# On Nano (1,2)\* Generalized *α* - Closed Sets in Nano Bitopological Space

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Abstract: The purpose of this paper is to define and study a new class of set called Nano  $(1,2)^*$  generalized  $\alpha$ - closed sets in Nano bitopological spaces. Some of its properties are investigated.

Keywords: Nano (1,2)\* generalized  $\alpha$  - closed sets, Nano (1,2)\*  $\alpha$ -closure, Nano (1,2)\* $\alpha$ -interior.

### **1. Introduction**

O.Njastad[9] introduced and defined an  $\alpha$  -open and closed set. H.Maki et.al.,[8] introduced generalized  $\alpha$  -closed set. The concepts of Nano topology was introduced by Lellis Thivagar[6]. K.Bhuvaneswari et.al.,[2] introduced and studied the Nano generalized  $\alpha$  -closed sets in Nano topological spaces. In 1963, J.C.Kelly[5] introduced the study of bitopological spaces. In 1990, M.Jelic[4] introduced the concepts of  $\alpha$  -open sets in bitopological spaces. Qays Hatem Imran[10] introduced generalized  $\alpha$  -generalized closed sets in bitopological spaces. In this paper a new set called Nano (1,2)\* generalized  $\alpha$  -closed set in Nano bitopological spaces introduced and studied.

### 2. Preliminaries

**Definition: 2.1 [9]** A subset A of a space  $(X, \tau)$  is called  $\alpha$  - open if  $A \subset Int(cl(Int(A)))$ .

Let X be a topological space. A subset A of a X is called  $\alpha$  - closed if.  $cl(Int(cl(A))) \subseteq A$ .

**Definition:** 2.2 [8] A subset A of  $(X, \tau)$  is called generalized  $\alpha$  - closed (briefly  $g\alpha$  -closed) if  $\alpha Cl(A) \subseteq U$ whenever  $A \subseteq U$  and U is  $\alpha$  - open set in  $(X, \tau)$ .

**Definition: 2.3 [6]** Let U be a non-empty finite set of objects called the universe and R be an equivalence relation on U named as the indiscernibility relation. Then U is divided into disjoint equivalence classes. Elements belonging to the same equivalence class are said to be indiscernible with one another. The pair (U,R) is said to be the approximation space. Let  $X \subseteq U$ .

i. The lower approximation on X with respect to R is the set of all objects which can be for certain classified as X with respect to R and is denoted by  $L_R(X)$ . That is,

 $L_{R}(X) = U\{R(X) : R(X) \subseteq X, x \in U\}, \quad \text{where}$ 

R(X) denotes the equivalence class determined by  $x \in U$ .

ii. The upper approximation of X with respect to R is the of all objects which can be possibly classified as X with

respect to R and is denoted by  $U_R(X)$ . That is,  $U_R(X) = U\{R(X) : R(X) \cap X \neq \phi, x \in U\}$ .

iii. The boundary region of X with respect to R is the set of all objects which can be classified neither as X nor as not X with respect to R and it is denoted by  $B_R(X)$ .

That is,  $B_R(X) = U_R(X) - L_R(X)$ .

**Definition:** 2.4 [6] Let U be the universe, R be an equivalence relation on U and  $\mathcal{T}_{R}(X) = \{U, \phi, \underline{L}_{R}(X), \underline{U}_{R}(X), \underline{B}_{R}(X)\}$ . Where  $X \subseteq U$ .  $\mathcal{T}_{R}(X)$  satisfies the following axioms:

i) U and  $\phi \in \mathcal{T}_{R}(X)$ .

ii) The union of the elements of any sub collection of  $\tau_{R}(X)$  is in  $\tau_{R}(X)$ .

iii) The intersection of the elements of any finite sub collection of  $\mathcal{T}_{R}(X)$  is in  $\mathcal{T}_{R}(X)$ .

Then  $\mathcal{T}_R(X)$  is a topology on U called the Nano topology on U with respect to X.  $(U, \mathcal{T}_R(X))$  is called the Nano topological space. The elements of  $\mathcal{T}_R(X)$  are called as Nano open sets and  $[\mathcal{T}_R(X)]^c$  is called as Nano closed sets. **Definition:2.5** [7] Let  $(U, \mathcal{T}_R(X))$  be a Nano topological space and  $A \subseteq U$ . Then A is said to be a Nano $\alpha$  - open if  $A \subseteq NInt$  [Ncl(NInt(A))] The compliment of a Nano $\alpha$  -open set of a space X is called Nano $\alpha$  - closed set in X.

