If X is quasi μ-$\mathcal{H}$ -regular space, then Y is quasi λ- $f(\mathcal{H})$-regular space.

Proof: Let $y \in Y$ and A be any λπ -closed set. Since f is completely (μ, λ)-irresolute, $f^{-1}(A)$ is μπ -closed subset of X. Since f is a bijection, $f(x) = y$, then $y \neq f^{-1}(x)$ for every $x \in X$. Since $(X, \mu, \mathcal{H})$ is quasi μ-$\mathcal{H}$ -regular space, there exists μ-open sets U and V such that $x \in U$,

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$f^{-1}(A) - V \in \mathcal{H}$ and $U \cap V \in \mathcal{H}$. Since f is (μ, λ)-open, $f(U)$ and $f(V)$ are μ-open sets in Y. Also $y \in f(U)$, and $A - f(V) \in f(\mathcal{H})$ and $f(U) \cap f(V) = f(U \cap V) \in f(\mathcal{H})$. Hence by using lemma(2.5) Y is quasi λ- $f(\mathcal{H})$-regular space.

Lemma 2.7 [3]: If $\mathcal{H} \neq \phi$ is a hereditary class on (Y, λ) and f: $(X, \mu) \rightarrow (Y, \lambda)$, then $f^{-1}(\mathcal{H}) = \{f^{-1}(H): H \in \mathcal{H}\}$ is a hereditary class on (X, μ).

Theorem 2.8: Let (X, μ) be a generalized topological space and $(Y, \lambda, \mathcal{H})$ be hereditary generalized topology. A function f: $(X, \mu) \rightarrow (Y, \lambda)$ is injective, Almost (μ, λ)-closed and (μ, λ) -continuous. If Y is quasi λ-$\mathcal{H}$-regular space, then X is quasi μ -$f^{-1}(\mathcal{H})$-regular space.

Proof: Let $x \in X$ and A be any μπ -closed subset of X. Since f is Almost (μ, λ)-closed, $f(A)$ is λπ -closed subset of Y. Since $(Y, \lambda, \mathcal{H})$ is quasi λ-$\mathcal{H}$-regular space, there exists λ-open sets U and V such that $f(x) \in U$, $f(A) - V \in \mathcal{H}$ and $U \cap V \in \mathcal{H}$. Since f is (μ, λ)-continuous and injective, $f^{-1}(U)$ and $f^{-1}(V)$ are μ -open sets in X, such that $x \in f^{-1}(U)$, $A - f^{-1}(V) \in f^{-1}(\mathcal{H})$ and $f^{-1}(U) \cap f^{-1}(V) = f^{-1}(U \cap V) \in f^{-1}(\mathcal{H})$. Hence by using lemma (2.7) X is quasi μ -$f^{-1}(\mathcal{H})$-regular space.

Lemma 2.9 [3]: If $\mathcal{H} \neq \phi$ is a hereditary class on (Y, λ) and Y is a subset of X. Then $\mathcal{H}_Y = \{ Y \cap H: H \in \mathcal{H} \}$ is a hereditary class on Y.

Theorem 2.10: Let $(X, \mu, \mathcal{H})$ be a generalized topological space. If X is quasi μ -$\mathcal{H}$-regular space and $Y \subset X$ is μ π -closed set, then Y is quasi μ -$\mathcal{H}_Y$ –regular space.

Proof: Let $y \in Y$ and A be μ π -closed subset of Y and $y \notin A$. Since Y is μ π –closed set and $Y \subset X$, A is μ π -closed subset of X. Since X is quasi μ -$\mathcal{H}$-regular space, there exist μ- open sets U and V such that $A - U \in \mathcal{H}$, $x \in V$ and $U \cap V \in \mathcal{H}$. If $A - U = H \in \mathcal{H}$, then $A \subset (U \cup H)$. Since $A \subset F$, $A \subset (F \cap (U \cup H))$ and so $A \subset (F \cap U) \cup (F \cap H)$. Therefore, $A - (F \cap U) \subset (F \cap H) \in \mathcal{H}_Y$, $y \in (V \cap F)$. Hence $(F \cap U)$ and $(F \cap V)$ are μ -open sets in Y such that $A - (F \cap U) \in \mathcal{H}_Y$, $y \in V$ and $(F \cap U) \cap (F \cap V) \in \mathcal{H}_Y$. Hence Y is quasi μ -$\mathcal{H}_Y$ – regular space.

