Theorem 2.11
For any graph G, G=$W_{3,n}$ then$\chi_{d,2}(G) \leq n, n \geq 3$.

Proof:
Let W_n be a web graph.
Let G= $U \cup V \cup W$be the vertex set.
Let U={ u_0 , u_1,……, u_{n-1} } V={ v_0 , v_1,……, v_{n-1} } and W= { w_0 , w_1,……, w_{n-1} } respectively.
Let $E(U)$= $\{u_i u_{i+1};\ 0 \leq i \leq n-2\}$ and $E(V)$= $\{ v_j v_{j+1};\ 0 \leq j \leq n-2\}$ be the edge set of U and V
Let E(t)=$\begin{cases} u_k v_k;\ 0 \leq K \leq n-1 \\ v_k w_k\ ; 0 \leq K \leq n-1 \end{cases}$ be the edge between the vertex set UV and VW respectively. Thus for any web graph $W_{3,n} E(G)$=3n+n. Consider x=1 be the edge joining the vertex UV and VW.
Now the procedure for 2 -dominator coloring of G follows:
Let color the vertex $G \in U \cup V \cup W$ by C={0,2x+1,2(2x+1),…..,(M-1)(2x+1)} by M different colors in clockwise direction. Note M=$\lceil n/(2x+1) \rceil$,x=1. Also the remaining vertices of $G \in U \cup V \cup W$ colored by n-M different color 1,2,…., n-M in clockwise direction. To color the vertices of U, starts from the vertex u_0 using C = { 0,2x+1,2(2x+1)},….(M-1)(2x+1)} and it may or may not ends at the vertex u_{n-1} and the remaining verticies of G colored by n-M different colors. Next color the vertices of V starts from vertex v_1 using above said the M colors C={ 0,2x+1,2(2x+1)},……..(M-1)(2x+1)} and it also may or may not ends at the vertex v_{n-1} then the remaining vertices of v colored by the same said$n$-M different colors in same previous manner. Similarly to color the remaining vertices of W needs those same n-M colors using same procedure as said above. Likewise coloring there exist a cliques in G.
Hence every vertex of G dominates at least two color classes.
Thus for any W_n , $\chi_{d,2}(W_{3,n}) \leq M + n - m \leq n, n \geq 3$.

Theorem 2.12
If G=H_n ba a helm graph, then

$$\chi_{d,2}(H_n) = \begin{cases} 3, n = 6,8,10,12, \ldots \\ 4, n = 7,9,11, \ldots . \end{cases}$$

Proof:
By the above theorem $\chi_{d,2}(W_n) = \begin{cases} 3, n = 6,8,10,12, \ldots \\ 4, n = 7,9,11, \ldots . \end{cases}$ definition of helm graph, assign color c_0 to all the pendent verticesw_0 ,w_1,……, w_n in G, which was already colored to the root vertex u_0 in G. By definition of 2-dominator coloring $\chi_{d,2}(H_n) = \begin{cases} 3, n = 6,8,10,12, \ldots \\ 4, n = 7,9,11, \ldots . \end{cases}$

Cayley Graph
Let G be a group, and let S be subset of G. Then the Cayley Digraph D(G, S) on G with connection set S is defined as follows:
1. The vertices are the elements of G
2. There is an arc joining g and h if and only if h = s_g for some s∈S.
We can extend this idea to a Cayley Color graph, where S is a generating set for G, each s_i∈ S is assigned a color, and if g = s_{ih}, then the arc connecting them is colored s_i .

Theorem 2.12
Let G be a group with generating set S and let φ be a color-preserving permutation on V (D(G, S)). Then φ is a color preserving automorphism of D(G, S) if and only if φ(gh) = φ(g)h.
Suppose that φ(gh) = φ(g)h.

Proof:
To show that φ is color-preserving, we need to show that if gh−1 = s, then φ(gh−1) = s. Suppose gh−1 = s. Then φ(gh−1) =φ(g)h −1 =φ(g)g −1 s =φ(gg−1)s =s.

3. Conclusion

In this paper, we found out some result of various graphs like,Central Graph of Star Graph C [$S_{1,n}$], Star Graph: $S_{1,n}$, Barbell Graph B (K_n, K_n) , Ladder Graph, Banana Tree $B_{m,n}$, Fire cracks graph: $F_{n,k}$, n,k ≥ 2, Wind Mill Graph $D_n^{(m)}$, Gear Graph G_n, Wheel Graph, Prism Graph $Y_{2,n}$, Web Graph W_n, Helm Graph H_n, Cayley Graph using 2-dominator coloring.This will be initiative study of extension of 2-Dominator coloring.

References

[1] S. Arumugam, Jay Bagga and K. RajaChandrasekar, 2012,"On Dominator ColoringsinGraphs", Proc. Indian Acad. Sci. (Math. Sci.)Vol. 122, No. 4, November, pp. 561–571.
[2] E.Cockayne,S.HedetniemiandS. Hedetniemi,"Dominating partitions of Graphs", unpublished manuscript.
[3] R. Gera, S Horton, C. Rasmussen, "Dominator Colorings and Safe Clique Partitions", Congressus Numerantium , 2006.
[4] S. M. Hedetniemi, S. T. Hedetniemi, and A. A. McRae, Dominator Colorings of graphs, (2006). In preparation.
[5] T. W. Haynes, S. T. Hedetniemi, and P. J. Slater, "Domination in Graphs: Advanced Topics", Marcel Dekker, New York, 1998.
[6] Marcel Dekker, "Fundamentals of Domination in Graphs", New York,1998.
[7] Ralucca Michelle Gera, On Dominator Colorings in Graphs, Department Of Applied Mathematics, Naval Postgraduate School, Monterey, CA 93943, USA.
[8] T.Ramachandran, A.Naseer Ahmed, "Dominator Coloring Number of Middle Graph", IOSR Journal of Mathematics, Vol. 10, Iss. 6, Ver. IV, pp 38-40, December 2014.
[9] A.Sangeetha Devi,M.M.Shanmugapriya, 2-Dominator Coloring in Graphs,International Journal of Applied Engineering Research,Vol. 10, 43059-43062.2015,.
[10] E. Sampathkumar and L. Pushpa Latha, "Strong, weak Domination and Domination Balance in a Graph", Discrete Math. 161.235-242 (1996).

Licensed Under Creative Commons Attribution CC BY