**Definition: 2.6 [2]** Let  $(U, \mathcal{T}_R(X))$  be a Nano topological space and  $A \subseteq U$ . Then A is said to be Nano generalized  $\alpha$  - closed set if Nacl(A)  $\subseteq$  V whenever  $A \subseteq V$  and V is Nano  $\alpha$  - open in U.

**Definition:2.7** [4] A subset A of a bitopological space  $(X, \mathcal{T}_{1,2})$  is called

i) 
$$(1,2)^* \cdot \alpha \text{-open if } A \subseteq \mathcal{T}_{1,2}Int(\mathcal{T}_{1,2}cl(\mathcal{T}_{1,2}Int(A)))$$
  
ii)  $(1,2)^* \cdot \alpha \text{-closed}$  if  
 $A \subseteq \mathcal{T}_{1,2}cl(\mathcal{T}_{1,2}Int(\mathcal{T}_{1,2}cl(A)))$ 

**Definition:2.8** A subset of a bitopological space (X,  $T_{1,2}$ ) is called

i) (1,2)\* generalized  $\alpha$  - closed (briefly (1,2)\*g $\alpha$ -closed) if  $\tau_{1,2}\alpha Cl(A) \subseteq U$  whenever  $A \subseteq U$  and U is

 $(1,2)^*\alpha$  -open in X.

ii)(1,2)\* generalized  $\alpha$  - open (briefly (1,2)\* g $\alpha$ - open) if X-A is (1,2)\* generalized  $\alpha$  -closed.

# **Definition: 2.9 [3]** Let U be the universe, R be an equivalence relation on U and

 $\tau_{R_{1,2}}(X) = \bigcup \{ \tau_{R_1}(X), \tau_{R_2}(X) \} \text{ where } X \subseteq U.$ 

Then it is satisfies the following axioms:

i) U and  $\Phi \in \mathcal{T}_{R_{1,2}}(X)$ .

ii) The union of the elements of any sub-collection of

 $\tau_{R_{1,2}}(X)$  is in  $\tau_{R_{1,2}}(X)$ .

iii) The intersection of the elements of any finite sub collection of  $\mathcal{T}_{R_{12}}(X)$  is in  $\mathcal{T}_{R_{12}}(X)$ .

Then  $\mathcal{T}_{R_{1,2}}(X)$  is a topology on U called the Nano

bitopology on U with respect to X.  $(U, \mathcal{T}_{R_{1,2}}(X))$  is

called the Nano bit opological space. Elements of the Nano bit opology are known as Nano  $(1,2)^*$  open sets in U.

Elements of  $[\tau_{R_{1,2}}(X)]^{c}$  are called Nano (1,2)\* closed sets in  $\tau_{R_{1,2}}(X)$ .

**Definition:2.10 [3]** If  $(U, \mathcal{T}_{R_{12}}(X))$  is a Nano

bitopological space with respect to X where  $X \subseteq U$  and if  $A \subseteq U$ , then

(i) The Nano  $(1,2)^*$  closure of A is defined as the intersection of all Nano  $(1,2)^*$  closed sets containing A and initial and  $M_{1,2} = M_{1,2} = M_{1,2}$ 

it is denoted by  $N_{\mathcal{T}_{1,2}} cl(A)$ .  $N_{\mathcal{T}_{1,2}} cl(A)$  is the

smallest Nano (1,2)\* closed set containing A.
(ii) The Nano (1,2)\* interior of A is defined as the union of all Nano (1,2)\* open subsets of A contained in A and it is

denoted by  $N_{\mathcal{T}_{1,2}}Int(A)$ .  $N_{\mathcal{T}_{1,2}}Int(A)$  is the largest Nano (1,2)\* open subset of A.

# 3. Nano (1,2)\* Generalised $\alpha$ - Closed Sets

**Definition:3.1** Let  $(U, \mathcal{T}_{R_{1,2}}(X))$  be a Nano bitopological space and  $A \subseteq U$ . Then A is said to be a Nano  $(1,2)^* \alpha$  -

open if

$$A \subseteq N \mathcal{T}_{1,2} Int[N \mathcal{T}_{1,2} cl(N \mathcal{T}_{1,2} Int(A))].$$

The compliment of a Nano  $(1,2)^* \alpha$  - open set of a space X is called Nano  $(1,2)^* \alpha$  - closed set in X.