Definition 2.11: A generalized topological space (X, μ) with the hereditary class $\mathcal{H}$ is said to be Almost μ- $\mathcal{H}$- normal space if for every pair of disjoint μ-regular closed sets A and B there exist μ-open sets F and G such that $A - F \in \mathcal{H}$, $A - G \in \mathcal{H}$ and $F \cap G \in \mathcal{H}$.

Theorem 2.12: Let $(X, \mu, \mathcal{H})$ be a hereditary generalized topological space and (Y, λ) be a generalized topological space. Also let f: $(X, \mu) \rightarrow (Y, \lambda)$ is (μ, λ) R-pre-closed and (μ, λ)-continuous injective function. If Y is Almost λ- $\mathcal{H}$ -normal space, then X is Almost μ -$f^{-1}(\mathcal{H})$-normal space.

Proof: Let A and B be disjoint μ-regular-closed subsets of X. Since f is (μ, λ) R-pre-closed, $f(A)$ and $f(B)$ are disjoint λ-regular-closed. Since Y is Almost λ- $\mathcal{H}$-normal space, there exist λ-open sets U and V in Y such that $f(A) - U \in \mathcal{H}$, $f(B) - V \in \mathcal{H}$, $U \cap V \in \mathcal{H}$. Then $f^{-1}(f(A)) - f^{-1}(U) \in f^{-1}(\mathcal{H})$ and $f^{-1}(f(B)) - f^{-1}(V) \in f^{-1}(\mathcal{H})$ which implies $A - f^{-1}(U) \in f^{-1}(\mathcal{H})$, $B - f^{-1}(V) \in f^{-1}(\mathcal{H})$ and $f^{-1}(U) \cap f^{-1}(V) \in f^{-1}(\mathcal{H})$. Since f is (μ, λ)-continuous, $f^{-1}(U)$ and $f^{-1}(V)$ are μ -open subsets of X. Hence (X, μ) is Almost μ-$f^{-1}(\mathcal{H})$-normal space.

Definition 2.13: A generalized topological space (X, μ) with the hereditary class $\mathcal{H}$ is said to be quasi ultra μ - $\mathcal{H}$ normal space if for every pair of disjoint μπ-closed sets A and B there exist μ-clopen sets F and G such that $A - F \in \mathcal{H}$, $A - G \in \mathcal{H}$ and $F \cap G \in \mathcal{H}$.

Theorem 2.14: Let $(X, \mu, \mathcal{H})$ be a hereditary generalized topological space and (Y, λ) be a generalized topological space. Also let f: $(X, \mu) \rightarrow (Y, \lambda)$ is Almost (μ, λ)π -closed and perfectly (μ, λ) -continuous injective function. If Y is Quasi λ- $\mathcal{H}$ -normal space, then X is Quasi ultra μ- $f^{-1}(\mathcal{H})$- normal space.

Proof: Let F and G be disjoint μπ-closed subsets of X. Since f is Almost (μ, λ) π-closed, $f(F)$ and $f(G)$ are disjoint λπ–closed subsets of Y. Since Y is quasi λ-$\mathcal{H}$-normal space, there exist λ-open sets U and V in Y such that $f(F) - U \in \mathcal{H}$, $f(G) - V \in \mathcal{H}$ and $U \cap V \in \mathcal{H}$. Then $F - f^{-1}(U) \in f^{-1}(\mathcal{H})$, $G - f^{-1}(V) \in f^{-1}(\mathcal{H})$ and

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$f^{-1}(U) \cap f^{-1}(V) \in f^{-1}(\mathcal{H})$. Since f is perfectly (μ, λ) -continuous, $f^{-1}(U)$ and $f^{-1}(V)$ are μ-open and μ -closed subsets of $(X$, μ). Hence X is quasi ultra μ-$f^{-1}(\mathcal{H})$- normal space.

Remark 2.15: Let $(X$, μ, $\mathcal{H})$ be a hereditary generalized topological space and (Y, λ) be a generalized topological space. Also let f: $(X$, μ) → $(Y$, λ) is (μ, λ) π-pre-closed and (μ, λ)-continuous injective function. If Y is Quasi λ - $\mathcal{H}$-normal space, then X is Almost μ -$f^{-1}(\mathcal{H})$-normal space.