**Definition:3.2** If  $(U, \mathcal{T}_{R_{12}}(X))$  is a Nano bitopological

space with respect X where  $X \subseteq U$  and if  $A \subseteq U$ , then The Nano  $(1,2)^* \alpha$  - closure of a set A is defined as the intersection of all Nano  $(1,2)^* \alpha$  - closed sets containing A and it is denoted by  $N\tau_{1,2}\alpha cl(A) \cdot N\tau_{1,2}\alpha cl(A)$  is the

smallest Nano  $(1,2)^* \alpha$  -closed set containing A. The Nano  $(1,2)^* \alpha$  -interior of a set A is defined as the union of all Nano  $(1,2)^* \alpha$  -open subsets contained in A and it is denoted by  $N\tau_{1,2}\alpha Int(A) \cdot N\tau_{1,2}\alpha Int(A)$  is the

largest Nano (1,2)\*  $\alpha$  -open subset of A.

**Definition:3.3** A subset A of  $(U, \mathcal{T}_{R_{12}}(X))$  is a called a

Nano (1,2)\* generalized  $\alpha$  -closed set

if  $N_{\mathcal{T}_{1,2}} \alpha cl(A) \subseteq V$  whenever  $A \subseteq V$  and V is Nano

 $(1,2)^* \alpha$  - open in V.

**Theorem:3.4** Let  $(U, \mathcal{T}_{R_{1,2}}(X))$  be a Nano bitopological

space. If a subset of a Nano bitopological space

 $(U, \mathcal{T}_{R_{1,2}}(X))$  is Nano (1,2)\* closed set, then A is a Nano (1,2)\* $\alpha$  -closed set.

**Proof:** Let A is a Nano (1,2)\* closed set. Then

 $N_{\mathcal{T}_1}$ , cl(A) = A. To prove that

 $N \mathcal{T}_{1,2} cl[N \mathcal{T}_{1,2} Int(N \mathcal{T}_{1,2} cl(A))] \subseteq A$  which implies that A is a Nano (1,2)\*  $\alpha$  - closed

set.

$$N_{\mathcal{T}_{1,2}}cl[N_{\mathcal{T}_{1,2}}Int(N_{\mathcal{T}_{1,2}}cl(A))] = N_{\mathcal{T}_{1,2}}Int(A) \subseteq A$$

Hence A is a Nano  $(1,2)^*\alpha$  -closed set. **Remark: 3.5** The converse of the above theorem [3.4] is not true which has been seen from the following example.

### Example:

Let U={a,b,c,d}  $U / R_1 = \{\{a\}, \{d\}, \{b,c\}\} \text{ and } X_1 = \{b,c\} \text{ Then}$   $\mathcal{T}_{R_1}(X) = \{U, \phi, \{b,c\}\}.$   $U / R_2 = \{\{a\}, \{c\}, \{b,d\}\} \text{ and } X_2 = \{b,d\} \text{ Then}$   $\mathcal{T}_{R_2}(X) = \{U, \phi, \{a\}, \{c\}, \{b,d\}\}$  $\mathcal{T}_{R_{1,2}}(X) = \{U, \phi, \{b,c\}, \{b,d\}\} \text{ are Nano } (1,2)^* \text{ open}$ 

 $[\tau_{R_{1,2}}(X)]^c = \{U, \phi, \{a,d\}, \{a,c\}\}$  are Nano (1,2)\* closed sets.

Nano 
$$(1,2)^*$$
  $\alpha$  -open sets  
= { $U, \phi, \{b,c\}, \{b,d\}, \{a,b,c\}, \{b,c,d\}, \{a,b,d\}$ }

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Nano  $(1,2)^*$   $\alpha$  -closed sets = { $U, \phi, \{a, d\}, \{a, c\}, \{d\}, \{a\}, \{c\}$ }. Then { $U, \phi, \{a, d\}, \{a, c\}, \{d\}, \{a\}, \{c\}$ } are Nano  $(1,2)^* \alpha$  closed sets but are not Nano  $(1,2)^*$  closed sets.