Proof: Let F and G be disjoint μ-regular-closed subsets of X and hence μπ-open. Since f is Almost (μ, λ)π -closed, $f(F)$ and $f(G)$ are disjoint λπ-closed. Since Y is quasi λ-$\mathcal{H}$-normal space, there exist λ-open sets U and V in Y such that $f(F) - U \in \mathcal{H}$, $f(G) - V \in \mathcal{H}$ and $U \cap V \in \mathcal{H}$. Then $F - f^{-1}(U) \in f^{-1}(\mathcal{H})$, $G - f^{-1}(V) \in f^{-1}(\mathcal{H})$ and $f^{-1}(U) \cap f^{-1}(V) \in f^{-1}(\mathcal{H})$. Since f is (μ, λ) -continuous, $f^{-1}(U)$ and $f^{-1}(V)$ are μ-open subsets of $(X$, μ). Hence X is Almost μ -$f^{-1}(\mathcal{H})$-normal space.

Theorem 2.16: Let $(X$, μ, $\mathcal{H})$ be a hereditary generalized topological space and (Y, λ) be a generalized topological space. Also let f: $(X$, μ) → $(Y$, λ) be a bijection (μ, λ) -R-irresolute and (μ, λ)-open function. If X is Almost μ- $\mathcal{H}$ -normal space, then Y is Almost λ- $f(\mathcal{H})$-normal space.
Proof: Proof is similar to the proof of (2.6) .

3. Quasi μ_β- $\mathcal{H}$ -regular spaces and Quasi μ_β- $\mathcal{H}$ -normal spaces

Definition 3.1: A hereditary generalized topological space $(X$, μ, $\mathcal{H})$ is said to be Quasi μ_β- $\mathcal{H}$-regular space if for every μ π -closed set A and $x \notin A$, there exist μ β -open sets U and V such that $A - U \in \mathcal{H}$, $x \in V$ and $U \cap V \in \mathcal{H}$.

Definition 3.2: Let $(X$, μ, $\mathcal{H})$ be a hereditary generalized topological spaces. A space $(X$, μ, $\mathcal{H})$ is said to be quasi μ_β-$\mathcal{H}$- normal if for every pair of μ π -closed sets A and B of X, there exist μ β-open sets U and V such that $A - U \in \mathcal{H}$, $B - V \in \mathcal{H}$, and $U \cap V \in \mathcal{H}$.

Theorem 3.3: Let $(X$, μ, $\mathcal{H})$ be a hereditary generalized topological spaces. Then the followings are equivalent:
(a) X is a quasi μ_β- $\mathcal{H}$ -normal space.
(b) for every μπ -closed set F and μπ -open set G containing F, there exists a μ β -open set V such that $F - U \in \mathcal{H}$ and $c_{\mu_\beta}{}^*(V) - G \in \mathcal{H}$.
c) For each pair of disjoint μπ -closed sets A and B, there exists an μ β -open set U such that $A - U \in \mathcal{H}$ and $c_{\mu_\beta}{}^*(U) \cap B \in \mathcal{H}$.

Proof:
(a) ⇒ (b) Let F be μπ -closed set and G be a μπ -open subset of X. Since $X - G$ is μπ -closed and $F \subset G$, $F \cap (X - G) = \phi$. Since X is quasi μ_β- $\mathcal{H}$-normal space, there exist disjoint μ β -open sets U and V such that $F - V \in \mathcal{H}$ and $(X - G) - V \in \mathcal{H}$. Then $c_{\mu_\beta}{}^*(V) \subset (X - U)$ and $(X - G) \cap c_{\mu_\beta}{}^*(V) \subset (X - G) \cap (X - U)$. Hence $c_{\mu_\beta}{}^*(V) - G \in \mathcal{H}$.
(b) ⇒ (c) It is obvious.
(c) ⇒ (a) Let A and B be μπ -closed sets. By (c) there exists a μ β-open U such that $A - U \in \mathcal{H}$ and $c_{\mu_\beta}{}^*(U) \cap B \in \mathcal{H}$. Let $V = X - c_{\mu_\beta}{}^*(U)$. Since V is μ β-open and $V = X - c_{\mu_\beta}{}^*(U)$, $U \cap V \in \mathcal{H}$ Hence X is quasi μ_β- $\mathcal{H}$-normal space.

Theorem 3.4: Let $(X$, μ, $\mathcal{H})$ be a hereditary generalized topological space and $(Y$, λ) be a generalized topological space . Also let f: $(X$, μ) → $(Y$, λ) is a bijective function, completely (μ, λ)- continuous and (μ, λ) β- open. If X is quasi μ_β- $\mathcal{H}$- normal space, then Y is quasi λ_β- $f(\mathcal{H})$-normal space.