**Theorem:3.6** Let  $(U, \mathcal{T}_{R_{1,2}}(X))$  be a Nano bitopological space. If a subset A of a Nano bitopological space  $(U, \mathcal{T}_{R_{1,2}}(X))$  is Nano  $(1,2)^*$  closed set, then A is a Nano  $(1,2)^*$  generalized  $\alpha$  closed set.

**Proof:** Let A be a Nano  $(1,2)^*$  closed set of U and  $A \subseteq V$ , V is Nano  $(1,2)^* \alpha$  open in U. Since A is a Nano  $(1,2)^*$  closed,  $N_{\mathcal{T}_{1,2}} cl(A) = A$ ,  $A \subseteq V$ , this implies

$$N_{\mathcal{T}_{1,2}}cl(A) \subseteq V.$$
 Also  $N_{\mathcal{T}_{1,2}}\alpha cl(A) \subseteq N_{\mathcal{T}_{1,2}}cl(A)$ 

which implies  $N_{\tau_{1,2}}\alpha cl(A) \subseteq V$  whenever  $A \subseteq V$  and V is Nano (1,2)\*  $\alpha$  - open in U.

Therefore A is Nano (1,2)\* generalized  $\alpha$  - closed set.

**Remark:3.7** The converse of the above theorem [3.6] is not true which has been seen from the following example. Let  $U = \{a,b,c,d\}$ 

 $U/R_1 = \{\{a\}, \{d\}, \{b,c\}\}$  and  $X_1 =$ 

 $\mathcal{T}_{R_1}(X) = \{\mathbf{U}, \phi, \{\mathbf{b}, \mathbf{c}\}\}.$ 

 $U/R_2 = \{\{a\}, \{c\}, \{b,d\}\} \text{ and } X_2 = \{b,d\} \text{ Then } \mathcal{T}_{R_2}(X) = \{U, \phi, \{a\}, \{c\}, \{b,d\}\}$ 

 $\mathcal{T}_{R_{1,2}}(X) = \{U, \phi, \{b,c\}, \{b,d\}\}$  are Nano  $(1,2)^*$  open sets

 $[\tau_{R_{1,2}}(X)]^c = \{U, \phi, \{a,d\}, \{a,c\}\}$  are Nano (1,2)\* closed sets.

Nano  $(1,2)^*$  generalized  $\alpha$  -closed sets = { $U, \phi, \{a\}, \{a, c\}, \{a, d\}$ }.

Then  $\{U, \phi, \{a\}, \{a,c\}, \{a,d\}\}$  are Nano  $(1,2)^*$  generalized  $\alpha$  closed sets but are not Nano  $(1,2)^*$  closed sets.

**Theorem:3.8** Let A is  $(U, \mathcal{T}_{R_{1,2}}(X))$  be a Nano bitopological space. If a subset A of a Nano bitopological space  $(U, \mathcal{T}_{R_{1,2}}(X))$  is Nano  $(1,2)^*$  generalized  $\alpha$  closed set, then it is Nano  $(1,2)^*$  generalized closed set.

**Proof:** Let A be a Nano  $(1,2)^*$  generalized  $\alpha$  - closed set. Then  $N_{\mathcal{T}_{1,2}}\alpha cl(A) \subseteq V$  whenever  $A \subseteq V$ , V is Nano  $(1,2)^* \alpha$  open in U. But  $N_{\mathcal{T}_{1,2}}\alpha cl(A) \subseteq N_{\mathcal{T}_{1,2}}cl(A)$ , this implies  $N_{\mathcal{T}_{1,2}}cl(A) \subseteq V$  whenever  $A \subseteq V$  and V is Nano  $(1,2)^*$  open in U. Therefore A is a Nano  $(1,2)^*$  generalized closed set.

**Remark: 3.9** The converse of the above theorem [3.8] is not true which has been seen from the following example.

Let  $U = \{a, b, c, d\}$   $U / R_1 = \{\{a\}, \{d\}, \{b, c\}\}$  and  $X_1 = \{b, c\}$  Then  $\mathcal{T}_{R_1}(X) = \{U, \phi, \{b, c\}\}$   $U / R_2 = \{\{a\}, \{c\}, \{b, d\}\}$  and  $X_2 = \{b, d\}$  Then  $\mathcal{T}_{R_2}(X) = \{U, \phi, \{b, d\}\}$   $\mathcal{T}_{R_{1,2}}(X) = \{U, \phi, \{b, c\}, \{b, d\}\}$  are Nano (1,2)\* open sets.  $[\mathcal{T}_{R_{1,2}}(X)]^c = \{U, \phi, \{a, d\}, \{a, c\}\}$  are Nano (1,2)\*

closed sets.