Proof: Let A and B be λπ-closed subsets of Y. Since f is completely (μ, λ)-continuous, $f^{-1}(A)$ and $f^{-1}(B)$ are μπ -closed subsets of X. Since X is quasi μ_β- $\mathcal{H}$-normal space, there exist μ β-open sets U and V in X such that $f^{-1}(A) - U \in \mathcal{H}$, $f^{-1}(B) - V \in \mathcal{H}$ and $U \cap V \in \mathcal{H}$. Since f is bijective, $f(f^{-1}(A)) - f(U) \in f(\mathcal{H})$, $f(f^{-1}(B)) - f(V) \in f(\mathcal{H})$ and $f(U) \cap f(V) \in f(\mathcal{H})$ and hence $A - f(U) \in f(\mathcal{H})$, $B - f(V) \in f(\mathcal{H})$. Since f is (μ, λ) β- open, $f(U)$ and $f(V)$ are μβ -open sets in Y. Hence it follows that Y is quasi λ_β- $f(\mathcal{H})$-normal space.

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Theorem 3.5: Let $(X, \mu, \mathcal{H})$ be a hereditary generalized topological space and (Y, λ) be a generalized topological space. Also let $f: (X, \mu) \rightarrow (Y, \lambda)$ is an injective function, and Almost (μ, λ)-closed and $(\mu, \lambda)\beta$-irresolute. If Y is quasi λ_β-$\mathcal{H}$-normal space, then X is quasi μ_β-$f^{-1}(\mathcal{H})$-normal space.

Proof: Let A and B be $\mu\pi$ -closed subsets of X. Since f is Almost (μ, λ)-closed, $f(A)$ and $f(B)$ are disjoint $\lambda\pi$ -closed. Since Y is quasi λ_β-$\mathcal{H}$-normal space, there exist $\mu\ \beta$-open sets U and V in Y such that $f(A) - U \in \mathcal{H}$ and $f(B) - V \in \mathcal{H}$and $U \cap V \in \mathcal{H}$. Then $f^{-1}(f(A)) - f^{-1}(U) \in f^{-1}(\mathcal{H})$ and $f^{-1}(f(B)) - f^{-1}(V) \in f^{-1}(\mathcal{H})$ and $f^{-1}(U \cap V) = f^{-1}(U) \cap f^{-1}(V) \in f^{-1}(\mathcal{H})$ which implies $A - f^{-1}(U) \in f^{-1}(\mathcal{H})$ and $B - f^{-1}(V) \in f^{-1}(\mathcal{H})$. Since f is $(\mu, \lambda)\beta$-irresolute, $f^{-1}(U)$ and $f^{-1}(V)$ are $\mu\ \beta$-open subsets of X. Hence it follows that X is quasi μ_β-$f^{-1}(\mathcal{H})$-normal space.

Theorem 3.6: Let $(X, \mu, \mathcal{H})$ be a generalized topological space. If X is quasi μ_β- $\mathcal{H}$-normal space, and $Y \subset X$ is $\mu\pi$ -closed set, then Y is quasi μ_β- $\mathcal{H}_Y$-normal space.

Proof: Let A and B be disjoint $\mu\pi$-closed subsets of Y. Since Y is $\mu\pi$-closed set and $Y \subset X$, A and B are $\mu\pi$-closed subsets of X. Since X is quasi μ_β- $\mathcal{H}$-normal space, there exist $\mu\ \beta$-open sets U and V such that $A - U \in \mathcal{H}$, $B - V \in \mathcal{H}$and $U \cap V \in \mathcal{H}$. If $A - U = H \in \mathcal{H}$, $B - V = G \in \mathcal{H}$, then $A \subset (U \cup H)$ and $B \subset (V \cup G)$. Since $A \subset Y$, $A \subset Y \cap (U \cup H)$ and so $A \subset (Y \cap U) \cup (Y \cap H)$. Therefore $A - (Y \cap U) \subset (Y \cap H) \in \mathcal{H}_Y$. Similarly $B - (Y \cap V) \subset (Y \cap G) \in \mathcal{H}_Y$. Hence $Y \cap U$ and $Y \cap V$ are $\mu\ \beta$-open sets in Y such that $A - (Y \cap U) \in \mathcal{H}_Y$ and $B - (Y \cap V) \in \mathcal{H}_Y$. Hence Y is quasi μ_β- $\mathcal{H}_Y$-normal space.

4. Conclusion

In this paper we introduced new classes of spaces namely quasi μ- $\mathcal{H}$- regular space ,quasi μ_β- $\mathcal{H}$-regular space, quasi μ_β- $\mathcal{H}$- normal space, Almost μ-$\mathcal{H}$ -Normal spaces, Quasi ultra μ- $\mathcal{H}$-Normal space in hereditary generalized topological spaces and derived some properties by using some basic properties of (μ, λ)-continuity in generalized topological spaces.

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