Nano (1,2)\* generalized  $\alpha$  - closed sets = { $U, \phi, \{a\}, \{a, c\}, \{a, d\}$ }

Then

Then

{b,c}

 $\{\{a\}, \{a,b\}, \{a,c\}, \{a,d\}, \{c,d\}, \{a,b,c\}, \{a,c,d\}, \{a,b,d\}, \{b,are Nano (1,2)^* generalized closed sets but are not Nano (1,2)^* generalized <math>\alpha$  - closed sets.

**Theorem:3.10** The intersection of two Nano (1,2)\* generalized  $\alpha$  - closed set in  $(U, \mathcal{T}_{R_{1,2}}(X))$  are also Nano

(1,2)\* generalized  $\alpha$  - closed set in  $(U, \mathcal{T}_{R_{1,2}}(X))$ .

**Proof:** Let A and B be two Nano  $(1,2)^*$  generalized  $\alpha$ closed set in  $((U, \mathcal{T}_{R_{1,2}}(X)))$ . Let V be a Nano  $(1,2)^*$ open set in U. such that  $A \subseteq V$  and  $B \subseteq V$ . Then we have  $A \cap B \subseteq V$ , Since A and B are Nano  $(1,2)^*$ generalized  $\alpha$ - closed set in  $(U, \mathcal{T}_{R_{1,2}}(X))$ .  $N_{\mathcal{T}_{1,2}}\alpha cl(A) \subseteq V$  and  $N_{\mathcal{T}_{1,2}}\alpha cl(B) \subseteq V$ . Now,  $N_{\mathcal{T}_{1,2}}\alpha cl(A \cap B) \subset N_{\mathcal{T}_{1,2}}\alpha cl(A) \cap N_{\mathcal{T}_{1,2}}\alpha cl(B)$ , this implies  $N_{\mathcal{T}_{1,2}}\alpha cl(A \cap B) \subseteq V$  whenever  $A \cap B \subseteq V$  and V is Nano  $(1,2)^*$  open in  $(U, \mathcal{T}_{R_{1,2}}(X))$ . Thus  $A \cap B$  is a Nano  $(1,2)^*$  generalized  $\alpha$  - closed set in  $(U, \mathcal{T}_{R_{1,2}}(X))$ .

**Remark:3.11** The union of two Nano  $(1,2)^*$  generalized  $\alpha$  - closed set in  $(U, \mathcal{T}_{R_{1,2}}(X))$  are need not be Nano  $(1,2)^*$  generalized  $\alpha$  - closed set in  $(U, \mathcal{T}_{R_{1,2}}(X))$ . Let  $U = \{a, b, c, d\}$   $U / R_1 = \{\{a\}, \{d\}, \{b, c\}\}$  and  $X_1 = \{b, c\}$  Then  $\mathcal{T}_{R_1}(X) = \{U, \phi, \{b, c\}\}$   $U / R_2 = \{\{a\}, \{c\}, \{b, d\}\}$  and  $X_2 = \{b, d\}$  Then  $\mathcal{T}_{R_2}(X) = \{U, \phi, \{b, d\}\}$   $\tau_{R_{1,2}}(X) = \{U, \phi, \{b, c\}, \{b, d\}\} \text{ are Nano } (1,2)^* \text{ open sets.}$   $[\tau_{R_{1,2}}(X)]^c = \{U, \phi, \{a, d\}, \{a, c\}\} \text{ are Nano } (1,2)^*$ 

closed sets.

Nano (1,2)\*  $\alpha$  - closed sets

 $= \{U, \phi, \{a, d\}, \{a, c\}, \{d\}, \{a\}, \{c\}\}.$ 

Nano (1,2)\* generalized  $\alpha$  - closed sets

 $= \{U, \phi, \{a\}, \{a, c\}, \{a, d\}\}.$ 

Hence the union of two Nano (1,2)\* generalized  $\alpha$  - closed set in  $(U, \mathcal{T}_{R_{12}}(X))$  are need not be Nano (1,2)\*

generalized  $\alpha$  - closed set in  $(U, \mathcal{T}_{R_{1,2}}(X))$ .

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